Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions\1.1 Binomial products\1.1.3 General"

Test results for the 3078 problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

- **Problem 240: Result more than twice size of optimal antiderivative.**

$$\int x^2 \left(a+b x^3\right)^3 \, dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{\left(a+b x^3\right)^4}{12 b}$$

Result (type 1, 43 leaves):

$$\frac{a^3 x^3}{3}+\frac{1}{2} a^2 b x^6+\frac{1}{3} a b^2 x^9+\frac{b^3 x^{12}}{12}$$

- **Problem 245: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^3\right)^3}{x^{13}} \, dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{\left(a+b x^3\right)^4}{12 a x^{12}}$$

Result (type 1, 43 leaves):

$$-\frac{a^3}{12 x^{12}}-\frac{a^2 b}{3 x^9}-\frac{a b^2}{2 x^6}-\frac{b^3}{3 x^3}$$

- **Problem 262: Result more than twice size of optimal antiderivative.**

$$\int x^5 \left(a+b x^3\right)^5 \, dx$$

Optimal (type 1, 34 leaves, 3 steps):

$$-\frac{a\left(a+b x^3\right)^6}{18 b^2}+\frac{\left(a+b x^3\right)^7}{21 b^2}$$

Result (type 1, 69 leaves):

$$\frac{a^5 x^6}{6}+\frac{5}{9} a^4 b x^9+\frac{5}{6} a^3 b^2 x^{12}+\frac{2}{3} a^2 b^3 x^{15}+\frac{5}{18} a b^4 x^{18}+\frac{b^5 x^{21}}{21}$$

- **Problem 263: Result more than twice size of optimal antiderivative.**

$$\int x^2 \left(a+b x^3\right)^5 \, dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{\left(a+b x^3\right)^6}{18 b}$$

Result (type 1, 69 leaves):

$$\frac{a^5 x^3}{3}+\frac{5}{6} a^4 b x^6+\frac{10}{9} a^3 b^2 x^9+\frac{5}{6} a^2 b^3 x^{12}+\frac{1}{3} a b^4 x^{15}+\frac{b^5 x^{18}}{18}$$

- **Problem 270: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^3\right)^5}{x^{19}} \, dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{\left(a+b x^3\right)^6}{18 a x^{18}}$$

Result (type 1, 69 leaves):

$$-\frac{a^5}{18 x^{18}}-\frac{a^4 b}{3 x^{15}}-\frac{5 a^3 b^2}{6 x^{12}}-\frac{10 a^2 b^3}{9 x^9}-\frac{5 a b^4}{6 x^6}-\frac{b^5}{3 x^3}$$

- **Problem 289: Result more than twice size of optimal antiderivative.**

$$\int x^8 \left(a+b x^3\right)^8 \, dx$$

Optimal (type 1, 53 leaves, 3 steps):

$$\frac{a^2\left(a+b x^3\right)^9}{27 b^3}-\frac{a\left(a+b x^3\right)^{10}}{15 b^3}+\frac{\left(a+b x^3\right)^{11}}{33 b^3}$$

Result (type 1, 108 leaves):

$$\frac{a^8 x^9}{9}+\frac{2}{3} a^7 b x^{12}+\frac{28}{15} a^6 b^2 x^{15}+\frac{28}{9} a^5 b^3 x^{18}+\frac{10}{3} a^4 b^4 x^{21}+\frac{7}{3} a^3 b^5 x^{24}+\frac{28}{27} a^2 b^6 x^{27}+\frac{4}{15} a b^7 x^{30}+\frac{b^8 x^{33}}{33}$$

- **Problem 290: Result more than twice size of optimal antiderivative.**

$$\int x^5 \left(a+b x^3\right)^8 \, dx$$

Optimal (type 1, 34 leaves, 3 steps):

$$-\frac{a \left(a+b x^3\right)^9}{27 b^2}+\frac{\left(a+b x^3\right)^{10}}{30 b^2}$$

Result (type 1, 108 leaves):

$$\frac{a^8 x^6}{6}+\frac{8}{9} a^7 b x^9+\frac{7}{3} a^6 b^2 x^{12}+\frac{56}{15} a^5 b^3 x^{15}+\frac{35}{9} a^4 b^4 x^{18}+\frac{8}{3} a^3 b^5 x^{21}+\frac{7}{6} a^2 b^6 x^{24}+\frac{8}{27} a b^7 x^{27}+\frac{b^8 x^{30}}{30}$$

- **Problem 291: Result more than twice size of optimal antiderivative.**

$$\int x^2 \left(a+b x^3\right)^8 \, dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{\left(a+b x^3\right)^9}{27 b}$$

Result (type 1, 108 leaves):

$$\frac{a^8 x^3}{3}+\frac{4}{3} a^7 b x^6+\frac{28}{9} a^6 b^2 x^9+\frac{14}{3} a^5 b^3 x^{12}+\frac{14}{3} a^4 b^4 x^{15}+\frac{28}{9} a^3 b^5 x^{18}+\frac{4}{3} a^2 b^6 x^{21}+\frac{1}{3} a b^7 x^{24}+\frac{b^8 x^{27}}{27}$$

- **Problem 301: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^3\right)^8}{x^{28}} \, dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{\left(a+b x^3\right)^9}{27 a x^{27}}$$

Result (type 1, 108 leaves):

$$-\frac{a^8}{27 x^{27}}-\frac{a^7 b}{3 x^{24}}-\frac{4 a^6 b^2}{3 x^{21}}-\frac{28 a^5 b^3}{9 x^{18}}-\frac{14 a^4 b^4}{3 x^{15}}-\frac{14 a^3 b^5}{3 x^{12}}-\frac{28 a^2 b^6}{9 x^9}-\frac{4 a b^7}{3 x^6}-\frac{b^8}{3 x^3}$$

- **Problem 302: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^3\right)^8}{x^{31}} \, dx$$

Optimal (type 1, 40 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^9}{30 a x^{30}}+\frac{b \left(a+b x^3\right)^9}{270 a^2 x^{27}}$$

Result (type 1, 108 leaves):

$$-\frac{a^8}{30 x^{30}}-\frac{8 a^7 b}{27 x^{27}}-\frac{7 a^6 b^2}{6 x^{24}}-\frac{8 a^5 b^3}{3 x^{21}}-\frac{35 a^4 b^4}{9 x^{18}}-\frac{56 a^3 b^5}{15 x^{15}}-\frac{7 a^2 b^6}{3 x^{12}}-\frac{8 a b^7}{9 x^9}-\frac{b^8}{6 x^6}$$

- **Problem 364: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{1+a-b x^3} \, dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(1+a)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (1+a)^{2/3} b^{1/3}}-\frac{\text{Log}\left[(1+a)^{1/3}-b^{1/3} x\right]}{3 (1+a)^{2/3} b^{1/3}}+\frac{\text{Log}\left[(1+a)^{2/3}+(1+a)^{1/3} b^{1/3} x+b^{2/3} x^2\right]}{6 (1+a)^{2/3} b^{1/3}}$$

Result (type 3, 124 leaves):

$$\frac{1}{6 (1+a)^{2/3} b^{1/3}}(-1)^{2/3} \left(-2 \sqrt{3} \text{ArcTan}\left[\frac{-1+\frac{2 (-1)^{1/3} b^{1/3} x}{(1+a)^{1/3}}}{\sqrt{3}}\right]-2 \text{Log}\left[(1+a)^{1/3}+(-1)^{1/3} b^{1/3} x\right]+\text{Log}\left[(1+a)^{2/3}-(-1)^{1/3} (1+a)^{1/3} b^{1/3} x+(-1)^{2/3} b^{2/3} x^2\right]\right)$$

- **Problem 366: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{-1+a-b x^3} \, dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2 b^{1/3} x}{(1-a)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (1-a)^{2/3} b^{1/3}}-\frac{\text{Log}\left[(1-a)^{1/3}+b^{1/3} x\right]}{3 (1-a)^{2/3} b^{1/3}}+\frac{\text{Log}\left[(1-a)^{2/3}-(1-a)^{1/3} b^{1/3} x+b^{2/3} x^2\right]}{6 (1-a)^{2/3} b^{1/3}}$$

Result (type 3, 124 leaves):

$$\frac{1}{6 (-1+a)^{2/3} b^{1/3}}(-1)^{2/3} \left(-2 \sqrt{3} \text{ArcTan}\left[\frac{-1+\frac{2 (-1)^{1/3} b^{1/3} x}{(-1+a)^{1/3}}}{\sqrt{3}}\right]-2 \text{Log}\left[(-1+a)^{1/3}+(-1)^{1/3} b^{1/3} x\right]+\text{Log}\left[(-1+a)^{2/3}-(-1)^{1/3} (-1+a)^{1/3} b^{1/3} x+(-1)^{2/3} b^{2/3} x^2\right]\right)$$

Problem 376: Result unnecessarily involves imaginary or complex numbers.

$$\int x^6 \sqrt{a+b x^3}\, dx$$

Optimal (type 4, 275 leaves, 4 steps):

$$-\frac{48 a^2 x \sqrt{a+b x^3}}{935 b^2}+\frac{6 a x^4 \sqrt{a+b x^3}}{187 b}+\frac{2}{17} x^7 \sqrt{a+b x^3}+$$
$$\left(32\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^3 \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(935 b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 184 leaves):

$$\sqrt{a+b x^3} \left(-\frac{48 a^2 x}{935 b^2}+\frac{6 a x^4}{187 b}+\frac{2 x^7}{17}\right)+\frac{1}{935 (-b)^{1/3} b^2 \sqrt{a+b x^3}}$$
$$32\, i\, 3^{3/4} a^{10/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 377: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \sqrt{a+b x^3}\, dx$$

Optimal (type 4, 251 leaves, 3 steps):

$$\frac{6 a x \sqrt{a+b x^3}}{55 b}+\frac{2}{11} x^4 \sqrt{a+b x^3}-$$
$$\left(4\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^2 \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(55 b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 168 leaves):

$$\frac{2 x \sqrt{a+b x^3}\left(3 a+5 b x^3\right)}{55 b}+\frac{1}{55(-b)^{4/3} \sqrt{a+b x^3}}$$

$$4 i 3^{3/4} a^{7/3} \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+b x^3} \, dx$$

Optimal (type 4, 227 leaves, 2 steps):

$$\frac{2}{5} x \sqrt{a+b x^3}+\left(2 \times 3^{3/4} \sqrt{2+\sqrt{3}} \; a\left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(5 b^{1/3} \sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 155 leaves):

$$\frac{2}{5} x \sqrt{a+b x^3}+\frac{1}{5(-b)^{1/3} \sqrt{a+b x^3}}$$

$$2 i 3^{3/4} a^{4/3} \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 379: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}}{x^3} \, dx$$

Optimal (type 4, 228 leaves, 2 steps):

$$-\frac{\sqrt{a+b x^3}}{2 x^2}+\left(3^{3/4} \sqrt{2+\sqrt{3}}\, b^{2/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(2 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 158 leaves) :

$$-\frac{\sqrt{a+b x^3}}{2 x^2}+\frac{1}{2 (-b)^{1/3} \sqrt{a+b x^3}}$$

$$i\, 3^{3/4} a^{1/3} b \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}}{x^6}\, dx$$

Optimal (type 4, 253 leaves, 3 steps) :

$$-\frac{\sqrt{a+b x^3}}{5 x^5}-\frac{3 b \sqrt{a+b x^3}}{20 a x^2}-$$

$$\left(3^{3/4} \sqrt{2+\sqrt{3}}\, b^{5/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(20 a \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 173 leaves) :

$$\left(-\frac{1}{5\,x^5}-\frac{3\,b}{20\,a\,x^2}\right)\sqrt{a+b\,x^3}-\frac{1}{20\,a^{2/3}\,(-b)^{1/3}\,\sqrt{a+b\,x^3}}$$

$$i\,3^{3/4}\,b^2\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]$$

- **Problem 381: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x^3}}{x^9}\,dx$$

Optimal (type 4, 277 leaves, 4 steps):

$$-\frac{\sqrt{a+b\,x^3}}{8\,x^8}-\frac{3\,b\,\sqrt{a+b\,x^3}}{80\,a\,x^5}+\frac{21\,b^2\,\sqrt{a+b\,x^3}}{320\,a^2\,x^2}+$$

$$\left(7\times3^{3/4}\,\sqrt{2+\sqrt{3}}\;b^{8/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(320\,a^2\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 181 leaves):

$$\frac{1}{320\,a^2\,x^8\,\sqrt{a+b\,x^3}}\left(-40\,a^3-52\,a^2\,b\,x^3+9\,a\,b^2\,x^6+21\,b^3\,x^9-\right.$$

$$\left.7\,i\,3^{3/4}\,a^{1/3}\,(-b)^{8/3}\,x^8\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 382: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^7\,\sqrt{a+b\,x^3}\,dx$$

Optimal (type 4, 535 leaves, 6 steps):

$$-\frac{60\,a^2\,x^2\,\sqrt{a+b\,x^3}}{1729\,b^2}+\frac{6\,a\,x^5\,\sqrt{a+b\,x^3}}{247\,b}+\frac{2}{19}\,x^8\,\sqrt{a+b\,x^3}+\frac{240\,a^3\,\sqrt{a+b\,x^3}}{1729\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(120\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{10/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\bigg/$$

$$\left(1729\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\frac{80\,\sqrt{2}\,\,3^{3/4}\,a^{10/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{1729\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}$$

Result (type 4, 238 leaves) :

$$-\frac{1}{1729\,(-b)^{8/3}\,\sqrt{a+b\,x^3}}$$

$$2\left((-b)^{2/3}\,\left(a+b\,x^3\right)\,\left(30\,a^2\,x^2-21\,a\,b\,x^5-91\,b^2\,x^8\right)+40\,(-1)^{2/3}\,3^{3/4}\,a^{11/3}\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 383: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4\,\sqrt{a+b\,x^3}\,dx$$

Optimal (type 4, 511 leaves, 5 steps) :

$$\frac{6\,a\,x^2\sqrt{a+b\,x^3}}{91\,b}+\frac{2}{13}x^5\sqrt{a+b\,x^3}-\frac{24\,a^2\sqrt{a+b\,x^3}}{91\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(12\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{7/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)-$$

$$\frac{8\sqrt{2}\;3^{3/4}\,a^{7/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]}{91\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}}$$

Result (type 4, 231 leaves) :

$$\frac{2\sqrt{a+b\,x^3}\left(3\,a\,x^2+7\,b\,x^5\right)}{91\,b}+\frac{1}{91\,(-b)^{5/3}\sqrt{a+b\,x^3}}8\,(-1)^{1/6}\,3^{3/4}\,a^{8/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(-\mathrm{i}\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 384: Result unnecessarily involves imaginary or complex numbers.**

$$\int x\sqrt{a+b\,x^3}\,dx$$

Optimal (type 4, 487 leaves, 4 steps) :

$$\frac{2}{7} x^2 \sqrt{a+b x^3}+\frac{6 a \sqrt{a+b x^3}}{7 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\ a^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],\ -7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(7 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)+$$

$$\frac{2 \sqrt{2}\ 3^{3/4} a^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],\ -7-4 \sqrt{3}\right]}{7 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}}$$

Result (type 4, 218 leaves) :

$$\frac{2}{7} x^2 \sqrt{a+b x^3}+\frac{1}{7 (-b)^{2/3} \sqrt{a+b x^3}} 2 (-1)^{1/6} 3^{3/4} a^{5/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ (-1)^{1/3}\right]+(-1)^{1/3}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ (-1)^{1/3}\right]\right)$$

- **Problem 385: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+b x^3}}{x^2}\, dx$$

Optimal (type 4, 479 leaves, 4 steps) :

$$-\frac{\sqrt{a+b x^3}}{x}+\frac{3 b^{1/3}\sqrt{a+b x^3}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\; a^{1/3} b^{1/3}\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)+$$

$$\left(\sqrt{2}\; 3^{3/4} a^{1/3} b^{1/3}\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 214 leaves) :

$$-\frac{\sqrt{a+b x^3}}{x}+\frac{1}{(-b)^{2/3}\sqrt{a+b x^3}}(-1)^{1/6}\, 3^{3/4} a^{2/3} b\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-\mathrm{i}\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 386: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b x^3}}{x^5}\,dx$$

Optimal (type 4, 511 leaves, 5 steps) :

$$-\frac{\sqrt{a+b x^3}}{4 x^4}-\frac{3 b \sqrt{a+b x^3}}{8 a x}+\frac{3 b^{4/3} \sqrt{a+b x^3}}{8 a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\ b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\bigg/$$

$$\left(16 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\frac{3^{3/4} b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{4 \sqrt{2}\ a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 231 leaves) :

$$-\frac{\sqrt{a+b x^3} \left(2 a+3 b x^3\right)}{8 a x^4}+\frac{1}{8 a^{1/3} \sqrt{a+b x^3}}(-1)^{1/6}\, 3^{3/4} (-b)^{4/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 394: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^6 \left(a+b x^3\right)^{3/2} dx$$

Optimal (type 4, 296 leaves, 5 steps) :

$$-\frac{432 a^3 x \sqrt{a+b x^3}}{21505 b^2}+\frac{54 a^2 x^4 \sqrt{a+b x^3}}{4301 b}+\frac{18}{391} a x^7 \sqrt{a+b x^3}+\frac{2}{23} x^7 \left(a+b x^3\right)^{3/2}+$$

$$\left(288\times 3^{3/4} \sqrt{2+\sqrt{3}}\ a^4 \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\bigg/$$

$$\left(21505 b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 195 leaves) :

$$\sqrt{a+b x^3}\left(-\frac{432 a^3 x}{21505 b^2}+\frac{54 a^2 x^4}{4301 b}+\frac{52 a x^7}{391}+\frac{2 b x^{10}}{23}\right)+$$
$$\left(288 \mathrm{i}\, 3^{3/4} a^{13/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/$$
$$\left(21505 (-b)^{1/3} b^2\sqrt{a+b x^3}\right)$$

- **Problem 395: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3\left(a+b x^3\right)^{3/2} dx$$

Optimal (type 4, 272 leaves, 4 steps) :

$$\frac{54 a^2 x\sqrt{a+b x^3}}{935 b}+\frac{18}{187} a x^4\sqrt{a+b x^3}+\frac{2}{17} x^4\left(a+b x^3\right)^{3/2}-$$
$$\left(36\times 3^{3/4}\sqrt{2+\sqrt{3}}\, a^3\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(935 b^{4/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 178 leaves) :

$$-\frac{1}{935 (-b)^{4/3}\sqrt{a+b x^3}} 2\left((-b)^{1/3}\left(a+b x^3\right)\left(27 a^2 x+100 a b x^4+55 b^2 x^7\right)-\right.$$
$$\left.18 \mathrm{i}\, 3^{3/4} a^{10/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 396: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^3\right)^{3/2} \, dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$\frac{18}{55} a x \sqrt{a+b x^3}+\frac{2}{11} x \left(a+b x^3\right)^{3/2}+$$
$$\left(18\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^2 \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$
$$\left(55 b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 166 leaves):

$$\sqrt{a+b x^3} \left(\frac{28 a x}{55}+\frac{2 b x^4}{11}\right)+\frac{1}{55 (-b)^{1/3} \sqrt{a+b x^3}}$$
$$18\, i\, 3^{3/4} a^{7/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2}}{x^3} \, dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$\frac{9}{10} b x \sqrt{a+b x^3}-\frac{\left(a+b x^3\right)^{3/2}}{2 x^2}+$$
$$\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a b^{2/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$
$$\left(10 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 167 leaves):

$$\left(-\frac{a}{2 x^2}+\frac{2 b x}{5}\right) \sqrt{a+b x^3}+\frac{1}{10 (-b)^{1/3} \sqrt{a+b x^3}}$$

$$9 \, i \, 3^{3/4} a^{4/3} b \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

- **Problem 398: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+b x^3\right)^{3/2}}{x^6} \, dx$$

Optimal (type 4, 247 leaves, 3 steps):

$$-\frac{9 b \sqrt{a+b x^3}}{20 x^2}-\frac{\left(a+b x^3\right)^{3/2}}{5 x^5}+$$

$$\left(9 \times 3^{3/4} \sqrt{2+\sqrt{3}} \, b^{5/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(20 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 167 leaves):

$$-\frac{\sqrt{a+b x^3} \left(4 a+13 b x^3\right)}{20 x^5}+\frac{1}{20 \sqrt{a+b x^3}}$$

$$9 \, i \, 3^{3/4} a^{1/3} (-b)^{5/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

- **Problem 399: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^7 \left(a+b x^3\right)^{3/2} \, dx$$

Optimal (type 4, 556 leaves, 7 steps):

$$-\frac{108\,a^3\,x^2\,\sqrt{a+b\,x^3}}{8645\,b^2}+\frac{54\,a^2\,x^5\,\sqrt{a+b\,x^3}}{6175\,b}+\frac{18}{475}\,a\,x^8\,\sqrt{a+b\,x^3}+\frac{432\,a^4\,\sqrt{a+b\,x^3}}{8645\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+\frac{2}{25}\,x^8\,\left(a+b\,x^3\right)^{3/2}-$$

$$\left(216\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{13/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(8645\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)+$$

$$\left(144\,\sqrt{2}\,\,3^{3/4}\,a^{13/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(8645\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 253 leaves) :

$$\frac{2\,x^2\,\sqrt{a+b\,x^3}\,\left(-270\,a^3+189\,a^2\,b\,x^3+2548\,a\,b^2\,x^6+1729\,b^3\,x^9\right)}{43\,225\,b^2}+$$

$$\frac{1}{8645\,(-b)^{8/3}\,\sqrt{a+b\,x^3}}144\,(-1)^{1/6}\,3^{3/4}\,a^{14/3}\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(-i\,\sqrt{3}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 400: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4\,\left(a+b\,x^3\right)^{3/2}\,dx$$

Optimal (type 4, 532 leaves, 6 steps) :

$$\frac{54\,a^2\,x^2\,\sqrt{a+b\,x^3}}{1729\,b}+\frac{18}{247}\,a\,x^5\,\sqrt{a+b\,x^3}-\frac{216\,a^3\,\sqrt{a+b\,x^3}}{1729\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+\frac{2}{19}\,x^5\,\left(a+b\,x^3\right)^{3/2}+$$

$$\left(108\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{10/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(1729\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\frac{72\,\sqrt{2}\,3^{3/4}\,a^{10/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{1729\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}$$

Result (type 4, 238 leaves) :

$$-\frac{1}{1729\,(-b)^{5/3}\,\sqrt{a+b\,x^3}}$$

$$2\left((-b)^{2/3}\,\left(a+b\,x^3\right)\,\left(27\,a^2\,x^2+154\,a\,b\,x^5+91\,b^2\,x^8\right)+36\,(-1)^{2/3}\,3^{3/4}\,a^{11/3}\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 401: Result unnecessarily involves imaginary or complex numbers.**

$$\int x\,\left(a+b\,x^3\right)^{3/2}\,dx$$

Optimal (type 4, 508 leaves, 5 steps) :

$$\frac{18}{91} a x^2 \sqrt{a+b x^3}+\frac{54 a^2 \sqrt{a+b x^3}}{91 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2}{13} x^2 \left(a+b x^3\right)^{3/2}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\ a^{7/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)+$$

$$\frac{18 \sqrt{2}\ 3^{3/4} a^{7/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{91 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}}$$

Result (type 4, 229 leaves) :

$$\sqrt{a+b x^3} \left(\frac{32 a x^2}{91}+\frac{2 b x^5}{13}\right)+\frac{1}{91 (-b)^{2/3} \sqrt{a+b x^3}} 18 (-1)^{1/6} 3^{3/4} a^{8/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 402: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+b x^3\right)^{3/2}}{x^2}\, dx$$

Optimal (type 4, 504 leaves, 5 steps) :

$$\frac{9}{7} b x^2 \sqrt{a+b x^3}+\frac{27 a b^{1/3} \sqrt{a+b x^3}}{7 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{\left(a+b x^3\right)^{3/2}}{x}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\; a^{4/3} b^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(14 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(9 \sqrt{2}\ 3^{3/4} a^{4/3} b^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 228 leaves) :

$$\left(-\frac{a}{x}+\frac{2 b x^2}{7}\right) \sqrt{a+b x^3}+\frac{1}{7 (-b)^{2/3} \sqrt{a+b x^3}} 9 (-1)^{1/6} 3^{3/4} a^{5/3} b \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 403: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+b x^3\right)^{3/2}}{x^5} \, dx$$

Optimal (type 4, 505 leaves, 5 steps) :

$$-\frac{9 b \sqrt{a+b x^3}}{8 x}+\frac{27 b^{4/3} \sqrt{a+b x^3}}{8 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{\left(a+b x^3\right)^{3/2}}{4 x^4}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{1/3} b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(16 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\frac{9\times 3^{3/4} a^{1/3} b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{4 \sqrt{2} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 228 leaves) :

$$-\frac{\sqrt{a+b x^3} \left(2 a+11 b x^3\right)}{8 x^4}+\frac{1}{8 \sqrt{a+b x^3}} 9 (-1)^{1/6} 3^{3/4} a^{2/3} (-b)^{4/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 411: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^6}{\sqrt{a+b x^3}}\, dx$$

Optimal (type 4, 254 leaves, 3 steps) :

$$-\frac{16 a x \sqrt{a+b x^3}}{55 b^2}+\frac{2 x^4 \sqrt{a+b x^3}}{11 b}+\frac{32 \sqrt{2+\sqrt{3}}\, a^2 \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{55\times 3^{1/4} b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 174 leaves) :

$$\sqrt{a+b x^3}\left(-\frac{16 a x}{55 b^2}+\frac{2 x^4}{11 b}\right)+$$

$$\left(32 i a^{7/3} \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\Big/$$

$$\left(55 \times 3^{1/4} (-b)^{1/3} b^2 \sqrt{a+b x^3}\right)$$

Problem 412: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{a+b x^3}} \, dx$$

Optimal (type 4, 230 leaves, 2 steps):

$$\frac{2 x \sqrt{a+b x^3}}{5 b}-\frac{4 \sqrt{2+\sqrt{3}}\, a \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]}{5 \times 3^{1/4} b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 158 leaves):

$$\frac{2 x \sqrt{a+b x^3}}{5 b}+\frac{1}{5 \times 3^{1/4} (-b)^{4/3} \sqrt{a+b x^3}}$$

$$4 i a^{4/3} \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Problem 413: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+b x^3}} \, dx$$

Optimal (type 4, 207 leaves, 1 step):

$$\frac{2\sqrt{2+\sqrt{3}}\,\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],\,-7-4\sqrt{3}\right]}{3^{1/4}\,b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \sqrt{a+b\,x^3}}$$

Result (type 4, 136 leaves) :

$$\frac{1}{3^{1/4}\,(-b)^{1/3}\sqrt{a+b\,x^3}}2\,\mathrm{i}\,a^{1/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\ \sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]$$

■ Problem 414: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 234 leaves, 2 steps) :

$$-\frac{\sqrt{a+b\,x^3}}{2\,a\,x^2}-\frac{\sqrt{2+\sqrt{3}}\ b^{2/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],\,-7-4\sqrt{3}\right]}{2\times 3^{1/4}\,a\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \sqrt{a+b\,x^3}}$$

Result (type 4, 161 leaves) :

$$-\frac{\sqrt{a+b\,x^3}}{2\,a\,x^2}-\left(\mathrm{i}\,b\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\ \sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(2\times 3^{1/4}\,a^{2/3}\,(-b)^{1/3}\sqrt{a+b\,x^3}\right)$$

■ Problem 415: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 256 leaves, 3 steps) :

$$-\frac{\sqrt{a+b x^3}}{5 a x^5}+\frac{7 b \sqrt{a+b x^3}}{20 a^2 x^2}+\frac{7 \sqrt{2+\sqrt{3}}\, b^{5/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{20\times 3^{1/4} a^2 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 170 leaves) :

$$\frac{1}{60 a^2 x^5 \sqrt{a+b x^3}}\left(-12 a^2+9 a b x^3+21 b^2 x^6+\right.$$

$$\left.7 i\, 3^{3/4} a^{1/3} (-b)^{5/3} x^5 \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 416: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{a+b x^3}}\, dx$$

Optimal (type 4, 514 leaves, 5 steps) :

$$-\frac{20 a x^2 \sqrt{a+b x^3}}{91 b^2}+\frac{2 x^5 \sqrt{a+b x^3}}{13 b}+\frac{80 a^2 \sqrt{a+b x^3}}{91 b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(40\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{7/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\frac{80 \sqrt{2}\, a^{7/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{91\times 3^{1/4} b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 228 leaves) :

$$-\frac{1}{273\,(-b)^{8/3}\sqrt{a+b\,x^3}}2\left(3\,(-b)^{2/3}\,(a+b\,x^3)\,(10\,a\,x^2-7\,b\,x^5)+40\,(-1)^{2/3}\,3^{3/4}\,a^{8/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$
$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 417: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 490 leaves, 4 steps):

$$\frac{2\,x^2\sqrt{a+b\,x^3}}{7\,b}-\frac{8\,a\sqrt{a+b\,x^3}}{7\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$
$$\left(4\times3^{1/4}\sqrt{2-\sqrt{3}}\,a^{4/3}\,(a^{1/3}+b^{1/3}\,x)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(7\,b^{5/3}\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-\frac{8\sqrt{2}\,a^{4/3}\,(a^{1/3}+b^{1/3}\,x)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]}{7\times3^{1/4}\,b^{5/3}\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}$$

Result (type 4, 221 leaves):

$$\frac{2\,x^2\sqrt{a+b\,x^3}}{7\,b}+\frac{1}{7\times3^{1/4}\,(-b)^{5/3}\sqrt{a+b\,x^3}}8\,(-1)^{1/6}\,a^{5/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$
$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 418: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 462 leaves, 3 steps):

$$\frac{2\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+\frac{2\sqrt{2}\;a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}}$$

Result (type 4, 197 leaves):

$$\frac{1}{3^{1/4}\,(-b)^{2/3}\sqrt{a+b\,x^3}}\,2\,(-1)^{1/6}\,a^{2/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\left(-i\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 419: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^2\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 484 leaves, 4 steps):

$$-\frac{\sqrt{a+b x^3}}{a x}+\frac{b^{1/3} \sqrt{a+b x^3}}{a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\left(3^{1/4} \sqrt{2-\sqrt{3}}\, b^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(2 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\frac{\sqrt{2}\, b^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{3^{1/4} a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 217 leaves) :

$$-\frac{\sqrt{a+b x^3}}{a x}+\left((-1)^{1/6} b \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Big/ \left(3^{1/4} a^{1/3} (-b)^{2/3} \sqrt{a+b x^3}\right)$$

- **Problem 420: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^5 \sqrt{a+b x^3}}\, dx$$

Optimal (type 4, 514 leaves, 5 steps) :

$$-\frac{\sqrt{a+b x^3}}{4 a x^4}+\frac{5 b \sqrt{a+b x^3}}{8 a^2 x}-\frac{5 b^{4/3} \sqrt{a+b x^3}}{8 a^2 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+$$

$$\left(5\times 3^{1/4} \sqrt{2-\sqrt{3}}\; b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(16 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\frac{5 b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{4 \sqrt{2}\; 3^{1/4} a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 231 leaves) :

$$\frac{\sqrt{a+b x^3} \left(-2 a+5 b x^3\right)}{8 a^2 x^4}-\frac{1}{8\times 3^{1/4} a^{4/3} \sqrt{a+b x^3}} 5 (-1)^{1/6} (-b)^{4/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 428: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^6}{\left(a+b x^3\right)^{3/2}}\, dx$$

Optimal (type 4, 251 leaves, 3 steps) :

$$-\frac{2 x^4}{3 b \sqrt{a+b x^3}}+\frac{16 x \sqrt{a+b x^3}}{15 b^2}-\frac{32 \sqrt{2+\sqrt{3}}\; a \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]}{15\times 3^{1/4} b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 161 leaves) :

$$\frac{1}{45\,(-b)^{7/3}\sqrt{a+b\,x^3}}\left(6\,(-b)^{1/3}\,x\,\left(8\,a+3\,b\,x^3\right)-\right.$$

$$\left.32\,i\,3^{3/4}\,a^{4/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 429: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 229 leaves, 2 steps) :

$$-\frac{2\,x}{3\,b\,\sqrt{a+b\,x^3}}+\frac{4\,\sqrt{2+\sqrt{3}}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{3\times 3^{1/4}\,b^{4/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}}$$

Result (type 4, 151 leaves) :

$$\frac{1}{9\,(-b)^{4/3}\sqrt{a+b\,x^3}}$$

$$\left(6\,(-b)^{1/3}\,x-4\,i\,3^{3/4}\,a^{1/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 430: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 232 leaves, 2 steps) :

$$\frac{2x}{3a\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\,\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\,a\,b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\sqrt{a+bx^3}}$$

Result (type 4, 154 leaves) :

$$\frac{1}{9a(-b)^{1/3}\sqrt{a+bx^3}}\left(6(-b)^{1/3}x+2\,i\,3^{3/4}a^{1/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 431: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 255 leaves, 3 steps) :

$$\frac{2}{3ax^2\sqrt{a+bx^3}}-\frac{7\sqrt{a+bx^3}}{6a^2x^2}-\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{6\times 3^{1/4}\,a^2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\sqrt{a+bx^3}}$$

Result (type 4, 170 leaves) :

$$\frac{1}{18a^2(-b)^{1/3}x^2\sqrt{a+bx^3}}\left(-3(-b)^{1/3}\left(3a+7bx^3\right)-\right.$$

$$\left.7\,i\,3^{3/4}a^{1/3}bx^2\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 432: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6 \left(a+b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 277 leaves, 4 steps):

$$\frac{2}{3 a x^5 \sqrt{a+b x^3}}-\frac{13 \sqrt{a+b x^3}}{15 a^2 x^5}+\frac{91 b \sqrt{a+b x^3}}{60 a^3 x^2}+$$
$$\left(91 \sqrt{2+\sqrt{3}}\, b^{5/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$
$$\left(60 \times 3^{1/4} a^3 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 183 leaves):

$$\left(3 (-b)^{1/3} \left(-12 a^2+39 a b x^3+91 b^2 x^6\right)+91 i\, 3^{3/4} a^{1/3} b^2 x^5 \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\right.$$
$$\left.\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right) \Big/ \left(180 a^3 (-b)^{1/3} x^5 \sqrt{a+b x^3}\right)$$

Problem 433: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\left(a+b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 511 leaves, 5 steps):

$$-\frac{2x^5}{3b\sqrt{a+bx^3}}+\frac{20x^2\sqrt{a+bx^3}}{21b^2}-\frac{80a\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$
$$\left(40\sqrt{2-\sqrt{3}}\;a^{4/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(7\times3^{3/4}b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)-$$
$$\frac{80\sqrt{2}\;a^{4/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{21\times3^{1/4}b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}}$$

Result (type 4, 221 leaves) :

$$\frac{1}{63(-b)^{8/3}\sqrt{a+bx^3}}2\left(3(-b)^{2/3}x^2\left(10a+3bx^3\right)+40(-1)^{2/3}3^{3/4}a^{5/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$
$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 434: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 487 leaves, 4 steps) :

$$-\frac{2x^2}{3b\sqrt{a+bx^3}}+\frac{8\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\frac{4\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3^{3/4}b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}}+$$

$$\frac{8\sqrt{2}\;a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}}$$

Result (type 4, 216 leaves) :

$$\frac{1}{9b\sqrt{a+bx^3}}2\left(-3x^2+1\Big/(-b)^{2/3}4\,(-1)^{1/6}\,3^{3/4}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left.\left(-i\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 435: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x}{\left(a+bx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 489 leaves, 4 steps) :

$$\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+\frac{\sqrt{2-\sqrt{3}}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}}-$$

$$\frac{2\sqrt{2}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3\times3^{1/4}a^{2/3}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}}$$

Result (type 4, 212 leaves):

$$\frac{1}{9a\sqrt{a+bx^3}}2\left(3x^2+1\Big/(-b)^{2/3}(-1)^{2/3}3^{3/4}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 436: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 513 leaves, 5 steps):

$$\frac{2}{3\,a\,x\,\sqrt{a+b\,x^3}}-\frac{5\,\sqrt{a+b\,x^3}}{3\,a^2\,x}+\frac{5\,b^{1/3}\,\sqrt{a+b\,x^3}}{3\,a^2\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\frac{5\,\sqrt{2-\sqrt{3}}\,b^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{2\times 3^{3/4}\,a^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}+$$

$$\frac{5\,\sqrt{2}\,b^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{3\times 3^{1/4}\,a^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}$$

Result (type 4, 226 leaves) :

$$\frac{1}{9\,a^2\,(-b)^{2/3}\,x\,\sqrt{a+b\,x^3}}\left(-3\,(-b)^{2/3}\,\left(3\,a+5\,b\,x^3\right)-5\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,b\,x\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 437: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 535 leaves, 6 steps) :

$$\frac{2}{3 a x^4 \sqrt{a+b x^3}}-\frac{11 \sqrt{a+b x^3}}{12 a^2 x^4}+\frac{55 b \sqrt{a+b x^3}}{24 a^3 x}-\frac{55 b^{4/3} \sqrt{a+b x^3}}{24 a^3 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+$$

$$\left(55 \sqrt{2-\sqrt{3}}\ b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(16\times 3^{3/4} a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)-$$

$$\frac{55 b^{4/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]}{12 \sqrt{2}\ 3^{1/4} a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}}$$

Result (type 4, 241 leaves) :

$$\frac{1}{72 a^3 (-b)^{2/3} x^4 \sqrt{a+b x^3}}$$

$$\left(3 (-b)^{2/3} \left(-6 a^2+33 a b x^3+55 b^2 x^6\right)+55 (-1)^{2/3} 3^{3/4} a^{2/3} b^2 x^4 \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\ \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{5/6}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)$$

- **Problem 446: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^6}{\sqrt{1+x^3}}\, dx$$

Optimal (type 4, 136 leaves, 3 steps) :

$$-\frac{16}{55}x\sqrt{1+x^3}+\frac{2}{11}x^4\sqrt{1+x^3}+\frac{32\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{55\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Result (type 4, 108 leaves) :

$$\frac{1}{165\sqrt{1+x^3}}$$

$$2\left(3x\left(-8-3x^3+5x^6\right)+16(-1)^{1/6}3^{3/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 447: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 120 leaves, 2 steps) :

$$\frac{2}{5}x\sqrt{1+x^3}-\frac{4\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{5\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Result (type 4, 100 leaves) :

$$\frac{1}{15\sqrt{1+x^3}}\left(6\left(x+x^4\right)-4(-1)^{1/6}3^{3/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 448: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 103 leaves, 1 step) :

$$\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Result (type 4, 88 leaves) :

$$\frac{1}{3^{1/4}\sqrt{1+x^3}}2\,(-1)^{1/6}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 449: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^3\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 122 leaves, 2 steps) :

$$-\frac{\sqrt{1+x^3}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{2\times3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Result (type 4, 104 leaves) :

$$-\frac{1}{6x^2\sqrt{1+x^3}}\left(3+3x^3+(-1)^{1/6}3^{3/4}x^2\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 450: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^6\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 138 leaves, 3 steps) :

$$-\frac{\sqrt{1+x^3}}{5x^5}+\frac{7\sqrt{1+x^3}}{20x^2}+\frac{7\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{20\times3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Result (type 4, 110 leaves) :

$$\frac{1}{60\,x^5\sqrt{1+x^3}}\left(-12+9\,x^3+21\,x^6+7\,(-1)^{1/6}\,3^{3/4}\,x^5\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\,\sqrt{1+(-1)^{1/3}\,x+(-1)^{2/3}\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}\,(1+x)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 451: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^7}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 262 leaves, 5 steps) :

$$-\frac{20}{91}\,x^2\sqrt{1+x^3}+\frac{2}{13}\,x^5\sqrt{1+x^3}+\frac{80\sqrt{1+x^3}}{91\left(1+\sqrt{3}+x\right)}-\frac{40\times3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],\,-7-4\sqrt{3}\right]}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}+$$

$$\frac{80\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],\,-7-4\sqrt{3}\right]}{91\times3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Result (type 4, 145 leaves) :

$$\frac{1}{273\sqrt{1+x^3}}2\left(3\,x^2\left(1+x^3\right)\left(-10+7\,x^3\right)-40\times3^{3/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\,\sqrt{1+(-1)^{1/3}\,x+(-1)^{2/3}\,x^2}\right.$$
$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}\,(1+x)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}\,(1+x)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 452: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 246 leaves, 4 steps) :

$$\frac{2}{7}x^2\sqrt{1+x^3}-\frac{8\sqrt{1+x^3}}{7\left(1+\sqrt{3}+x\right)}+\frac{4\times 3^{1/4}\sqrt{2-\sqrt{3}}\ (1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}-$$

$$\frac{8\sqrt{2}\ (1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{7\times 3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Result (type 4, 138 leaves) :

$$\frac{1}{21\sqrt{1+x^3}}2\left(3x^2\left(1+x^3\right)+4\times 3^{3/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\right.$$

$$\left.\left(\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 453: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 224 leaves, 3 steps) :

$$\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\ (1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+$$

$$\frac{2\sqrt{2}\ (1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Result (type 4, 123 leaves) :

$$-\frac{1}{3^{1/4}\sqrt{1+x^3}}2\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}$$
$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 454: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 238 leaves, 4 steps) :

$$-\frac{\sqrt{1+x^3}}{x}+\frac{\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+$$
$$\frac{\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Result (type 4, 138 leaves) :

$$\frac{1}{3\sqrt{1+x^3}}\left(-\frac{3\left(1+x^3\right)}{x}-3^{3/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\right.$$
$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 455: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^5\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 262 leaves, 5 steps) :

$$-\frac{\sqrt{1+x^3}}{4x^4}+\frac{5\sqrt{1+x^3}}{8x}-\frac{5\sqrt{1+x^3}}{8\left(1+\sqrt{3}+x\right)}+\frac{5\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{16\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}-$$

$$\frac{5\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{4\sqrt{2}\,3^{1/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Result (type 4, 145 leaves) :

$$\frac{1}{24\sqrt{1+x^3}}\left(\frac{3\left(1+x^3\right)\left(-2+5x^3\right)}{x^4}+5\times 3^{3/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\right.$$
$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 464: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^6}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 152 leaves, 3 steps) :

$$-\frac{16}{55}x\sqrt{1-x^3}-\frac{2}{11}x^4\sqrt{1-x^3}-\frac{32\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{55\times 3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Result (type 4, 93 leaves) :

$$\frac{1}{165\sqrt{1-x^3}}2\left(3x\left(-8+3x^3+5x^6\right)+16\,i\,3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\sqrt{1+x+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 465: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 134 leaves, 2 steps):

$$-\frac{2}{5}x\sqrt{1-x^3}-\frac{4\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{5\times 3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Result (type 4, 86 leaves):

$$\frac{2\left(3x\left(-1+x^3\right)+2\,\mathrm{i}\,3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)}{15\sqrt{1-x^3}}$$

Problem 466: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 115 leaves, 1 step):

$$-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Result (type 4, 73 leaves):

$$\frac{2\,\mathrm{i}\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]}{3^{1/4}\sqrt{1-x^3}}$$

Problem 467: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^3\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 136 leaves, 2 steps):

$$-\frac{\sqrt{1-x^3}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{2\times 3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Result (type 4, 90 leaves):

$$\frac{-3+3x^3+i\,3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\;x^2\sqrt{1+x+x^2}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]}{6x^2\sqrt{1-x^3}}$$

- **Problem 468: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^6\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 154 leaves, 3 steps):

$$-\frac{\sqrt{1-x^3}}{5x^5}-\frac{7\sqrt{1-x^3}}{20x^2}-\frac{7\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{20\times 3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Result (type 4, 95 leaves):

$$\frac{-12-9x^3+21x^6+7\,i\,3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\;x^5\sqrt{1+x+x^2}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]}{60x^5\sqrt{1-x^3}}$$

- **Problem 469: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^7}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\frac{80\sqrt{1-x^3}}{91\left(1+\sqrt{3}-x\right)}-\frac{20}{91}x^2\sqrt{1-x^3}-\frac{2}{13}x^5\sqrt{1-x^3}-\frac{40\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{91\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}+$$

$$\frac{80\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{91\times 3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Result (type 4, 144 leaves):

$$\frac{1}{273\sqrt{1-x^3}}2\left(3x^2\left(-1+x^3\right)\left(10+7x^3\right)+40(-1)^{1/6}3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\sqrt{1+x+x^2}\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 470: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 276 leaves, 4 steps) :

$$\frac{8\sqrt{1-x^3}}{7\left(1+\sqrt{3}-x\right)}-\frac{2}{7}x^2\sqrt{1-x^3}-\frac{4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{7\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}+\frac{8\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{7\times 3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Result (type 4, 137 leaves) :

$$\frac{1}{21\sqrt{1-x^3}}2\left(3x^2\left(-1+x^3\right)+4(-1)^{1/6}3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\sqrt{1+x+x^2}\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 471: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 252 leaves, 3 steps) :

$$\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\ \sqrt{1-x^3}}+$$

$$\frac{2\sqrt{2}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\ \sqrt{1-x^3}}$$

Result (type 4, 122 leaves) :

$$\frac{1}{3^{1/4}\sqrt{1-x^3}}2\ (-1)^{1/6}\sqrt{(-1)^{5/6}\ (-1+x)}\ \sqrt{1+x+x^2}$$

$$\left(-i\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 472: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 270 leaves, 4 steps) :

$$-\frac{\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{\sqrt{1-x^3}}{x}+\frac{3^{1/4}\sqrt{2-\sqrt{3}}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{2\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\ \sqrt{1-x^3}}-$$

$$\frac{\sqrt{2}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\ \sqrt{1-x^3}}$$

Result (type 4, 133 leaves) :

$$\frac{1}{3\sqrt{1-x^3}}\left(\frac{3\left(-1+x^3\right)}{x}+(-1)^{2/3}\,3^{3/4}\sqrt{(-1)^{5/6}\,(-1+x)}\,\sqrt{1+x+x^2}\right.$$
$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 473: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^5\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$-\frac{5\sqrt{1-x^3}}{8\left(1+\sqrt{3}-x\right)}-\frac{\sqrt{1-x^3}}{4\,x^4}-\frac{5\sqrt{1-x^3}}{8\,x}+\frac{5\times3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],\,-7-4\sqrt{3}\right]}{16\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}-$$
$$\frac{5\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],\,-7-4\sqrt{3}\right]}{4\sqrt{2}\,3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Result (type 4, 145 leaves):

$$\frac{1}{24\,x^4\sqrt{1-x^3}}\left(3\left(-1+x^3\right)\left(2+5\,x^3\right)+1\Big/\left(\sqrt{(-1)^{5/6}\,(-1+x)}\right)5\times3^{3/4}\,(-1+x)\,x^4\sqrt{1+x+x^2}\right.$$
$$\left.\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 478: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{x\sqrt{-1+x^3}}\,dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\frac{2}{3}\,\text{ArcTan}\left[\sqrt{-1+x^3}\right]$$

Result (type 3, 36 leaves) :

$$\frac{2\sqrt{-1+x^3}\ \text{ArcTanh}\left[\sqrt{1-x^3}\right]}{3\sqrt{1-x^3}}$$

- **Problem 482: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 153 leaves, 3 steps) :

$$\frac{16}{55}x\sqrt{-1+x^3}+\frac{2}{11}x^4\sqrt{-1+x^3}-\frac{32\sqrt{2-\sqrt{3}}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],\ -7+4\sqrt{3}\right]}{55\times 3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\ \sqrt{-1+x^3}}$$

Result (type 4, 91 leaves) :

$$\frac{1}{165\sqrt{-1+x^3}}2\left(3x\left(-8+3x^3+5x^6\right)+16\,i\,3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\ \sqrt{1+x+x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\ (-1)^{1/3}\right]\right)$$

- **Problem 483: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 137 leaves, 2 steps) :

$$\frac{2}{5}x\sqrt{-1+x^3}-\frac{4\sqrt{2-\sqrt{3}}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],\ -7+4\sqrt{3}\right]}{5\times 3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\ \sqrt{-1+x^3}}$$

Result (type 4, 84 leaves) :

$$\frac{2\left(3x\left(-1+x^3\right)+2\,i\,3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\ \sqrt{1+x+x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\ (-1)^{1/3}\right]\right)}{15\sqrt{-1+x^3}}$$

Problem 484: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 120 leaves, 1 step):

$$-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Result (type 4, 71 leaves):

$$\frac{2\,i\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]}{3^{1/4}\sqrt{-1+x^3}}$$

Problem 485: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 139 leaves, 2 steps):

$$\frac{\sqrt{-1+x^3}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{2\times 3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Result (type 4, 90 leaves):

$$\frac{\sqrt{-1+x^3}}{2x^2}+\frac{i\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]}{2\times 3^{1/4}\sqrt{-1+x^3}}$$

Problem 486: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 155 leaves, 3 steps):

$$\frac{\sqrt{-1+x^3}}{5\,x^5}+\frac{7\sqrt{-1+x^3}}{20\,x^2}-\frac{7\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{20\times 3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\;\sqrt{-1+x^3}}$$

Result (type 4, 93 leaves) :

$$\frac{-12-9\,x^3+21\,x^6+7\,i\,3^{3/4}\sqrt{(-1)^{5/6}\,(-1+x)}\;x^5\sqrt{1+x+x^2}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]}{60\,x^5\sqrt{-1+x^3}}$$

- **Problem 487: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^7}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 294 leaves, 5 steps) :

$$-\frac{80\sqrt{-1+x^3}}{91\left(1-\sqrt{3}-x\right)}+\frac{20}{91}\,x^2\sqrt{-1+x^3}+\frac{2}{13}\,x^5\sqrt{-1+x^3}+\frac{40\times 3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{91\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\;\sqrt{-1+x^3}}-$$

$$\frac{80\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{91\times 3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\;\sqrt{-1+x^3}}$$

Result (type 4, 142 leaves) :

$$\frac{1}{273\sqrt{-1+x^3}}2\left(3\,x^2\left(-1+x^3\right)\left(10+7\,x^3\right)+40\,(-1)^{1/6}\,3^{3/4}\sqrt{(-1)^{5/6}\,(-1+x)}\;\sqrt{1+x+x^2}\right.$$

$$\left.\left(-i\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 488: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$-\frac{8\sqrt{-1+x^3}}{7\left(1-\sqrt{3}-x\right)}+\frac{2}{7}x^2\sqrt{-1+x^3}+\frac{4\times 3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{7\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-$$

$$\frac{8\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{7\times 3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Result (type 4, 135 leaves):

$$\frac{1}{21\sqrt{-1+x^3}}2\left(3x^2\left(-1+x^3\right)+4(-1)^{1/6}3^{3/4}\sqrt{(-1)^{5/6}(-1+x)}\sqrt{1+x+x^2}\right.$$

$$\left.\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 489: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 255 leaves, 3 steps):

$$-\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}-$$

$$\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Result (type 4, 120 leaves) :

$$\frac{1}{3^{1/4}\sqrt{-1+x^3}}2\,(-1)^{1/6}\sqrt{(-1)^{5/6}\,(-1+x)}\,\sqrt{1+x+x^2}$$

$$\left(-\mathrm{i}\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 490: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 269 leaves, 4 steps) :

$$\frac{\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt{-1+x^3}}{x}-\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{2\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}+$$

$$\frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Result (type 4, 130 leaves) :

$$\frac{\sqrt{-1+x^3}}{x}+\frac{1}{3^{1/4}\sqrt{-1+x^3}}(-1)^{2/3}\sqrt{(-1)^{5/6}(-1+x)}\sqrt{1+x+x^2}$$
$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 491: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^5\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\frac{5\sqrt{-1+x^3}}{8\left(1-\sqrt{3}-x\right)}+\frac{\sqrt{-1+x^3}}{4x^4}+\frac{5\sqrt{-1+x^3}}{8x}-\frac{5\times 3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{16\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}+$$
$$\frac{5\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{4\sqrt{2}\,3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Result (type 4, 140 leaves):

$$\frac{1}{24\sqrt{-1+x^3}}\left(\frac{3\left(-1+x^3\right)\left(2+5x^3\right)}{x^4}+1\Big/\left(\sqrt{(-1)^{5/6}(-1+x)}\right)5\times 3^{3/4}(-1+x)\sqrt{1+x+x^2}\right.$$
$$\left.\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 496: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{x\sqrt{-1-x^3}}\,dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\frac{2}{3}\text{ArcTan}\left[\sqrt{-1-x^3}\right]$$

Result (type 3, 34 leaves) :

$$\frac{2\sqrt{-1-x^3}\ \text{ArcTanh}\left[\sqrt{1+x^3}\right]}{3\sqrt{1+x^3}}$$

Problem 500: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 149 leaves, 3 steps) :

$$\frac{16}{55}x\sqrt{-1-x^3}-\frac{2}{11}x^4\sqrt{-1-x^3}+\frac{32\sqrt{2-\sqrt{3}}\ (1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{55\times 3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\ \sqrt{-1-x^3}}$$

Result (type 4, 115 leaves) :

$$\frac{1}{165\sqrt{-1-x^3}}$$

$$2\left(3x\left(-8-3x^3+5x^6\right)+16(-1)^{5/6}3^{3/4}\sqrt{-(-1)^{5/6}+i x}\ \sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 501: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 131 leaves, 2 steps) :

$$-\frac{2}{5}x\sqrt{-1-x^3}-\frac{4\sqrt{2-\sqrt{3}}\ (1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{5\times 3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\ \sqrt{-1-x^3}}$$

Result (type 4, 107 leaves) :

$$\frac{1}{15\sqrt{-1-x^3}}\left(6\left(x+x^4\right)-4(-1)^{5/6}3^{3/4}\sqrt{-(-1)^{5/6}+i x}\ \sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 502: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 112 leaves, 1 step):

$$\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 95 leaves):

$$\frac{2\,(-1)^{5/6}\sqrt{-(-1)^{5/6}+i\,x}\,\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]}{3^{1/4}\sqrt{-1-x^3}}$$

Problem 503: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 133 leaves, 2 steps):

$$\frac{\sqrt{-1-x^3}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{2\times 3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 111 leaves):

$$-\frac{1}{6x^2\sqrt{-1-x^3}}\left(3+3x^3+(-1)^{5/6}\,3^{3/4}\sqrt{-(-1)^{5/6}+i\,x}\;x^2\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 504: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 151 leaves, 3 steps):

$$\frac{\sqrt{-1-x^3}}{5\,x^5}-\frac{7\,\sqrt{-1-x^3}}{20\,x^2}+\frac{7\,\sqrt{2-\sqrt{3}}\,\left(1+x\right)\,\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\text{EllipticF}\!\left[\text{ArcSin}\!\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],\,-7+4\,\sqrt{3}\right]}{20\times 3^{1/4}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 117 leaves) :

$$\frac{1}{60\,x^5\,\sqrt{-1-x^3}}\left(-12+9\,x^3+21\,x^6+7\,(-1)^{5/6}\,3^{3/4}\,\sqrt{-(-1)^{5/6}+i\,x}\;x^5\,\sqrt{1-(-1)^{2/3}\,x-(-1)^{1/3}\,x^2}\;\text{EllipticF}\!\left[\text{ArcSin}\!\left[\frac{\sqrt{-(-1)^{1/6}\,\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 505: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^7}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 282 leaves, 5 steps) :

$$\frac{20}{91}\,x^2\,\sqrt{-1-x^3}-\frac{2}{13}\,x^5\,\sqrt{-1-x^3}-\frac{80\,\sqrt{-1-x^3}}{91\,\left(1-\sqrt{3}+x\right)}+\frac{40\times 3^{1/4}\,\sqrt{2+\sqrt{3}}\,\left(1+x\right)\,\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\text{EllipticE}\!\left[\text{ArcSin}\!\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],\,-7+4\,\sqrt{3}\right]}{91\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}-$$

$$\frac{80\,\sqrt{2}\,\left(1+x\right)\,\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\text{EllipticF}\!\left[\text{ArcSin}\!\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],\,-7+4\,\sqrt{3}\right]}{91\times 3^{1/4}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 164 leaves) :

$$\frac{1}{273\,\sqrt{-1-x^3}}2\left(3\,x^2\,\left(1+x^3\right)\,\left(-10+7\,x^3\right)+40\,(-1)^{5/6}\,3^{3/4}\,\sqrt{-(-1)^{5/6}+i\,x}\;\sqrt{1-(-1)^{2/3}\,x-(-1)^{1/3}\,x^2}\right.$$

$$\left.\left(-i\,\sqrt{3}\;\text{EllipticE}\!\left[\text{ArcSin}\!\left[\frac{\sqrt{-(-1)^{1/6}\,\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\!\left[\text{ArcSin}\!\left[\frac{\sqrt{-(-1)^{1/6}\,\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 506: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 264 leaves, 4 steps):

$$-\frac{2}{7}x^2\sqrt{-1-x^3}+\frac{8\sqrt{-1-x^3}}{7\left(1-\sqrt{3}+x\right)}-\frac{4\times 3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{7\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}+$$

$$\frac{8\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{7\times 3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 157 leaves):

$$\frac{1}{21\sqrt{-1-x^3}}2\left(3x^2\left(1+x^3\right)-4\,(-1)^{5/6}\,3^{3/4}\sqrt{-(-1)^{5/6}+i\,x}\,\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\right.$$

$$\left.\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

Problem 507: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 239 leaves, 3 steps):

$$-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}-$$

$$\frac{2\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 142 leaves) :

$$\frac{1}{3^{1/4}\sqrt{-1-x^3}}2\,(-1)^{5/6}\sqrt{-(-1)^{5/6}+\mathrm{i}\,x}\,\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}$$

$$\left(-\mathrm{i}\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 508: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 257 leaves, 4 steps) :

$$\frac{\sqrt{-1-x^3}}{x}-\frac{\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{2\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}-$$

$$\frac{\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 156 leaves) :

$$\frac{1}{3\sqrt{-1-x^3}}\left(-\frac{3\left(1+x^3\right)}{x}+(-1)^{5/6}\,3^{3/4}\sqrt{-(-1)^{5/6}+\mathrm{i}\,x}\,\sqrt{1-(-1)^{2/3}\,x-(-1)^{1/3}\,x^2}\right.$$
$$\left.\left(-\mathrm{i}\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 509: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^5\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 282 leaves, 5 steps):

$$\frac{\sqrt{-1-x^3}}{4x^4}-\frac{5\sqrt{-1-x^3}}{8x}+\frac{5\sqrt{-1-x^3}}{8\left(1-\sqrt{3}+x\right)}-\frac{5\times 3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],\,-7+4\sqrt{3}\right]}{16\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}+$$
$$\frac{5\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],\,-7+4\sqrt{3}\right]}{4\sqrt{2}\,3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 164 leaves):

$$\frac{1}{24\sqrt{-1-x^3}}\left(\frac{3\left(1+x^3\right)\left(-2+5x^3\right)}{x^4}-5\,(-1)^{5/6}\,3^{3/4}\sqrt{-(-1)^{5/6}+\mathrm{i}\,x}\,\sqrt{1-(-1)^{2/3}\,x-(-1)^{1/3}\,x^2}\right.$$
$$\left.\left(-\mathrm{i}\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 514: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x}\,dx$$

Optimal (type 3, 95 leaves, 6 steps):

$$(a+b x^3)^{1/3}-\frac{a^{1/3}\,\text{ArcTan}\left[\frac{a^{1/3}+2(a+b x^3)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}}-\frac{1}{2}a^{1/3}\,\text{Log}[x]+\frac{1}{2}a^{1/3}\,\text{Log}\left[a^{1/3}-(a+b x^3)^{1/3}\right]$$

Result (type 5, 61 leaves):

$$\frac{2(a+b x^3)-a\left(1+\frac{a}{b x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{a}{b x^3}\right]}{2(a+b x^3)^{2/3}}$$

■ Problem 515: Result unnecessarily involves higher level functions.

$$\int\frac{(a+b x^3)^{1/3}}{x^4}\,dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$-\frac{(a+b x^3)^{1/3}}{3x^3}-\frac{b\,\text{ArcTan}\left[\frac{a^{1/3}+2(a+b x^3)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{3\sqrt{3}\,a^{2/3}}-\frac{b\,\text{Log}[x]}{6a^{2/3}}+\frac{b\,\text{Log}\left[a^{1/3}-(a+b x^3)^{1/3}\right]}{6a^{2/3}}$$

Result (type 5, 67 leaves):

$$\frac{-2(a+b x^3)-b\left(1+\frac{a}{b x^3}\right)^{2/3}x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{a}{b x^3}\right]}{6x^3(a+b x^3)^{2/3}}$$

■ Problem 516: Result unnecessarily involves higher level functions.

$$\int x^4(a+b x^3)^{1/3}\,dx$$

Optimal (type 3, 120 leaves, 3 steps):

$$\frac{a x^2(a+b x^3)^{1/3}}{18b}+\frac{1}{6}x^5(a+b x^3)^{1/3}+\frac{a^2\,\text{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}\,b^{5/3}}+\frac{a^2\,\text{Log}\left[b^{1/3}x-(a+b x^3)^{1/3}\right]}{18b^{5/3}}$$

Result (type 5, 78 leaves):

$$\frac{x^2\left(a^2+4abx^3+3b^2x^6-a^2\left(1+\frac{b x^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]\right)}{18b(a+b x^3)^{2/3}}$$

■ Problem 517: Result unnecessarily involves higher level functions.

$$\int x(a+b x^3)^{1/3}\,dx$$

Optimal (type 3, 94 leaves, 2 steps):

$$\frac{1}{3} x^2 \left(a+b x^3\right)^{1/3}-\frac{a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}}-\frac{a \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 5, 63 leaves) :

$$\frac{x^2 \left(2 \left(a+b x^3\right)+a \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]\right)}{6 \left(a+b x^3\right)^{2/3}}$$

- **Problem 518: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^2}\, dx$$

Optimal (type 3, 88 leaves, 2 steps) :

$$-\frac{\left(a+b x^3\right)^{1/3}}{x}-\frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2} b^{1/3} \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]$$

Result (type 5, 66 leaves) :

$$\frac{-2 \left(a+b x^3\right)+b x^3 \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{2 x \left(a+b x^3\right)^{2/3}}$$

- **Problem 524: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left(a+b x^3\right)^{1/3} dx$$

Optimal (type 5, 33 leaves, 2 steps) :

$$\frac{x \left(a+b x^3\right)^{4/3} \operatorname{Hypergeometric2F1}\left[1,\frac{5}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{a}$$

Result (type 6, 196 leaves) :

$$\frac{3 \left((-1)^{2/3} a^{1/3}+b^{1/3} x\right) \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{4}{3},-\frac{1}{3},-\frac{1}{3},\frac{7}{3},-\frac{i \left((-1)^{2/3} a^{1/3}+b^{1/3} x\right)}{\sqrt{3}\, a^{1/3}},\frac{i+\sqrt{3}-\frac{2 i b^{1/3} x}{a^{1/3}}}{3 i+\sqrt{3}}\right]}{4\times 2^{1/3} b^{1/3} \left(\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}\right)^{1/3} \left(\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}\right)^{1/3}}$$

Problem 526: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^6} \, dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{4/3} \text{Hypergeometric2F1}\left[-\frac{1}{3},1,-\frac{2}{3},-\frac{b x^3}{a}\right]}{5 a x^5}$$

Result (type 5, 83 leaves):

$$\frac{-2 a^2-3 a b x^3-b^2 x^6-b^2 x^6 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{10 a x^5 \left(a+b x^3\right)^{2/3}}$$

Problem 531: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x} \, dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$\frac{1}{2} \left(a+b x^3\right)^{2/3}+\frac{a^{2/3} \text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3}}-\frac{1}{2} a^{2/3} \text{Log}[x]+\frac{1}{2} a^{2/3} \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]$$

Result (type 5, 58 leaves):

$$\frac{a+b x^3-2 a \left(1+\frac{a}{b x^3}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{a}{b x^3}\right]}{2 \left(a+b x^3\right)^{1/3}}$$

Problem 532: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^4} \, dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{3 x^3}+\frac{2 b \text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{1/3}}-\frac{b \text{Log}[x]}{3 a^{1/3}}+\frac{b \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{3 a^{1/3}}$$

Result (type 5, 67 leaves):

$$\frac{-a-b x^3-2 b \left(1+\frac{a}{b x^3}\right)^{1/3} x^3 \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{a}{b x^3}\right]}{3 x^3 \left(a+b x^3\right)^{1/3}}$$

Problem 533: Result more than twice size of optimal antiderivative.

$$\int x^4 \left(a+b x^3\right)^{2/3} \, dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{x^5 \left(a+b x^3\right)^{5/3} \text{Hypergeometric2F1}\left[1, \frac{10}{3}, \frac{8}{3}, -\frac{b x^3}{a}\right]}{5 a}$$

Result (type 5, 78 leaves):

$$\frac{x^2 \left(a^2+3 a b x^3+2 b^2 x^6-a^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]\right)}{14 b \left(a+b x^3\right)^{1/3}}$$

Problem 536: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^5} \, dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{5/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, -\frac{1}{3}, -\frac{b x^3}{a}\right]}{4 a x^4}$$

Result (type 5, 82 leaves):

$$\frac{-a^2-3 a b x^3-2 b^2 x^6+b^2 x^6 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]}{4 a x^4 \left(a+b x^3\right)^{1/3}}$$

Problem 538: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(a+b x^3\right)^{2/3} \, dx$$

Optimal (type 3, 91 leaves, 2 steps):

$$\frac{1}{3} x \left(a+b x^3\right)^{2/3}+\frac{2 a \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{1/3}}-\frac{a \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{3 b^{1/3}}$$

Result (type 6, 196 leaves):

$$\frac{3 \left((-1)^{2/3} a^{1/3}+b^{1/3} x\right) \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{5}{3}, -\frac{2}{3}, -\frac{2}{3}, \frac{8}{3}, -\frac{i \left((-1)^{2/3} a^{1/3}+b^{1/3} x\right)}{\sqrt{3} a^{1/3}}, \frac{i+\sqrt{3}-\frac{2 i b^{1/3} x}{a^{1/3}}}{3 i+\sqrt{3}}\right]}{5 \times 2^{2/3} b^{1/3} \left(\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}\right)^{2/3} \left(\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}\right)^{2/3}}$$

Problem 548: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(a+b x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 83 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{1/3}}-\frac{\text{Log}[x]}{2\,a^{1/3}}+\frac{\text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2\,a^{1/3}}$$

Result (type 5, 46 leaves):

$$-\frac{\left(1+\frac{a}{b x^3}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{a}{b x^3}\right]}{\left(a+b x^3\right)^{1/3}}$$

Problem 549: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4\left(a+b x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{3\,a\,x^3}-\frac{b\,\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{3\sqrt{3}\,a^{4/3}}+\frac{b\,\text{Log}[x]}{6\,a^{4/3}}-\frac{b\,\text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{6\,a^{4/3}}$$

Result (type 5, 69 leaves):

$$\frac{-a-b x^3+b\left(1+\frac{a}{b x^3}\right)^{1/3} x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{a}{b x^3}\right]}{3\,a\,x^3\left(a+b x^3\right)^{1/3}}$$

Problem 550: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7}{\left(a+b x^3\right)^{1/3}}\,dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$\frac{x^8\left(a+b x^3\right)^{2/3}\,\text{Hypergeometric2F1}\left[1,\frac{10}{3},\frac{11}{3},-\frac{b x^3}{a}\right]}{8\,a}$$

Result (type 5, 80 leaves):

$$\frac{x^2\left(-5 a^2-a b x^3+4 b^2 x^6+5 a^2\left(1+\frac{b x^3}{a}\right)^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]\right)}{28\,b^2\left(a+b x^3\right)^{1/3}}$$

Problem 554: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^3\right)^{1/3}} \, dx$$

Optimal (type 5, 38 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{2}{3},1,-\frac{1}{3},-\frac{b x^3}{a}\right]}{4 a x^4}$$

Result (type 5, 82 leaves):

$$\frac{-a^2+a b x^3+2 b^2 x^6-b^2 x^6 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{4 a^2 x^4 \left(a+b x^3\right)^{1/3}}$$

Problem 565: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x \left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 84 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3}}-\frac{\text{Log}[x]}{2 a^{2/3}}+\frac{\text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2 a^{2/3}}$$

Result (type 5, 48 leaves):

$$-\frac{\left(1+\frac{a}{b x^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{a}{b x^3}\right]}{2 \left(a+b x^3\right)^{2/3}}$$

Problem 566: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 \left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{3 a x^3}+\frac{2 b \text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3}}+\frac{b \text{Log}[x]}{3 a^{5/3}}-\frac{b \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{3 a^{5/3}}$$

Result (type 5, 69 leaves):

$$\frac{-a-b x^3+b \left(1+\frac{a}{b x^3}\right)^{2/3} x^3 \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{a}{b x^3}\right]}{3 a x^3 \left(a+b x^3\right)^{2/3}}$$

Problem 567: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$-\frac{5 a x^2 \left(a+b x^3\right)^{1/3}}{18 b^2}+\frac{x^5 \left(a+b x^3\right)^{1/3}}{6 b}-\frac{5 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}}-\frac{5 a^2 \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{8/3}}$$

Result (type 5, 80 leaves):

$$\frac{x^2 \left(-5 a^2-2 a b x^3+3 b^2 x^6+5 a^2 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]\right)}{18 b^2 \left(a+b x^3\right)^{2/3}}$$

Problem 568: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 97 leaves, 2 steps):

$$\frac{x^2 \left(a+b x^3\right)^{1/3}}{3 b}+\frac{2 a \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3}}+\frac{a \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{3 b^{5/3}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(a+b x^3-a \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]\right)}{3 b \left(a+b x^3\right)^{2/3}}$$

Problem 569: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 72 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}}-\frac{\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3}}$$

Result (type 5, 52 leaves):

$$\frac{x^2 \left(\frac{a+b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]}{2 \left(a+b x^3\right)^{2/3}}$$

- **Problem 574: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^6}{\left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 5, 38 leaves, 2 steps) :

$$\frac{x^7 \left(a+b x^3\right)^{1/3} \text{Hypergeometric2F1}\left[1, \frac{8}{3}, \frac{10}{3}, -\frac{b x^3}{a}\right]}{7 a}$$

Result (type 5, 78 leaves) :

$$\frac{-2 a^2 x - a b x^4 + b^2 x^7 + 2 a^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{5 b^2 \left(a+b x^3\right)^{2/3}}$$

- **Problem 576: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 5, 33 leaves, 2 steps) :

$$\frac{x \left(a+b x^3\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}\right]}{a}$$

Result (type 5, 177 leaves) :

$$\frac{1}{b^{1/3} \left(a+b x^3\right)^{2/3}} 3 \times 2^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x\right) \left(\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}\right)^{2/3}$$
$$\left(\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i + \sqrt{3}}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{\left(1+i \sqrt{3}\right) a^{1/3} + \left(1-i \sqrt{3}\right) b^{1/3} x}{2 \left(a^{1/3}+b^{1/3} x\right)}\right]$$

- **Problem 578: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^6 \left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 5, 38 leaves, 2 steps) :

$$-\frac{\left(a+b x^3\right)^{1/3} \text{Hypergeometric2F1}\left[-\frac{4}{3}, 1, -\frac{2}{3}, -\frac{b x^3}{a}\right]}{5 a x^5}$$

Result (type 5, 82 leaves) :

$$\frac{-a^2+a\,b\,x^3+2\,b^2\,x^6+2\,b^2\,x^6\left(1+\frac{b\,x^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{5\,a^2\,x^5\left(a+b\,x^3\right)^{2/3}}$$

- **Problem 581: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\left(1-x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 53 leaves, 1 step) :

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2}\,\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 5, 20 leaves) :

$$\frac{1}{2}\,x^2\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},x^3\right]$$

- **Problem 635: Result more than twice size of optimal antiderivative.**

$$\int x^3\left(a+b\,x^4\right)^3\,dx$$

Optimal (type 1, 16 leaves, 1 step) :

$$\frac{\left(a+b\,x^4\right)^4}{16\,b}$$

Result (type 1, 43 leaves) :

$$\frac{a^3\,x^4}{4}+\frac{3}{8}\,a^2\,b\,x^8+\frac{1}{4}\,a\,b^2\,x^{12}+\frac{b^3\,x^{16}}{16}$$

- **Problem 776: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4\,\sqrt{a+c\,x^4}\,dx$$

Optimal (type 4, 127 leaves, 3 steps) :

$$\frac{2\,a\,x\,\sqrt{a+c\,x^4}}{21\,c}+\frac{1}{7}\,x^5\,\sqrt{a+c\,x^4}-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{21\,c^{5/4}\,\sqrt{a+c\,x^4}}$$

Result (type 4, 106 leaves) :

$$\frac{2 a^2 x + 5 a c x^5 + 3 c^2 x^9 + \frac{2 i a^2 \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{21 c \sqrt{a+c x^4}}$$

Problem 777: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+c x^4}\, dx$$

Optimal (type 4, 105 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{a+c x^4} + \frac{a^{3/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 89 leaves):

$$\frac{x \left(a+c x^4\right) - \frac{2 i a \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{3 \sqrt{a+c x^4}}$$

Problem 778: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+c x^4}}{x^4}\, dx$$

Optimal (type 4, 107 leaves, 2 steps):

$$-\frac{\sqrt{a+c x^4}}{3 x^3} + \frac{c^{3/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 a^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 92 leaves):

$$\frac{-\frac{a+c x^4}{x^3}-\frac{2 i c \sqrt{1+\frac{c x^4}{a}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{3 \sqrt{a+c x^4}}$$

Problem 779: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+c x^4}}{x^8}\, dx$$

Optimal (type 4, 129 leaves, 3 steps):

$$-\frac{\sqrt{a+c x^4}}{7 x^7}-\frac{2 c \sqrt{a+c x^4}}{21 a x^3}-\frac{c^{7/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{21 a^{5/4} \sqrt{a+c x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-\frac{3 a^2}{x^7}-\frac{5 a c}{x^3}-2 c^2 x+\frac{2 i c^2 \sqrt{1+\frac{c x^4}{a}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{21 a \sqrt{a+c x^4}}$$

Problem 780: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{a+c x^4}\, dx$$

Optimal (type 4, 234 leaves, 4 steps):

$$\frac{1}{5} x^3 \sqrt{a+c x^4}+\frac{2 a x \sqrt{a+c x^4}}{5 \sqrt{c} \left(\sqrt{a}+\sqrt{c}\, x^2\right)}-\frac{2 a^{5/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 c^{3/4} \sqrt{a+c x^4}}+$$

$$\frac{a^{5/4} \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 c^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 121 leaves):

$$\frac{x^3\left(a+c x^4\right)+\frac{2 i a \sqrt{1+\frac{c x^4}{a}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)}{\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2}}}{5 \sqrt{a+c x^4}}$$

Problem 781: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+c x^4}}{x^2}\, dx$$

Optimal (type 4, 224 leaves, 4 steps):

$$-\frac{\sqrt{a+c x^4}}{x}+\frac{2 \sqrt{c}\, x \sqrt{a+c x^4}}{\sqrt{a}+\sqrt{c}\, x^2}-\frac{2 a^{1/4} c^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{a+c x^4}}+$$

$$\frac{a^{1/4} c^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{\sqrt{a+c x^4}}$$

Result (type 4, 119 leaves):

$$\frac{-\frac{a+c x^4}{x}+\frac{2 i c \sqrt{1+\frac{c x^4}{a}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)}{\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2}}}{\sqrt{a+c x^4}}$$

Problem 782: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+c x^4}}{x^6}\, dx$$

Optimal (type 4, 258 leaves, 5 steps):

$$-\frac{\sqrt{a+c x^4}}{5 x^5}-\frac{2 c \sqrt{a+c x^4}}{5 a x}+\frac{2 c^{3/2} x \sqrt{a+c x^4}}{5 a \left(\sqrt{a}+\sqrt{c} x^2\right)}-\frac{2 c^{5/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{3/4} \sqrt{a+c x^4}}+$$

$$\frac{c^{5/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 133 leaves) :

$$\frac{1}{5 \sqrt{a+c x^4}}$$

$$\left(-\frac{\left(a+c x^4\right) \left(a+2 c x^4\right)}{a x^5}-2 i \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{1+\frac{c x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]\right)\right)$$

- **Problem 796: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4 \left(a+c x^4\right)^{3/2} dx$$

Optimal (type 4, 148 leaves, 4 steps) :

$$\frac{4 a^2 x \sqrt{a+c x^4}}{77 c}+\frac{6}{77} a x^5 \sqrt{a+c x^4}+\frac{1}{11} x^5 \left(a+c x^4\right)^{3/2}-\frac{2 a^{11/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{77 c^{5/4} \sqrt{a+c x^4}}$$

Result (type 4, 117 leaves) :

$$\frac{4 a^3 x+17 a^2 c x^5+20 a c^2 x^9+7 c^3 x^{13}+\frac{4 i a^3 \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{77 c \sqrt{a+c x^4}}$$

- **Problem 797: Result unnecessarily involves imaginary or complex numbers.**

$$\int \left(a+c x^4\right)^{3/2} dx$$

Optimal (type 4, 122 leaves, 3 steps) :

$$\frac{2}{7} a x \sqrt{a+c x^4}+\frac{1}{7} x \left(a+c x^4\right)^{3/2}+\frac{2 a^{7/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{7 c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 102 leaves) :

$$\frac{3 a^2 x+4 a c x^5+c^2 x^9-\frac{4 i a^2 \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{7 \sqrt{a+c x^4}}$$

- **Problem 798: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+c x^4\right)^{3/2}}{x^4} \, dx$$

Optimal (type 4, 124 leaves, 3 steps) :

$$\frac{2}{3} c x \sqrt{a+c x^4}-\frac{\left(a+c x^4\right)^{3/2}}{3 x^3}+\frac{2 a^{3/4} c^{3/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{3 \sqrt{a+c x^4}}$$

Result (type 4, 96 leaves) :

$$\frac{-\frac{a^2}{x^3}+c^2 x^5-\frac{4 i a c \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{3 \sqrt{a+c x^4}}$$

- **Problem 799: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+c x^4\right)^{3/2}}{x^8} \, dx$$

Optimal (type 4, 126 leaves, 3 steps) :

$$-\frac{2 c \sqrt{a+c x^4}}{7 x^3}-\frac{\left(a+c x^4\right)^{3/2}}{7 x^7}+\frac{2 c^{7/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{7 a^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-\frac{a^2+4 a c x^4+3 c^2 x^8}{x^7}-\frac{4 i c^2 \sqrt{1+\frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{7 \sqrt{a+c x^4}}$$

Problem 800: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(a+c x^4\right)^{3/2} \, dx$$

Optimal (type 4, 255 leaves, 5 steps):

$$\frac{2}{15} a x^3 \sqrt{a+c x^4}+\frac{4 a^2 x \sqrt{a+c x^4}}{15 \sqrt{c} \left(\sqrt{a}+\sqrt{c} x^2\right)}+\frac{1}{9} x^3 \left(a+c x^4\right)^{3/2}-$$

$$\frac{4 a^{9/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 c^{3/4} \sqrt{a+c x^4}}+\frac{2 a^{9/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 c^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 133 leaves):

$$\frac{\left(a+c x^4\right) \left(11 a x^3+5 c x^7\right)+\frac{12 i a^2 \sqrt{1+\frac{c x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right)}{\left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2}}}{45 \sqrt{a+c x^4}}$$

Problem 801: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+c x^4\right)^{3/2}}{x^2} \, dx$$

Optimal (type 4, 251 leaves, 5 steps):

$$\frac{6}{5}\,c\,x^3\sqrt{a+c\,x^4}+\frac{12\,a\,\sqrt{c}\,x\,\sqrt{a+c\,x^4}}{5\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\left(a+c\,x^4\right)^{3/2}}{x}-\frac{12\,a^{5/4}\,c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\sqrt{a+c\,x^4}}+$$

$$\frac{6\,a^{5/4}\,c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\sqrt{a+c\,x^4}}$$

Result (type 4, 136 leaves) :

$$\left(-\frac{a}{x}+\frac{c\,x^3}{5}\right)\sqrt{a+c\,x^4}+\frac{12\,i\,a\,c\sqrt{1+\frac{c\,x^4}{a}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)}{5\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{a+c\,x^4}}$$

■ Problem 802: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+c\,x^4\right)^{3/2}}{x^6}\,dx$$

Optimal (type 4, 252 leaves, 5 steps) :

$$-\frac{6\,c\sqrt{a+c\,x^4}}{5\,x}+\frac{12\,c^{3/2}\,x\sqrt{a+c\,x^4}}{5\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\left(a+c\,x^4\right)^{3/2}}{5\,x^5}-\frac{12\,a^{1/4}\,c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\sqrt{a+c\,x^4}}+$$

$$\frac{6\,a^{1/4}\,c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\sqrt{a+c\,x^4}}$$

Result (type 4, 132 leaves) :

$$\frac{-\frac{\left(a+c\,x^4\right)\left(a+7\,c\,x^4\right)}{x^5}+\frac{12\,i\,c^2\sqrt{1+\frac{c\,x^4}{a}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)}{\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}}}{5\sqrt{a+c\,x^4}}$$

- **Problem 803: Result unnecessarily involves imaginary or complex numbers.**

$$\int \left(1+x^4\right)^{3/2} dx$$

Optimal (type 4, 72 leaves, 3 steps):

$$\frac{2}{7} x \sqrt{1+x^4}+\frac{1}{7} x \left(1+x^4\right)^{3/2}+\frac{2 \left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{2}\right]}{7 \sqrt{1+x^4}}$$

Result (type 4, 55 leaves):

$$\frac{3 x+4 x^5+x^9-4 (-1)^{1/4} \sqrt{1+x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right],-1\right]}{7 \sqrt{1+x^4}}$$

- **Problem 809: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{1+x^4}\, dx$$

Optimal (type 4, 58 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{1+x^4}+\frac{\left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{2}\right]}{3 \sqrt{1+x^4}}$$

Result (type 4, 48 leaves):

$$\frac{x+x^5-2 (-1)^{1/4} \sqrt{1+x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right],-1\right]}{3 \sqrt{1+x^4}}$$

- **Problem 821: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^8}{\sqrt{a+b x^4}} dx$$

Optimal (type 4, 130 leaves, 3 steps):

$$-\frac{5 a x \sqrt{a+b x^4}}{21 b^2}+\frac{x^5 \sqrt{a+b x^4}}{7 b}+\frac{5 a^{7/4} \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{42 b^{9/4} \sqrt{a+b x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-5 a^2 x - 2 a b x^5 + 3 b^2 x^9 - \frac{5 i a^2 \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{21 b^2 \sqrt{a + b x^4}}$$

Problem 822: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a + b x^4}} \, dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\frac{x \sqrt{a + b x^4}}{3 b} - \frac{a^{3/4} \left(\sqrt{a} + \sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{6 b^{5/4} \sqrt{a + b x^4}}$$

Result (type 4, 92 leaves):

$$\frac{x \left(a + b x^4\right) + \frac{i a \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{3 b \sqrt{a + b x^4}}$$

Problem 823: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b x^4}} \, dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(\sqrt{a} + \sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} b^{1/4} \sqrt{a + b x^4}}$$

Result (type 4, 74 leaves):

$$-\frac{i\sqrt{1+\frac{b x^4}{a}}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ \sqrt{a+b x^4}}$$

- **Problem 824: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^4\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 110 leaves, 2 steps):

$$-\frac{\sqrt{a+b x^4}}{3 a x^3}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\ x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2\ \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{6 a^{5/4}\sqrt{a+b x^4}}$$

Result (type 4, 95 leaves):

$$\frac{-\frac{a+b x^4}{x^3}+\frac{i\ b\sqrt{1+\frac{b x^4}{a}}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}}{3 a\sqrt{a+b x^4}}$$

- **Problem 825: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^8\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 132 leaves, 3 steps):

$$-\frac{\sqrt{a+b x^4}}{7 a x^7}+\frac{5 b\sqrt{a+b x^4}}{21 a^2 x^3}+\frac{5 b^{7/4}\left(\sqrt{a}+\sqrt{b}\ x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2\ \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{42 a^{9/4}\sqrt{a+b x^4}}$$

Result (type 4, 106 leaves):

$$\frac{-\frac{3a^2}{x^7}+\frac{2ab}{x^3}+5b^2x-\frac{5\,i\,b^2\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}}{21a^2\sqrt{a+bx^4}}$$

- **Problem 826: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{10}}{\sqrt{a+bx^4}}\,dx$$

Optimal (type 4, 261 leaves, 5 steps):

$$-\frac{7ax^3\sqrt{a+bx^4}}{45b^2}+\frac{x^7\sqrt{a+bx^4}}{9b}+\frac{7a^2x\sqrt{a+bx^4}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-$$

$$\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{15b^{11/4}\sqrt{a+bx^4}}+\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{30b^{11/4}\sqrt{a+bx^4}}$$

Result (type 4, 136 leaves):

$$\frac{\left(a+bx^4\right)\left(-7ax^3+5bx^7\right)+\frac{21\,i\,a^2\sqrt{1+\frac{bx^4}{a}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)}{\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}}}{45b^2\sqrt{a+bx^4}}$$

- **Problem 827: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^6}{\sqrt{a+bx^4}}\,dx$$

Optimal (type 4, 237 leaves, 4 steps):

$$\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3ax\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{5b^{7/4}\sqrt{a+bx^4}}-$$

$$\frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{10b^{7/4}\sqrt{a+bx^4}}$$

Result (type 4, 168 leaves) :

$$\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{1}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\sqrt{a+bx^4}}$$

$$3a^{3/2}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 828: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^2}{\sqrt{a+bx^4}}\,dx$$

Optimal (type 4, 210 leaves, 3 steps) :

$$\frac{x\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{a^{1/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{a+bx^4}}+$$

$$\frac{a^{1/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{2b^{3/4}\sqrt{a+bx^4}}$$

Result (type 4, 104 leaves) :

$$\frac{i \sqrt{1+\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right] - \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right]\right)}{\left(\frac{i \sqrt{b}}{\sqrt{a}}\right)^{3/2} \sqrt{a+b x^4}}$$

- **Problem 829: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^2 \sqrt{a+b x^4}} \, dx$$

Optimal (type 4, 232 leaves, 4 steps) :

$$-\frac{\sqrt{a+b x^4}}{a x} + \frac{\sqrt{b}\, x \sqrt{a+b x^4}}{a \left(\sqrt{a}+\sqrt{b}\, x^2\right)} - \frac{b^{1/4} \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4} \sqrt{a+b x^4}} +$$

$$\frac{b^{1/4} \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} \sqrt{a+b x^4}}$$

Result (type 4, 121 leaves) :

$$\frac{1}{\sqrt{a+b x^4}} \left(-\frac{a+b x^4}{a x} - i \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{1+\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right] - \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right]\right)\right)$$

- **Problem 830: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^6 \sqrt{a+b x^4}} \, dx$$

Optimal (type 4, 261 leaves, 5 steps) :

$$-\frac{\sqrt{a+b x^4}}{5 a x^5}+\frac{3 b \sqrt{a+b x^4}}{5 a^2 x}-\frac{3 b^{3/2} x \sqrt{a+b x^4}}{5 a^2 \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{3 b^{5/4} \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 a^{7/4} \sqrt{a+b x^4}}-$$
$$\frac{3 b^{5/4} \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{10 a^{7/4} \sqrt{a+b x^4}}$$

Result (type 4, 135 leaves) :

$$\frac{1}{5 a^2 \sqrt{a+b x^4}}\left(\frac{\left(a+b x^4\right) \left(-a+3 b x^4\right)}{x^5}+3 i a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{1+\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)\right)$$

Problem 841: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{\sqrt{a-b x^4}} \, dx$$

Optimal (type 4, 100 leaves, 4 steps) :

$$-\frac{5 a x \sqrt{a-b x^4}}{21 b^2}-\frac{x^5 \sqrt{a-b x^4}}{7 b}+\frac{5 a^{9/4} \sqrt{1-\frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{21 b^{9/4} \sqrt{a-b x^4}}$$

Result (type 4, 122 leaves) :

$$\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x \left(-5 a^2+2 a b x^4+3 b^2 x^8\right)-5 i a^2 \sqrt{1-\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{21 \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} b^2 \sqrt{a-b x^4}}$$

Problem 842: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a-b x^4}} \, dx$$

Optimal (type 4, 77 leaves, 3 steps) :

$$-\frac{x\sqrt{a-b\,x^4}}{3\,b}+\frac{a^{5/4}\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,-1\right]}{3\,b^{5/4}\sqrt{a-b\,x^4}}$$

Result (type 4, 108 leaves) :

$$\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\;x\left(-a+b\,x^4\right)-\mathrm{i}\,a\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\;x\right],\,-1\right]}{3\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\;b\sqrt{a-b\,x^4}}$$

Problem 843: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{a-b\,x^4}}\,dx$$

Optimal (type 4, 53 leaves, 2 steps) :

$$\frac{a^{1/4}\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,-1\right]}{b^{1/4}\sqrt{a-b\,x^4}}$$

Result (type 4, 72 leaves) :

$$-\frac{\mathrm{i}\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\;x\right],\,-1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{a-b\,x^4}}$$

Problem 844: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^4\sqrt{a-b\,x^4}}\,dx$$

Optimal (type 4, 79 leaves, 3 steps) :

$$-\frac{\sqrt{a-b\,x^4}}{3\,a\,x^3}+\frac{b^{3/4}\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,-1\right]}{3\,a^{3/4}\sqrt{a-b\,x^4}}$$

Result (type 4, 90 leaves) :

$$\frac{-\frac{a}{x^3}+b\,x-\frac{i\,b\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}}{3\,a\,\sqrt{a-b\,x^4}}$$

- **Problem 845: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^8\,\sqrt{a-b\,x^4}}\,dx$$

Optimal (type 4, 102 leaves, 4 steps):

$$-\frac{\sqrt{a-b\,x^4}}{7\,a\,x^7}-\frac{5\,b\,\sqrt{a-b\,x^4}}{21\,a^2\,x^3}+\frac{5\,b^{7/4}\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{21\,a^{7/4}\,\sqrt{a-b\,x^4}}$$

Result (type 4, 104 leaves):

$$\frac{-\frac{3\,a^2}{x^7}-\frac{2\,a\,b}{x^3}+5\,b^2\,x-\frac{5\,i\,b^2\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}}{21\,a^2\,\sqrt{a-b\,x^4}}$$

- **Problem 846: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{10}}{\sqrt{a-b\,x^4}}\,dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$-\frac{7\,a\,x^3\,\sqrt{a-b\,x^4}}{45\,b^2}-\frac{x^7\,\sqrt{a-b\,x^4}}{9\,b}+\frac{7\,a^{11/4}\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{15\,b^{11/4}\,\sqrt{a-b\,x^4}}-\frac{7\,a^{11/4}\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{15\,b^{11/4}\,\sqrt{a-b\,x^4}}$$

Result (type 4, 134 leaves):

$$\frac{\left(-a+b x^4\right) \left(7 a x^3+5 b x^7\right)+\frac{21 i a^2 \sqrt{1-\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)}{\left(-\frac{\sqrt{b}}{\sqrt{a}}\right)^{3/2}}}{45 b^2 \sqrt{a-b x^4}}$$

Problem 847: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{a-b x^4}} \, dx$$

Optimal (type 4, 135 leaves, 7 steps):

$$-\frac{x^3 \sqrt{a-b x^4}}{5 b}+\frac{3 a^{7/4} \sqrt{1-\frac{b x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{5 b^{7/4} \sqrt{a-b x^4}}-\frac{3 a^{7/4} \sqrt{1-\frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{5 b^{7/4} \sqrt{a-b x^4}}$$

Result (type 4, 120 leaves):

$$\frac{-a x^3+b x^7+\frac{3 i a \sqrt{1-\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)}{\left(-\frac{\sqrt{b}}{\sqrt{a}}\right)^{3/2}}}{5 b \sqrt{a-b x^4}}$$

Problem 848: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a-b x^4}} \, dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$\frac{a^{3/4} \sqrt{1-\frac{b x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{b^{3/4} \sqrt{a-b x^4}}-\frac{a^{3/4} \sqrt{1-\frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{b^{3/4} \sqrt{a-b x^4}}$$

Result (type 4, 100 leaves):

$$\frac{i \sqrt{1-\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)}{\left(-\frac{\sqrt{b}}{\sqrt{a}}\right)^{3/2} \sqrt{a-b x^4}}$$

- **Problem 849: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^2 \sqrt{a - b x^4}} \, dx$$

Optimal (type 4, 128 leaves, 7 steps):

$$-\frac{\sqrt{a - b x^4}}{a x} - \frac{b^{1/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{a^{1/4} \sqrt{a - b x^4}} + \frac{b^{1/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{a^{1/4} \sqrt{a - b x^4}}$$

Result (type 4, 115 leaves):

$$\frac{1}{\sqrt{a - b x^4}} \left(-\frac{1}{x} + \frac{b x^3}{a} - i \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{1 - \frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)\right)$$

- **Problem 850: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^6 \sqrt{a - b x^4}} \, dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$-\frac{\sqrt{a - b x^4}}{5 a x^5} - \frac{3 b \sqrt{a - b x^4}}{5 a^2 x} - \frac{3 b^{5/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{5 a^{5/4} \sqrt{a - b x^4}} + \frac{3 b^{5/4} \sqrt{1 - \frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{5 a^{5/4} \sqrt{a - b x^4}}$$

Result (type 4, 131 leaves):

$$\frac{1}{5 a^2 \sqrt{a - b x^4}} \left(\frac{\left(-a + b x^4\right) \left(a + 3 b x^4\right)}{x^5} - 3 i a \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} b \sqrt{1 - \frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)\right)$$

- **Problem 861: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^{12}}{\left(a + b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 151 leaves, 4 steps):

$$-\frac{x^9}{2 b \sqrt{a+b x^4}}-\frac{15 a x \sqrt{a+b x^4}}{14 b^3}+\frac{9 x^5 \sqrt{a+b x^4}}{14 b^2}+\frac{15 a^{7/4} \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{28 b^{13/4} \sqrt{a+b x^4}}$$

Result (type 4, 106 leaves) :

$$\frac{-15 a^2 x-6 a b x^5+2 b^2 x^9-\frac{15 i a^2 \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{14 b^3 \sqrt{a+b x^4}}$$

■ Problem 862: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8}{\left(a+b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 129 leaves, 3 steps) :

$$-\frac{x^5}{2 b \sqrt{a+b x^4}}+\frac{5 x \sqrt{a+b x^4}}{6 b^2}-\frac{5 a^{3/4} \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{12 b^{9/4} \sqrt{a+b x^4}}$$

Result (type 4, 93 leaves) :

$$\frac{5 a x+2 b x^5+\frac{5 i a \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}}{6 b^2 \sqrt{a+b x^4}}$$

■ Problem 863: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\left(a+b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 108 leaves, 2 steps) :

$$-\frac{x}{2 b \sqrt{a+b x^4}}+\frac{\left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} b^{5/4} \sqrt{a+b x^4}}$$

Result (type 4, 102 leaves) :

$$-\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\ x+i \sqrt{1+\frac{b x^4}{a}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\ x\right], -1\right]}{2 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\ b \sqrt{a+b x^4}}$$

■ Problem 864: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 108 leaves, 2 steps) :

$$\frac{x}{2 a \sqrt{a+b x^4}}+\frac{\left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} b^{1/4} \sqrt{a+b x^4}}$$

Result (type 4, 102 leaves) :

$$\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\ x-i \sqrt{1+\frac{b x^4}{a}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\ x\right], -1\right]}{2 a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\ \sqrt{a+b x^4}}$$

■ Problem 865: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \left(a+b x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 131 leaves, 3 steps) :

$$\frac{1}{2 a x^3 \sqrt{a+b x^4}}-\frac{5 \sqrt{a+b x^4}}{6 a^2 x^3}-\frac{5 b^{3/4} \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{12 a^{9/4} \sqrt{a+b x^4}}$$

Result (type 4, 93 leaves) :

$$\frac{-\frac{2a}{x^3}-5bx+\frac{5\,i\,b\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}}{6a^2\sqrt{a+bx^4}}$$

- **Problem 866: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^8\left(a+bx^4\right)^{3/2}}\,dx$$

Optimal (type 4, 153 leaves, 4 steps) :

$$\frac{1}{2ax^7\sqrt{a+bx^4}}-\frac{9\sqrt{a+bx^4}}{14a^2x^7}+\frac{15b\sqrt{a+bx^4}}{14a^3x^3}+\frac{15b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{28a^{13/4}\sqrt{a+bx^4}}$$

Result (type 4, 106 leaves) :

$$\frac{-\frac{2a^2}{x^7}+\frac{6ab}{x^3}+15b^2x-\frac{15\,i\,b^2\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}}{14a^3\sqrt{a+bx^4}}$$

- **Problem 867: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{14}}{\left(a+bx^4\right)^{3/2}}\,dx$$

Optimal (type 4, 282 leaves, 6 steps) :

$$-\frac{x^{11}}{2b\sqrt{a+bx^4}}-\frac{77ax^3\sqrt{a+bx^4}}{90b^3}+\frac{11x^7\sqrt{a+bx^4}}{18b^2}+\frac{77a^2x\sqrt{a+bx^4}}{30b^{7/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-$$

$$\frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{30b^{15/4}\sqrt{a+bx^4}}+\frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{60b^{15/4}\sqrt{a+bx^4}}$$

Result (type 4, 183 leaves) :

$$\frac{1}{90\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{7/2}\sqrt{a+b x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}\,x^3\left(-77 a^2-22 a b x^4+10 b^2 x^8\right)+\right.$$

$$\left.231\,a^{5/2}\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-231\,a^{5/2}\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 868: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{10}}{\left(a+b x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 258 leaves, 5 steps) :

$$-\frac{x^7}{2 b\sqrt{a+b x^4}}+\frac{7 x^3\sqrt{a+b x^4}}{10 b^2}-\frac{21 a x\sqrt{a+b x^4}}{10 b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{21 a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{10\,b^{11/4}\sqrt{a+b x^4}}-\frac{21 a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{20\,b^{11/4}\sqrt{a+b x^4}}$$

Result (type 4, 172 leaves) :

$$\frac{1}{10\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{5/2}\sqrt{a+b x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}\,x^3\left(7 a+2 b x^4\right)-\right.$$

$$\left.21\,a^{3/2}\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+21\,a^{3/2}\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 869: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{6}}{\left(a+b x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 236 leaves, 4 steps) :

$$-\frac{x^3}{2\,b\,\sqrt{a+b\,x^4}}+\frac{3\,x\,\sqrt{a+b\,x^4}}{2\,b^{3/2}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3\,a^{1/4}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{2\,b^{7/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{3\,a^{1/4}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{4\,b^{7/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 163 leaves) :

$$\frac{1}{2\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}\,x^3+3\,\sqrt{a}\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]-3\,\sqrt{a}\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 870: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2}{\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 239 leaves, 4 steps) :

$$\frac{x^3}{2\,a\,\sqrt{a+b\,x^4}}-\frac{x\,\sqrt{a+b\,x^4}}{2\,a\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{2\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{4\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 163 leaves) :

$$\left(i\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b}\,x^3-\sqrt{a}\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\sqrt{a}\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)\Big/$$

$$\left(2\,a^{3/2}\left(\frac{i\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+b\,x^4}\right)$$

- **Problem 871: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 260 leaves, 5 steps) :

$$\frac{1}{2\,a\,x\sqrt{a+b\,x^4}}-\frac{3\sqrt{a+b\,x^4}}{2\,a^2\,x}+\frac{3\sqrt{b}\,x\sqrt{a+b\,x^4}}{2\,a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3\,b^{1/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{7/4}\sqrt{a+b\,x^4}}+$$

$$\frac{3\,b^{1/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{7/4}\sqrt{a+b\,x^4}}$$

Result (type 4, 178 leaves) :

$$\frac{1}{2\,a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\sqrt{a+b\,x^4}}\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(2\,a+3\,b\,x^4\right)+\right.$$

$$\left.3\sqrt{a}\sqrt{b}\,x\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-3\sqrt{a}\sqrt{b}\,x\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 872: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^6\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 282 leaves, 6 steps) :

$$\frac{1}{2\,a\,x^5\sqrt{a+b\,x^4}}-\frac{7\sqrt{a+b\,x^4}}{10\,a^2\,x^5}+\frac{21\,b\sqrt{a+b\,x^4}}{10\,a^3\,x}-\frac{21\,b^{3/2}\,x\sqrt{a+b\,x^4}}{10\,a^3\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{21\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{10\,a^{11/4}\sqrt{a+b\,x^4}}-\frac{21\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{20\,a^{11/4}\sqrt{a+b\,x^4}}$$

Result (type 4, 192 leaves) :

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-2\,a^2+14\,a\,b\,x^4+21\,b^2\,x^8\right)-21\sqrt{a}\,b^{3/2}\,x^5\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\right.$$

$$\left.21\sqrt{a}\,b^{3/2}\,x^5\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Bigg/\left(10\,a^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^5\sqrt{a+b\,x^4}\right)$$

- **Problem 873: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(a+b\,x^4\right)^{5/2}}\,dx$$

Optimal (type 4, 127 leaves, 3 steps) :

$$\frac{x}{6\,a\left(a+b\,x^4\right)^{3/2}}+\frac{5\,x}{12\,a^2\sqrt{a+b\,x^4}}+\frac{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{24\,a^{9/4}\,b^{1/4}\sqrt{a+b\,x^4}}$$

Result (type 4, 99 leaves) :

$$\frac{7\,a\,x+5\,b\,x^5-\frac{5\,i\left(a+b\,x^4\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}}{12\,a^2\left(a+b\,x^4\right)^{3/2}}$$

- **Problem 927: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^8}{\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 74 leaves, 3 steps):

$$-\frac{5}{21}x\sqrt{1+x^4}+\frac{1}{7}x^5\sqrt{1+x^4}+\frac{5\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{42\sqrt{1+x^4}}$$

Result (type 4, 57 leaves):

$$-\frac{5x+2x^5-3x^9+5(-1)^{1/4}\sqrt{1+x^4}\ \text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]}{21\sqrt{1+x^4}}$$

- **Problem 928: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4}{\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 58 leaves, 2 steps):

$$\frac{1}{3}x\sqrt{1+x^4}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{6\sqrt{1+x^4}}$$

Result (type 4, 47 leaves):

$$\frac{x+x^5+(-1)^{1/4}\sqrt{1+x^4}\ \text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]}{3\sqrt{1+x^4}}$$

- **Problem 929: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 43 leaves, 1 step):

$$\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{1+x^4}}$$

Result (type 4, 21 leaves):

$$-(-1)^{1/4}\,\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]$$

Problem 930: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{1+x^4}} \, dx$$

Optimal (type 4, 60 leaves, 2 steps):

$$-\frac{\sqrt{1+x^4}}{3 x^3}-\frac{\left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{6 \sqrt{1+x^4}}$$

Result (type 4, 55 leaves):

$$\frac{-1-x^4+(-1)^{1/4} x^3 \sqrt{1+x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right],-1\right]}{3 x^3 \sqrt{1+x^4}}$$

Problem 931: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^8 \sqrt{1+x^4}} \, dx$$

Optimal (type 4, 76 leaves, 3 steps):

$$-\frac{\sqrt{1+x^4}}{7 x^7}+\frac{5 \sqrt{1+x^4}}{21 x^3}+\frac{5 \left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{42 \sqrt{1+x^4}}$$

Result (type 4, 61 leaves):

$$\frac{-3+2 x^4+5 x^8-5 (-1)^{1/4} x^7 \sqrt{1+x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right],-1\right]}{21 x^7 \sqrt{1+x^4}}$$

Problem 932: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}}{\sqrt{1+x^4}} \, dx$$

Optimal (type 4, 140 leaves, 5 steps):

$$-\frac{7}{45} x^3 \sqrt{1+x^4}+\frac{1}{9} x^7 \sqrt{1+x^4}+\frac{7 x \sqrt{1+x^4}}{15 \left(1+x^2\right)}-\frac{7 \left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{15 \sqrt{1+x^4}}+\frac{7 \left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{30 \sqrt{1+x^4}}$$

Result (type 4, 72 leaves):

$$\frac{1}{45}\left(\frac{x^3\left(-7-2x^4+5x^8\right)}{\sqrt{1+x^4}}-21\,(-1)^{3/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]+21\,(-1)^{3/4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

- **Problem 933: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^6}{\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 124 leaves, 4 steps):

$$\frac{1}{5}x^3\sqrt{1+x^4}-\frac{3x\sqrt{1+x^4}}{5\left(1+x^2\right)}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{1+x^4}}-\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{10\sqrt{1+x^4}}$$

Result (type 4, 73 leaves):

$$\frac{1}{5}\left(3\,(-1)^{3/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]+\frac{x^3+x^7-3\,(-1)^{3/4}\sqrt{1+x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]}{\sqrt{1+x^4}}\right)$$

- **Problem 934: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^2}{\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 103 leaves, 3 steps):

$$\frac{x\sqrt{1+x^4}}{1+x^2}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{1+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{1+x^4}}$$

Result (type 4, 37 leaves):

$$(-1)^{3/4}\left(-\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]+\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

- **Problem 935: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^2\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 117 leaves, 4 steps):

$$-\frac{\sqrt{1+x^4}}{x}+\frac{x\sqrt{1+x^4}}{1+x^2}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{1+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{1+x^4}}$$

Result (type 4, 70 leaves) :

$$-\frac{1}{x\sqrt{1+x^4}}-\frac{x^3}{\sqrt{1+x^4}}-(-1)^{3/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]+(-1)^{3/4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]$$

- **Problem 936: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{x^6\sqrt{1+x^4}}\,dx$$

Optimal (type 4, 140 leaves, 5 steps) :

$$-\frac{\sqrt{1+x^4}}{5x^5}+\frac{3\sqrt{1+x^4}}{5x}-\frac{3x\sqrt{1+x^4}}{5\left(1+x^2\right)}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{1+x^4}}-\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{10\sqrt{1+x^4}}$$

Result (type 4, 94 leaves) :

$$\frac{1}{5x^5\sqrt{1+x^4}}\left(-1+2x^4+3x^8+3(-1)^{3/4}x^5\sqrt{1+x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]-3(-1)^{3/4}x^5\sqrt{1+x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

- **Problem 947: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{12}}{\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 90 leaves, 4 steps) :

$$-\frac{x^9}{2\sqrt{1+x^4}}-\frac{15}{14}x\sqrt{1+x^4}+\frac{9}{14}x^5\sqrt{1+x^4}+\frac{15\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{28\sqrt{1+x^4}}$$

Result (type 4, 57 leaves) :

$$-\frac{15x+6x^5-2x^9+15(-1)^{1/4}\sqrt{1+x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]}{14\sqrt{1+x^4}}$$

- **Problem 948: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^8}{\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 74 leaves, 3 steps) :

$$-\frac{x^5}{2\sqrt{1+x^4}}+\frac{5}{6}x\sqrt{1+x^4}-\frac{5\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{12\sqrt{1+x^4}}$$

Result (type 4, 52 leaves) :

$$\frac{5x+2x^5+5(-1)^{1/4}\sqrt{1+x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]}{6\sqrt{1+x^4}}$$

- **Problem 949: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4}{\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 58 leaves, 2 steps) :

$$-\frac{x}{2\sqrt{1+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 38 leaves) :

$$-\frac{x}{2\sqrt{1+x^4}}-\frac{1}{2}(-1)^{1/4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]$$

- **Problem 950: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 58 leaves, 2 steps) :

$$\frac{x}{2\sqrt{1+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 37 leaves) :

$$\frac{1}{2}\left(\frac{x}{\sqrt{1+x^4}}-(-1)^{1/4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

Problem 951: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \left(1+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 76 leaves, 3 steps):

$$\frac{1}{2 x^3 \sqrt{1+x^4}}-\frac{5 \sqrt{1+x^4}}{6 x^3}-\frac{5 \left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{12 \sqrt{1+x^4}}$$

Result (type 4, 46 leaves):

$$\frac{1}{6} \left(\frac{-2-5 x^4}{x^3 \sqrt{1+x^4}}+5 (-1)^{1/4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right], -1\right]\right)$$

Problem 952: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^8 \left(1+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\frac{1}{2 x^7 \sqrt{1+x^4}}-\frac{9 \sqrt{1+x^4}}{14 x^7}+\frac{15 \sqrt{1+x^4}}{14 x^3}+\frac{15 \left(1+x^2\right) \sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{2}\right]}{28 \sqrt{1+x^4}}$$

Result (type 4, 61 leaves):

$$\frac{-2+6 x^4+15 x^8-15 (-1)^{1/4} x^7 \sqrt{1+x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} x\right], -1\right]}{14 x^7 \sqrt{1+x^4}}$$

Problem 953: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{14}}{\left(1+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$-\frac{x^{11}}{2\sqrt{1+x^4}}-\frac{77}{90}x^3\sqrt{1+x^4}+\frac{11}{18}x^7\sqrt{1+x^4}+\frac{77x\sqrt{1+x^4}}{30\left(1+x^2\right)}-$$
$$\frac{77\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{30\sqrt{1+x^4}}+\frac{77\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{60\sqrt{1+x^4}}$$

Result (type 4, 72 leaves) :

$$\frac{1}{90}\left(\frac{x^3\left(-77-22x^4+10x^8\right)}{\sqrt{1+x^4}}-231\,(-1)^{3/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]+231\,(-1)^{3/4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

- **Problem 954: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{10}}{\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 140 leaves, 5 steps) :

$$-\frac{x^7}{2\sqrt{1+x^4}}+\frac{7}{10}x^3\sqrt{1+x^4}-\frac{21x\sqrt{1+x^4}}{10\left(1+x^2\right)}+\frac{21\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{10\sqrt{1+x^4}}-\frac{21\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{20\sqrt{1+x^4}}$$

Result (type 4, 75 leaves) :

$$\frac{1}{10}\left(\frac{7x^3}{\sqrt{1+x^4}}+\frac{2x^7}{\sqrt{1+x^4}}+21\,(-1)^{3/4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]-21\,(-1)^{3/4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

- **Problem 955: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{6}}{\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 124 leaves, 4 steps) :

$$-\frac{x^3}{2\sqrt{1+x^4}}+\frac{3x\sqrt{1+x^4}}{2\left(1+x^2\right)}-\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{1+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 61 leaves) :

$$\frac{1}{2}\left(-\frac{x^3}{\sqrt{1+x^4}}-3\,(-1)^{3/4}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]+3\,(-1)^{3/4}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]\right)$$

- **Problem 956: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2}{\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 124 leaves, 4 steps):

$$\frac{x^3}{2\sqrt{1+x^4}}-\frac{x\sqrt{1+x^4}}{2\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{1+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 59 leaves):

$$\frac{1}{2}\left(\frac{x^3}{\sqrt{1+x^4}}+(-1)^{3/4}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]-(-1)^{3/4}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]\right)$$

- **Problem 957: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^2\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 140 leaves, 5 steps):

$$\frac{1}{2x\sqrt{1+x^4}}-\frac{3\sqrt{1+x^4}}{2x}+\frac{3x\sqrt{1+x^4}}{2\left(1+x^2\right)}-\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{1+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{1+x^4}}$$

Result (type 4, 75 leaves):

$$\frac{1}{2}\left(-\frac{2}{x\sqrt{1+x^4}}-\frac{3x^3}{\sqrt{1+x^4}}-3\,(-1)^{3/4}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]+3\,(-1)^{3/4}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}\,x\right],-1\right]\right)$$

- **Problem 958: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^6\left(1+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 156 leaves, 6 steps):

$$\frac{1}{2x^5\sqrt{1+x^4}}-\frac{7\sqrt{1+x^4}}{10x^5}+\frac{21\sqrt{1+x^4}}{10x}-\frac{21x\sqrt{1+x^4}}{10(1+x^2)}+$$
$$\frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{10\sqrt{1+x^4}}-\frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{20\sqrt{1+x^4}}$$

Result (type 4, 94 leaves) :

$$\frac{1}{10x^5\sqrt{1+x^4}}\left(-2+14x^4+21x^8+21(-1)^{3/4}x^5\sqrt{1+x^4}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]-21(-1)^{3/4}x^5\sqrt{1+x^4}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]\right)$$

- **Problem 959: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{(1+x^4)^{5/2}}\,dx$$

Optimal (type 4, 72 leaves, 3 steps) :

$$\frac{x}{6(1+x^4)^{3/2}}+\frac{5x}{12\sqrt{1+x^4}}+\frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{2}\right]}{24\sqrt{1+x^4}}$$

Result (type 4, 52 leaves) :

$$\frac{7x+5x^5-5(-1)^{1/4}(1+x^4)^{3/2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{1/4}x\right],-1\right]}{12(1+x^4)^{3/2}}$$

- **Problem 974: Result more than twice size of optimal antiderivative.**

$$\int\frac{x}{\sqrt{-4+x^4}}\,dx$$

Optimal (type 3, 18 leaves, 3 steps) :

$$\frac{1}{2}\,\text{ArcTanh}\left[\frac{x^2}{\sqrt{-4+x^4}}\right]$$

Result (type 3, 42 leaves) :

$$-\frac{1}{4}\,\text{Log}\left[1-\frac{x^2}{\sqrt{-4+x^4}}\right]+\frac{1}{4}\,\text{Log}\left[1+\frac{x^2}{\sqrt{-4+x^4}}\right]$$

Problem 984: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{3-b\,x^4}}\,dx$$

Optimal (type 4, 54 leaves, 4 steps):

$$\frac{3^{1/4}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{3^{1/4}}\right],-1\right]}{b^{3/4}}-\frac{3^{1/4}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{3^{1/4}}\right],-1\right]}{b^{3/4}}$$

Result (type 4, 76 leaves):

$$\frac{i\,3^{1/4}\,\sqrt{-\sqrt{b}}\,\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b}}\,x}{3^{1/4}}\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{-\sqrt{b}}\,x}{3^{1/4}}\right],-1\right]\right)}{b}$$

Problem 993: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^4\right)^{1/4}}{x}\,dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$\left(a+b\,x^4\right)^{1/4}-\frac{1}{2}\,a^{1/4}\,\text{ArcTan}\left[\frac{\left(a+b\,x^4\right)^{1/4}}{a^{1/4}}\right]-\frac{1}{2}\,a^{1/4}\,\text{ArcTanh}\left[\frac{\left(a+b\,x^4\right)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 61 leaves):

$$\frac{3\,\left(a+b\,x^4\right)-a\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{a}{b\,x^4}\right]}{3\,\left(a+b\,x^4\right)^{3/4}}$$

Problem 994: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^4\right)^{1/4}}{x^5}\,dx$$

Optimal (type 3, 75 leaves, 6 steps):

$$-\frac{\left(a+b\,x^4\right)^{1/4}}{4\,x^4}-\frac{b\,\text{ArcTan}\left[\frac{\left(a+b\,x^4\right)^{1/4}}{a^{1/4}}\right]}{8\,a^{3/4}}-\frac{b\,\text{ArcTanh}\left[\frac{\left(a+b\,x^4\right)^{1/4}}{a^{1/4}}\right]}{8\,a^{3/4}}$$

Result (type 5, 67 leaves):

$$\frac{-3\left(a+b x^4\right)-b\left(1+\frac{a}{b x^4}\right)^{3/4} x^4\, \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{a}{b x^4}\right]}{12 x^4\left(a+b x^4\right)^{3/4}}$$

Problem 995: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^9}\, dx$$

Optimal (type 3, 101 leaves, 7 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{8 x^8}-\frac{b\left(a+b x^4\right)^{1/4}}{32 a x^4}+\frac{3 b^2 \text{ArcTan}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{7/4}}+\frac{3 b^2 \text{ArcTanh}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{7/4}}$$

Result (type 5, 82 leaves):

$$\frac{-4 a^2-5 a b x^4-b^2 x^8+b^2\left(1+\frac{a}{b x^4}\right)^{3/4} x^8\, \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{a}{b x^4}\right]}{32 a x^8\left(a+b x^4\right)^{3/4}}$$

Problem 996: Result unnecessarily involves higher level functions.

$$\int x^9\left(a+b x^4\right)^{1/4} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$-\frac{2 a^2 x^2\left(a+b x^4\right)^{1/4}}{77 b^2}+\frac{a x^6\left(a+b x^4\right)^{1/4}}{77 b}+\frac{1}{11} x^{10}\left(a+b x^4\right)^{1/4}+\frac{4 a^{7/2}\left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{77 b^{5/2}\left(a+b x^4\right)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{x^2\left(-2 a^3-a^2 b x^4+8 a b^2 x^8+7 b^3 x^{12}+2 a^3\left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{77 b^2\left(a+b x^4\right)^{3/4}}$$

Problem 997: Result unnecessarily involves higher level functions.

$$\int x^5\left(a+b x^4\right)^{1/4} dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$\frac{a x^2\left(a+b x^4\right)^{1/4}}{21 b}+\frac{1}{7} x^6\left(a+b x^4\right)^{1/4}-\frac{2 a^{5/2}\left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{21 b^{3/2}\left(a+b x^4\right)^{3/4}}$$

Result (type 5, 78 leaves):

$$\frac{x^2 \left(a^2+4 a b x^4+3 b^2 x^8-a^2 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{21 b \left(a+b x^4\right)^{3/4}}$$

Problem 998: Result unnecessarily involves higher level functions.

$$\int x \left(a+b x^4\right)^{1/4} \, dx$$

Optimal (type 4, 79 leaves, 4 steps):

$$\frac{1}{3} x^2 \left(a+b x^4\right)^{1/4}+\frac{a^{3/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{3 \sqrt{b} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 63 leaves):

$$\frac{x^2 \left(2 \left(a+b x^4\right)+a \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{6 \left(a+b x^4\right)^{3/4}}$$

Problem 999: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^3} \, dx$$

Optimal (type 4, 79 leaves, 4 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{2 x^2}+\frac{\sqrt{a} \sqrt{b} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 66 leaves):

$$\frac{-2 \left(a+b x^4\right)+b x^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{4 x^2 \left(a+b x^4\right)^{3/4}}$$

Problem 1000: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^7} \, dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{6 x^6}-\frac{b \left(a+b x^4\right)^{1/4}}{12 a x^2}-\frac{b^{3/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{12 \sqrt{a} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 85 leaves):

$$-\frac{2\left(2 a^2+3 a b x^4+b^2 x^8\right)-b^2 x^8\left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{24 a x^6\left(a+b x^4\right)^{3/4}}$$

- **Problem 1001: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^{11}} \, dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{10 x^{10}}-\frac{b\left(a+b x^4\right)^{1/4}}{60 a x^6}+\frac{b^2\left(a+b x^4\right)^{1/4}}{24 a^2 x^2}+\frac{b^{5/2}\left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{24 a^{3/2}\left(a+b x^4\right)^{3/4}}$$

Result (type 5, 94 leaves):

$$\frac{-24 a^3-28 a^2 b x^4+6 a b^2 x^8+10 b^3 x^{12}+5 b^3 x^{12}\left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{240 a^2 x^{10}\left(a+b x^4\right)^{3/4}}$$

- **Problem 1002: Result unnecessarily involves higher level functions.**

$$\int x^6\left(a+b x^4\right)^{1/4} \, dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$\frac{a x^3\left(a+b x^4\right)^{1/4}}{32 b}+\frac{1}{8} x^7\left(a+b x^4\right)^{1/4}+\frac{3 a^2 \text{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{64 b^{7/4}}-\frac{3 a^2 \text{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{64 b^{7/4}}$$

Result (type 5, 78 leaves):

$$\frac{x^3\left(a^2+5 a b x^4+4 b^2 x^8-a^2\left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{32 b\left(a+b x^4\right)^{3/4}}$$

- **Problem 1003: Result unnecessarily involves higher level functions.**

$$\int x^2\left(a+b x^4\right)^{1/4} \, dx$$

Optimal (type 3, 77 leaves, 5 steps):

$$\frac{1}{4} x^3\left(a+b x^4\right)^{1/4}-\frac{a \text{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{8 b^{3/4}}+\frac{a \text{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{8 b^{3/4}}$$

Result (type 5, 63 leaves):

$$\frac{x^3 \left(3 \left(a+b x^4\right)+a \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{12 \left(a+b x^4\right)^{3/4}}$$

Problem 1004: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^2} \, dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{x}-\frac{1}{2} b^{1/4} \text{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]+\frac{1}{2} b^{1/4} \text{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]$$

Result (type 5, 66 leaves):

$$\frac{-3 \left(a+b x^4\right)+b x^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{3 x \left(a+b x^4\right)^{3/4}}$$

Problem 1009: Result unnecessarily involves higher level functions.

$$\int x^{12} \left(a+b x^4\right)^{1/4} \, dx$$

Optimal (type 4, 150 leaves, 8 steps):

$$\frac{3 a^3 x \left(a+b x^4\right)^{1/4}}{112 b^3}-\frac{3 a^2 x^5 \left(a+b x^4\right)^{1/4}}{280 b^2}+\frac{a x^9 \left(a+b x^4\right)^{1/4}}{140 b}+\frac{1}{14} x^{13} \left(a+b x^4\right)^{1/4}+\frac{3 a^{7/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{112 b^{5/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 101 leaves):

$$\frac{1}{560 b^3 \left(a+b x^4\right)^{3/4}} \left(15 a^4 x+9 a^3 b x^5-2 a^2 b^2 x^9+44 a b^3 x^{13}+40 b^4 x^{17}-15 a^4 x \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]\right)$$

Problem 1010: Result unnecessarily involves higher level functions.

$$\int x^8 \left(a+b x^4\right)^{1/4} \, dx$$

Optimal (type 4, 126 leaves, 7 steps):

$$-\frac{a^2 x \left(a+b x^4\right)^{1/4}}{24 b^2}+\frac{a x^5 \left(a+b x^4\right)^{1/4}}{60 b}+\frac{1}{10} x^9 \left(a+b x^4\right)^{1/4}-\frac{a^{5/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{24 b^{3/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 90 leaves):

$$\frac{-5\,a^3\,x-3\,a^2\,b\,x^5+14\,a\,b^2\,x^9+12\,b^3\,x^{13}+5\,a^3\,x\,\left(1+\frac{b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]}{120\,b^2\,\left(a+b\,x^4\right)^{3/4}}$$

Problem 1011: Result unnecessarily involves higher level functions.

$$\int x^4\,\left(a+b\,x^4\right)^{1/4}\,dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{a\,x\,\left(a+b\,x^4\right)^{1/4}}{12\,b}+\frac{1}{6}\,x^5\,\left(a+b\,x^4\right)^{1/4}+\frac{a^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{12\,\sqrt{b}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 76 leaves):

$$\frac{x\,\left(a^2+3\,a\,b\,x^4+2\,b^2\,x^8-a^2\,\left(1+\frac{b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]\right)}{12\,b\,\left(a+b\,x^4\right)^{3/4}}$$

Problem 1012: Result unnecessarily involves higher level functions.

$$\int\left(a+b\,x^4\right)^{1/4}\,dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$\frac{1}{2}\,x\,\left(a+b\,x^4\right)^{1/4}-\frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{2\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 58 leaves):

$$\frac{x\,\left(a+b\,x^4+a\,\left(1+\frac{b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]\right)}{2\,\left(a+b\,x^4\right)^{3/4}}$$

Problem 1013: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^4\right)^{1/4}}{x^4}\,dx$$

Optimal (type 4, 82 leaves, 5 steps):

$$-\frac{\left(a+b\,x^4\right)^{1/4}}{3\,x^3}-\frac{b^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{3\,\sqrt{a}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 66 leaves):

$$\frac{-a-b x^4+b x^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]}{3 x^3 \left(a+b x^4\right)^{3/4}}$$

Problem 1014: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^8} \, dx$$

Optimal (type 4, 104 leaves, 6 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{7 x^7}-\frac{b \left(a+b x^4\right)^{1/4}}{21 a x^3}+\frac{2 b^{5/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{21 a^{3/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 83 leaves):

$$\frac{-3 a^2-4 a b x^4-b^2 x^8-2 b^2 x^8 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]}{21 a x^7 \left(a+b x^4\right)^{3/4}}$$

Problem 1015: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^{12}} \, dx$$

Optimal (type 4, 128 leaves, 7 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{11 x^{11}}-\frac{b \left(a+b x^4\right)^{1/4}}{77 a x^7}+\frac{2 b^2 \left(a+b x^4\right)^{1/4}}{77 a^2 x^3}-\frac{4 b^{7/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{77 a^{5/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 93 leaves):

$$\frac{-7 a^3-8 a^2 b x^4+a b^2 x^8+2 b^3 x^{12}+4 b^3 x^{12} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]}{77 a^2 x^{11} \left(a+b x^4\right)^{3/4}}$$

Problem 1016: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{x^{16}} \, dx$$

Optimal (type 4, 152 leaves, 8 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{15 x^{15}}-\frac{b \left(a+b x^4\right)^{1/4}}{165 a x^{11}}+\frac{2 b^2 \left(a+b x^4\right)^{1/4}}{231 a^2 x^7}-\frac{4 b^3 \left(a+b x^4\right)^{1/4}}{231 a^3 x^3}+\frac{8 b^{9/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{231 a^{7/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 105 leaves) :

$$\frac{1}{1155\, a^3\, x^{15}\, \left(a+b\, x^4\right)^{3/4}}\left(-77\, a^4-84\, a^3\, b\, x^4+3\, a^2\, b^2\, x^8-10\, a\, b^3\, x^{12}-20\, b^4\, x^{16}-40\, b^4\, x^{16}\, \left(1+\frac{b\, x^4}{a}\right)^{3/4}\, \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b\, x^4}{a}\right]\right)$$

- **Problem 1022: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b\, x^4\right)^{3/4}}{x}\, dx$$

Optimal (type 3, 70 leaves, 6 steps) :

$$\frac{1}{3}\, \left(a+b\, x^4\right)^{3/4}+\frac{1}{2}\, a^{3/4}\, \text{ArcTan}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]-\frac{1}{2}\, a^{3/4}\, \text{ArcTanh}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 58 leaves) :

$$\frac{a+b\, x^4-3\, a\, \left(1+\frac{a}{b\, x^4}\right)^{1/4}\, \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b\, x^4}\right]}{3\, \left(a+b\, x^4\right)^{1/4}}$$

- **Problem 1023: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b\, x^4\right)^{3/4}}{x^5}\, dx$$

Optimal (type 3, 75 leaves, 6 steps) :

$$-\frac{\left(a+b\, x^4\right)^{3/4}}{4\, x^4}+\frac{3\, b\, \text{ArcTan}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{8\, a^{1/4}}-\frac{3\, b\, \text{ArcTanh}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{8\, a^{1/4}}$$

Result (type 5, 67 leaves) :

$$\frac{-a-b\, x^4-3\, b\, \left(1+\frac{a}{b\, x^4}\right)^{1/4}\, x^4\, \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b\, x^4}\right]}{4\, x^4\, \left(a+b\, x^4\right)^{1/4}}$$

- **Problem 1024: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b\, x^4\right)^{3/4}}{x^9}\, dx$$

Optimal (type 3, 101 leaves, 7 steps) :

$$-\frac{\left(a+b\, x^4\right)^{3/4}}{8\, x^8}-\frac{3\, b\, \left(a+b\, x^4\right)^{3/4}}{32\, a\, x^4}-\frac{3\, b^2\, \text{ArcTan}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{64\, a^{5/4}}+\frac{3\, b^2\, \text{ArcTanh}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{64\, a^{5/4}}$$

Result (type 5, 83 leaves) :

$$\frac{-4 a^2-7 a b x^4-3 b^2 x^8+3 b^2 \left(1+\frac{a}{b x^4}\right)^{1/4} x^8 \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b x^4}\right]}{32 a x^8 \left(a+b x^4\right)^{1/4}}$$

Problem 1025: Result unnecessarily involves higher level functions.

$$\int x^9 \left(a+b x^4\right)^{3/4} \, dx$$

Optimal (type 4, 149 leaves, 7 steps) :

$$\frac{4 a^3 x^2}{65 b^2 \left(a+b x^4\right)^{1/4}}-\frac{2 a^2 x^2 \left(a+b x^4\right)^{3/4}}{65 b^2}+\frac{a x^6 \left(a+b x^4\right)^{3/4}}{39 b}+\frac{1}{13} x^{10} \left(a+b x^4\right)^{3/4}-\frac{4 a^{7/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{65 b^{5/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 91 leaves) :

$$\frac{x^2 \left(-6 a^3-a^2 b x^4+20 a b^2 x^8+15 b^3 x^{12}+6 a^3 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{195 b^2 \left(a+b x^4\right)^{1/4}}$$

Problem 1026: Result unnecessarily involves higher level functions.

$$\int x^5 \left(a+b x^4\right)^{3/4} \, dx$$

Optimal (type 4, 125 leaves, 6 steps) :

$$-\frac{2 a^2 x^2}{15 b \left(a+b x^4\right)^{1/4}}+\frac{a x^2 \left(a+b x^4\right)^{3/4}}{15 b}+\frac{1}{9} x^6 \left(a+b x^4\right)^{3/4}+\frac{2 a^{5/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{15 b^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 80 leaves) :

$$\frac{x^2 \left(3 a^2+8 a b x^4+5 b^2 x^8-3 a^2 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{45 b \left(a+b x^4\right)^{1/4}}$$

Problem 1027: Result unnecessarily involves higher level functions.

$$\int x \left(a+b x^4\right)^{3/4} \, dx$$

Optimal (type 4, 98 leaves, 5 steps) :

$$\frac{3 a x^2}{5 \left(a+b x^4\right)^{1/4}}+\frac{1}{5} x^2 \left(a+b x^4\right)^{3/4}-\frac{3 a^{3/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{5 \sqrt{b} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 64 leaves) :

$$\frac{x^2 \left(2 \left(a+b x^4\right)+3 a \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{10 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1028: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{3/4}}{x^3} \, dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{3 b x^2}{2 \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{2 x^2}-\frac{3 \sqrt{a} \sqrt{b} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 67 leaves):

$$\frac{-2 \left(a+b x^4\right)+3 b x^4 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]}{4 x^2 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1029: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{3/4}}{x^7} \, dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\frac{b^2 x^2}{4 a \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{6 x^6}-\frac{b \left(a+b x^4\right)^{3/4}}{4 a x^2}-\frac{b^{3/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{4 \sqrt{a} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 86 leaves):

$$\frac{-2 \left(2 a^2+5 a b x^4+3 b^2 x^8\right)+3 b^2 x^8 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]}{24 a x^6 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1030: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{3/4}}{x^{11}} \, dx$$

Optimal (type 4, 149 leaves, 7 steps):

$$-\frac{3 b^3 x^2}{40 a^2 \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{10 x^{10}}-\frac{b \left(a+b x^4\right)^{3/4}}{20 a x^6}+\frac{3 b^2 \left(a+b x^4\right)^{3/4}}{40 a^2 x^2}+\frac{3 b^{5/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{40 a^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 94 leaves) :

$$\frac{-8 a^3 - 12 a^2 b x^4 + 2 a b^2 x^8 + 6 b^3 x^{12} - 3 b^3 x^{12} \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^4}{a}\right]}{80 a^2 x^{10} \left(a + b x^4\right)^{1/4}}$$

Problem 1040: Result unnecessarily involves higher level functions.

$$\int x^{10} \left(a + b x^4\right)^{3/4} dx$$

Optimal (type 4, 150 leaves, 8 steps) :

$$\frac{3 a^3 x^3}{80 b^2 \left(a + b x^4\right)^{1/4}} - \frac{a^2 x^3 \left(a + b x^4\right)^{3/4}}{40 b^2} + \frac{3 a x^7 \left(a + b x^4\right)^{3/4}}{140 b} + \frac{1}{14} x^{11} \left(a + b x^4\right)^{3/4} + \frac{3 a^{7/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{80 b^{5/2} \left(a + b x^4\right)^{1/4}}$$

Result (type 5, 91 leaves) :

$$\frac{x^3 \left(-7 a^3 - a^2 b x^4 + 26 a b^2 x^8 + 20 b^3 x^{12} + 7 a^3 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right)}{280 b^2 \left(a + b x^4\right)^{1/4}}$$

Problem 1041: Result unnecessarily involves higher level functions.

$$\int x^6 \left(a + b x^4\right)^{3/4} dx$$

Optimal (type 4, 126 leaves, 7 steps) :

$$-\frac{3 a^2 x^3}{40 b \left(a + b x^4\right)^{1/4}} + \frac{a x^3 \left(a + b x^4\right)^{3/4}}{20 b} + \frac{1}{10} x^7 \left(a + b x^4\right)^{3/4} - \frac{3 a^{5/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{40 b^{3/2} \left(a + b x^4\right)^{1/4}}$$

Result (type 5, 78 leaves) :

$$\frac{x^3 \left(a^2 + 3 a b x^4 + 2 b^2 x^8 - a^2 \left(1 + \frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right)}{20 b \left(a + b x^4\right)^{1/4}}$$

Problem 1042: Result unnecessarily involves higher level functions.

$$\int x^2 \left(a + b x^4\right)^{3/4} dx$$

Optimal (type 4, 99 leaves, 6 steps) :

$$\frac{a x^3}{4 \left(a + b x^4\right)^{1/4}} + \frac{1}{6} x^3 \left(a + b x^4\right)^{3/4} + \frac{a^{3/2} \left(1 + \frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{4 \sqrt{b} \left(a + b x^4\right)^{1/4}}$$

Result (type 5, 60 leaves) :

$$\frac{x^3 \left(a+b x^4+a \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{6 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1043: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{3/4}}{x^2} \, dx$$

Optimal (type 4, 97 leaves, 6 steps):

$$\frac{3 b x^3}{2 \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{x}+\frac{3 \sqrt{a} \sqrt{b} \left(1+\frac{a}{b x^4}\right)^{1/4} x \, \text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 63 leaves):

$$\frac{-a-b x^4+b x^4 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{x \left(a+b x^4\right)^{1/4}}$$

- **Problem 1044: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{3/4}}{x^6} \, dx$$

Optimal (type 4, 99 leaves, 6 steps):

$$-\frac{3 b}{5 x \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{5 x^5}+\frac{3 b^{3/2} \left(1+\frac{a}{b x^4}\right)^{1/4} x \, \text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{5 \sqrt{a} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 83 leaves):

$$\frac{-a^2-4 a b x^4-3 b^2 x^8+2 b^2 x^8 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{5 a x^5 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1045: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{3/4}}{x^{10}} \, dx$$

Optimal (type 4, 126 leaves, 7 steps):

$$\frac{2 b^2}{15 a x \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{9 x^9}-\frac{b \left(a+b x^4\right)^{3/4}}{15 a x^5}-\frac{2 b^{5/2} \left(1+\frac{a}{b x^4}\right)^{1/4} x \, \text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{15 a^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 94 leaves) :

$$\frac{-5 a^3-8 a^2 b x^4+3 a b^2 x^8+6 b^3 x^{12}-4 b^3 x^{12} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{45 a^2 x^9 \left(a+b x^4\right)^{1/4}}$$

Problem 1046: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{3/4}}{x^{14}} \, dx$$

Optimal (type 4, 150 leaves, 8 steps) :

$$-\frac{4 b^3}{65 a^2 x \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{13 x^{13}}-\frac{b \left(a+b x^4\right)^{3/4}}{39 a x^9}+\frac{2 b^2 \left(a+b x^4\right)^{3/4}}{65 a^2 x^5}+\frac{4 b^{7/2} \left(1+\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{65 a^{5/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 104 leaves) :

$$\frac{1}{195 a^3 x^{13} \left(a+b x^4\right)^{1/4}}\left(-15 a^4-20 a^3 b x^4+a^2 b^2 x^8-6 a b^3 x^{12}-12 b^4 x^{16}+8 b^4 x^{16} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)$$

Problem 1052: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{5/4}}{x} \, dx$$

Optimal (type 3, 83 leaves, 7 steps) :

$$a \left(a+b x^4\right)^{1/4}+\frac{1}{5} \left(a+b x^4\right)^{5/4}-\frac{1}{2} a^{5/4} \text{ArcTan}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]-\frac{1}{2} a^{5/4} \text{ArcTanh}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 76 leaves) :

$$\frac{3 \left(6 a^2+7 a b x^4+b^2 x^8\right)-5 a^2 \left(1+\frac{a}{b x^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{a}{b x^4}\right]}{15 \left(a+b x^4\right)^{3/4}}$$

Problem 1053: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{5/4}}{x^5} \, dx$$

Optimal (type 3, 91 leaves, 7 steps) :

$$\frac{5}{4} b \left(a+b x^4\right)^{1/4}-\frac{\left(a+b x^4\right)^{5/4}}{4 x^4}-\frac{5}{8} a^{1/4} b \,\text{ArcTan}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]-\frac{5}{8} a^{1/4} b \,\text{ArcTanh}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 73 leaves) :

$$\left(b-\frac{a}{4 x^4}\right) \left(a+b x^4\right)^{1/4}-\frac{5 a b \left(1+\frac{a}{b x^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{a}{b x^4}\right]}{12 \left(a+b x^4\right)^{3/4}}$$

- **Problem 1054: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{5/4}}{x^9} \, dx$$

Optimal (type 3, 98 leaves, 7 steps):

$$-\frac{5 b \left(a+b x^4\right)^{1/4}}{32 x^4}-\frac{\left(a+b x^4\right)^{5/4}}{8 x^8}-\frac{5 b^2 \text{ArcTan}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{3/4}}-\frac{5 b^2 \text{ArcTanh}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{3/4}}$$

Result (type 5, 85 leaves):

$$\left(-\frac{a}{8 x^8}-\frac{9 b}{32 x^4}\right) \left(a+b x^4\right)^{1/4}-\frac{5 b^2 \left(\frac{a+b x^4}{b x^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{a}{b x^4}\right]}{96 \left(a+b x^4\right)^{3/4}}$$

- **Problem 1055: Result unnecessarily involves higher level functions.**

$$\int x^9 \left(a+b x^4\right)^{5/4} \, dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$-\frac{2 a^3 x^2 \left(a+b x^4\right)^{1/4}}{231 b^2}+\frac{a^2 x^6 \left(a+b x^4\right)^{1/4}}{231 b}+\frac{1}{33} a x^{10} \left(a+b x^4\right)^{1/4}+\frac{1}{15} x^{10} \left(a+b x^4\right)^{5/4}+\frac{4 a^{9/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{231 b^{5/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 102 leaves):

$$\frac{1}{1155 b^2 \left(a+b x^4\right)^{3/4}} x^2 \left(-10 a^4-5 a^3 b x^4+117 a^2 b^2 x^8+189 a b^3 x^{12}+77 b^4 x^{16}+10 a^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]\right)$$

- **Problem 1056: Result unnecessarily involves higher level functions.**

$$\int x^5 \left(a+b x^4\right)^{5/4} \, dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$\frac{5 a^2 x^2 \left(a+b x^4\right)^{1/4}}{231 b}+\frac{5}{77} a x^6 \left(a+b x^4\right)^{1/4}+\frac{1}{11} x^6 \left(a+b x^4\right)^{5/4}-\frac{10 a^{7/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{231 b^{3/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{x^2 \left(5 a^3+41 a^2 b x^4+57 a b^2 x^8+21 b^3 x^{12}-5 a^3 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{231 b \left(a+b x^4\right)^{3/4}}$$

Problem 1057: Result unnecessarily involves higher level functions.

$$\int x \left(a+b x^4\right)^{5/4} \, dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{5}{21} a x^2 \left(a+b x^4\right)^{1/4}+\frac{1}{7} x^2 \left(a+b x^4\right)^{5/4}+\frac{5 a^{5/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{21 \sqrt{b} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 77 leaves):

$$\frac{x^2 \left(16 a^2+22 a b x^4+6 b^2 x^8+5 a^2 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{42 \left(a+b x^4\right)^{3/4}}$$

Problem 1058: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{5/4}}{x^3} \, dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{5}{6} b x^2 \left(a+b x^4\right)^{1/4}-\frac{\left(a+b x^4\right)^{5/4}}{2 x^2}+\frac{5 a^{3/2} \sqrt{b} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{6 \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{-6 a^2-2 a b x^4+4 b^2 x^8+5 a b x^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{12 x^2 \left(a+b x^4\right)^{3/4}}$$

Problem 1059: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{5/4}}{x^7} \, dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$-\frac{5 b \left(a+b x^4\right)^{1/4}}{12 x^2}-\frac{\left(a+b x^4\right)^{5/4}}{6 x^6}+\frac{5 \sqrt{a} b^{3/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{12 \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 85 leaves):

$$\left(-\frac{a}{6x^6}-\frac{7b}{12x^2}\right)\left(a+bx^4\right)^{1/4}+\frac{5b^2x^2\left(\frac{a+bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right]}{24\left(a+bx^4\right)^{3/4}}$$

- **Problem 1060: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+bx^4\right)^{5/4}}{x^{11}}\,dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$-\frac{b\left(a+bx^4\right)^{1/4}}{12x^6}-\frac{b^2\left(a+bx^4\right)^{1/4}}{24ax^2}-\frac{\left(a+bx^4\right)^{5/4}}{10x^{10}}-\frac{b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{24\sqrt{a}\left(a+bx^4\right)^{3/4}}$$

Result (type 5, 97 leaves):

$$\frac{-2\left(12a^3+34a^2bx^4+27ab^2x^8+5b^3x^{12}\right)-5b^3x^{12}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right]}{240ax^{10}\left(a+bx^4\right)^{3/4}}$$

- **Problem 1061: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+bx^4\right)^{5/4}}{x^{15}}\,dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$-\frac{b\left(a+bx^4\right)^{1/4}}{28x^{10}}-\frac{b^2\left(a+bx^4\right)^{1/4}}{168ax^6}+\frac{5b^3\left(a+bx^4\right)^{1/4}}{336a^2x^2}-\frac{\left(a+bx^4\right)^{5/4}}{14x^{14}}+\frac{5b^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{336a^{3/2}\left(a+bx^4\right)^{3/4}}$$

Result (type 5, 105 leaves):

$$\frac{1}{672a^2x^{14}\left(a+bx^4\right)^{3/4}}\left(-48a^4-120a^3bx^4-76a^2b^2x^8+6ab^3x^{12}+10b^4x^{16}+5b^4x^{16}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right]\right)$$

- **Problem 1062: Result unnecessarily involves higher level functions.**

$$\int x^{10}\left(a+bx^4\right)^{5/4}\,dx$$

Optimal (type 3, 148 leaves, 8 steps):

$$-\frac{35a^3x^3\left(a+bx^4\right)^{1/4}}{6144b^2}+\frac{5a^2x^7\left(a+bx^4\right)^{1/4}}{1536b}+\frac{5}{192}ax^{11}\left(a+bx^4\right)^{1/4}+\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}-\frac{35a^4\text{ArcTan}\left[\frac{b^{1/4}x}{\left(a+bx^4\right)^{1/4}}\right]}{4096b^{11/4}}+\frac{35a^4\text{ArcTanh}\left[\frac{b^{1/4}x}{\left(a+bx^4\right)^{1/4}}\right]}{4096b^{11/4}}$$

Result (type 5, 102 leaves):

$$\frac{1}{6144 b^2 \left(a+b x^4\right)^{3/4}} x^3 \left(-35 a^4-15 a^3 b x^4+564 a^2 b^2 x^8+928 a b^3 x^{12}+384 b^4 x^{16}+35 a^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)$$

Problem 1063: Result unnecessarily involves higher level functions.

$$\int x^6 \left(a+b x^4\right)^{5/4} \, dx$$

Optimal (type 3, 124 leaves, 7 steps):

$$\frac{5 a^2 x^3 \left(a+b x^4\right)^{1/4}}{384 b}+\frac{5}{96} a x^7 \left(a+b x^4\right)^{1/4}+\frac{1}{12} x^7 \left(a+b x^4\right)^{5/4}+\frac{5 a^3 \text{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{256 b^{7/4}}-\frac{5 a^3 \text{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{256 b^{7/4}}$$

Result (type 5, 91 leaves):

$$\frac{x^3 \left(5 a^3+57 a^2 b x^4+84 a b^2 x^8+32 b^3 x^{12}-5 a^3 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{384 b \left(a+b x^4\right)^{3/4}}$$

Problem 1064: Result unnecessarily involves higher level functions.

$$\int x^2 \left(a+b x^4\right)^{5/4} \, dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$\frac{5}{32} a x^3 \left(a+b x^4\right)^{1/4}+\frac{1}{8} x^3 \left(a+b x^4\right)^{5/4}-\frac{5 a^2 \text{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{64 b^{3/4}}+\frac{5 a^2 \text{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{64 b^{3/4}}$$

Result (type 5, 77 leaves):

$$\frac{x^3 \left(27 a^2+39 a b x^4+12 b^2 x^8+5 a^2 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{96 \left(a+b x^4\right)^{3/4}}$$

Problem 1065: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{5/4}}{x^2} \, dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$\frac{5}{4} b x^3 \left(a+b x^4\right)^{1/4}-\frac{\left(a+b x^4\right)^{5/4}}{x}-\frac{5}{8} a b^{1/4} \text{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]+\frac{5}{8} a b^{1/4} \text{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]$$

Result (type 5, 79 leaves):

$$\frac{-12\,a^2-9\,a\,b\,x^4+3\,b^2\,x^8+5\,a\,b\,x^4\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b\,x^4}{a}\right]}{12\,x\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1066: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^4\right)^{5/4}}{x^6}\,dx$$

Optimal (type 3, 92 leaves, 6 steps) :

$$-\frac{b\left(a+b\,x^4\right)^{1/4}}{x}-\frac{\left(a+b\,x^4\right)^{5/4}}{5\,x^5}-\frac{1}{2}\,b^{5/4}\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]+\frac{1}{2}\,b^{5/4}\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]$$

Result (type 5, 81 leaves) :

$$\frac{-3\left(a^2+7\,a\,b\,x^4+6\,b^2\,x^8\right)+5\,b^2\,x^8\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b\,x^4}{a}\right]}{15\,x^5\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1071: Result unnecessarily involves higher level functions.**

$$\int x^{12}\left(a+b\,x^4\right)^{5/4}\,dx$$

Optimal (type 4, 171 leaves, 9 steps) :

$$\frac{5\,a^4\,x\left(a+b\,x^4\right)^{1/4}}{672\,b^3}-\frac{a^3\,x^5\left(a+b\,x^4\right)^{1/4}}{336\,b^2}+\frac{a^2\,x^9\left(a+b\,x^4\right)^{1/4}}{504\,b}+$$
$$\frac{5}{252}\,a\,x^{13}\left(a+b\,x^4\right)^{1/4}+\frac{1}{18}\,x^{13}\left(a+b\,x^4\right)^{5/4}+\frac{5\,a^{9/2}\left(1+\frac{a}{b\,x^4}\right)^{3/4}x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{672\,b^{5/2}\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 112 leaves) :

$$\frac{1}{2016\,b^3\left(a+b\,x^4\right)^{3/4}}$$
$$\left(15\,a^5\,x+9\,a^4\,b\,x^5-2\,a^3\,b^2\,x^9+156\,a^2\,b^3\,x^{13}+264\,a\,b^4\,x^{17}+112\,b^5\,x^{21}-15\,a^5\,x\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b\,x^4}{a}\right]\right)$$

- **Problem 1072: Result unnecessarily involves higher level functions.**

$$\int x^8\left(a+b\,x^4\right)^{5/4}\,dx$$

Optimal (type 4, 147 leaves, 8 steps) :

$$-\frac{5 a^3 x \left(a+b x^4\right)^{1/4}}{336 b^2}+\frac{a^2 x^5 \left(a+b x^4\right)^{1/4}}{168 b}+\frac{1}{28} a x^9 \left(a+b x^4\right)^{1/4}+\frac{1}{14} x^9 \left(a+b x^4\right)^{5/4}-\frac{5 a^{7/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{336 b^{3/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 101 leaves) :

$$\frac{1}{336 b^2 \left(a+b x^4\right)^{3/4}}\left(-5 a^4 x-3 a^3 b x^5+38 a^2 b^2 x^9+60 a b^3 x^{13}+24 b^4 x^{17}+5 a^4 x \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]\right)$$

- **Problem 1073: Result unnecessarily involves higher level functions.**

$$\int x^4 \left(a+b x^4\right)^{5/4} dx$$

Optimal (type 4, 123 leaves, 7 steps) :

$$\frac{a^2 x \left(a+b x^4\right)^{1/4}}{24 b}+\frac{1}{12} a x^5 \left(a+b x^4\right)^{1/4}+\frac{1}{10} x^5 \left(a+b x^4\right)^{5/4}+\frac{a^{5/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{24 \sqrt{b} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 90 leaves) :

$$\frac{5 a^3 x+27 a^2 b x^5+34 a b^2 x^9+12 b^3 x^{13}-5 a^3 x \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]}{120 b \left(a+b x^4\right)^{3/4}}$$

- **Problem 1074: Result unnecessarily involves higher level functions.**

$$\int \left(a+b x^4\right)^{5/4} dx$$

Optimal (type 4, 97 leaves, 6 steps) :

$$\frac{5}{12} a x \left(a+b x^4\right)^{1/4}+\frac{1}{6} x \left(a+b x^4\right)^{5/4}-\frac{5 a^{3/2} \sqrt{b} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{12 \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 76 leaves) :

$$\frac{7 a^2 x+9 a b x^5+2 b^2 x^9+5 a^2 x \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]}{12 \left(a+b x^4\right)^{3/4}}$$

- **Problem 1075: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{5/4}}{x^4} dx$$

Optimal (type 4, 99 leaves, 6 steps) :

$$\frac{5}{6}\,b\,x\,\left(a+b\,x^4\right)^{1/4}-\frac{\left(a+b\,x^4\right)^{5/4}}{3\,x^3}-\frac{5\,\sqrt{a}\,b^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{6\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 80 leaves) :

$$\left(-\frac{a}{3\,x^3}+\frac{b\,x}{2}\right)\,\left(a+b\,x^4\right)^{1/4}+\frac{5\,a\,b\,x\,\left(\frac{a+b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]}{6\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1076: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^4\right)^{5/4}}{x^8}\,dx$$

Optimal (type 4, 101 leaves, 6 steps) :

$$-\frac{5\,b\,\left(a+b\,x^4\right)^{1/4}}{21\,x^3}-\frac{\left(a+b\,x^4\right)^{5/4}}{7\,x^7}-\frac{5\,b^{5/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{21\,\sqrt{a}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 80 leaves) :

$$\frac{-3\,a^2-11\,a\,b\,x^4-8\,b^2\,x^8+5\,b^2\,x^8\,\left(1+\frac{b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]}{21\,x^7\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1077: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^4\right)^{5/4}}{x^{12}}\,dx$$

Optimal (type 4, 125 leaves, 7 steps) :

$$-\frac{5\,b\,\left(a+b\,x^4\right)^{1/4}}{77\,x^7}-\frac{5\,b^2\,\left(a+b\,x^4\right)^{1/4}}{231\,a\,x^3}-\frac{\left(a+b\,x^4\right)^{5/4}}{11\,x^{11}}+\frac{10\,b^{7/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{231\,a^{3/2}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 94 leaves) :

$$\frac{-21\,a^3-57\,a^2\,b\,x^4-41\,a\,b^2\,x^8-5\,b^3\,x^{12}-10\,b^3\,x^{12}\,\left(1+\frac{b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]}{231\,a\,x^{11}\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1078: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^4\right)^{5/4}}{x^{16}}\,dx$$

Optimal (type 4, 149 leaves, 8 steps) :

$$-\frac{b\left(a+b x^4\right)^{1/4}}{33 x^{11}}-\frac{b^2\left(a+b x^4\right)^{1/4}}{231 a x^7}+\frac{2 b^3\left(a+b x^4\right)^{1/4}}{231 a^2 x^3}-\frac{\left(a+b x^4\right)^{5/4}}{15 x^{15}}-\frac{4 b^{9/2}\left(1+\frac{a}{b x^4}\right)^{3/4} x^3\,\text{EllipticF}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{231 a^{5/2}\left(a+b x^4\right)^{3/4}}$$

Result (type 5, 105 leaves) :

$$\frac{1}{1155 a^2 x^{15}\left(a+b x^4\right)^{3/4}}\left(-77 a^4-189 a^3 b x^4-117 a^2 b^2 x^8+5 a b^3 x^{12}+10 b^4 x^{16}+20 b^4 x^{16}\left(1+\frac{b x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]\right)$$

Problem 1085: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x\left(a+b x^4\right)^{1/4}}\,dx$$

Optimal (type 3, 55 leaves, 5 steps) :

$$\frac{\text{ArcTan}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{2 a^{1/4}}-\frac{\text{ArcTanh}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{2 a^{1/4}}$$

Result (type 5, 46 leaves) :

$$-\frac{\left(1+\frac{a}{b x^4}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b x^4}\right]}{\left(a+b x^4\right)^{1/4}}$$

Problem 1086: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^5\left(a+b x^4\right)^{1/4}}\,dx$$

Optimal (type 3, 78 leaves, 6 steps) :

$$-\frac{\left(a+b x^4\right)^{3/4}}{4 a x^4}-\frac{b\,\text{ArcTan}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{5/4}}+\frac{b\,\text{ArcTanh}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{5/4}}$$

Result (type 5, 69 leaves) :

$$\frac{-a-b x^4+b\left(1+\frac{a}{b x^4}\right)^{1/4} x^4\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b x^4}\right]}{4 a x^4\left(a+b x^4\right)^{1/4}}$$

Problem 1087: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^9\left(a+b x^4\right)^{1/4}}\,dx$$

Optimal (type 3, 104 leaves, 7 steps) :

$$-\frac{\left(a+b x^4\right)^{3/4}}{8 a x^8}+\frac{5 b \left(a+b x^4\right)^{3/4}}{32 a^2 x^4}+\frac{5 b^2 \text{ArcTan}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{9/4}}-\frac{5 b^2 \text{ArcTanh}\left[\frac{\left(a+b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{9/4}}$$

Result (type 5, 82 leaves) :

$$\frac{-4 a^2+a b x^4+5 b^2 x^8-5 b^2 \left(1+\frac{a}{b x^4}\right)^{1/4} x^8 \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b x^4}\right]}{32 a^2 x^8 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1088: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{13}}{\left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 152 leaves, 7 steps) :

$$-\frac{8 a^3 x^2}{39 b^3 \left(a+b x^4\right)^{1/4}}+\frac{4 a^2 x^2 \left(a+b x^4\right)^{3/4}}{39 b^3}-\frac{10 a x^6 \left(a+b x^4\right)^{3/4}}{117 b^2}+\frac{x^{10} \left(a+b x^4\right)^{3/4}}{13 b}+\frac{8 a^{7/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{39 b^{7/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 91 leaves) :

$$\frac{x^2 \left(12 a^3+2 a^2 b x^4-a b^2 x^8+9 b^3 x^{12}-12 a^3 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{117 b^3 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1089: Result unnecessarily involves higher level functions.**

$$\int \frac{x^9}{\left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 128 leaves, 6 steps) :

$$\frac{4 a^2 x^2}{15 b^2 \left(a+b x^4\right)^{1/4}}-\frac{2 a x^2 \left(a+b x^4\right)^{3/4}}{15 b^2}+\frac{x^6 \left(a+b x^4\right)^{3/4}}{9 b}-\frac{4 a^{5/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{15 b^{5/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 80 leaves) :

$$\frac{x^2 \left(-6 a^2-a b x^4+5 b^2 x^8+6 a^2 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{45 b^2 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1090: Result unnecessarily involves higher level functions.**

$$\int \frac{x^5}{\left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 104 leaves, 5 steps) :

$$-\frac{2 a x^2}{5 b \left(a+b x^4\right)^{1/4}}+\frac{x^2 \left(a+b x^4\right)^{3/4}}{5 b}+\frac{2 a^{3/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{5 b^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 64 leaves) :

$$\frac{x^2 \left(a+b x^4-a \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{5 b \left(a+b x^4\right)^{1/4}}$$

- **Problem 1091: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{\left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 74 leaves, 4 steps) :

$$\frac{x^2}{\left(a+b x^4\right)^{1/4}}-\frac{\sqrt{a} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{\sqrt{b} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 52 leaves) :

$$\frac{x^2 \left(\frac{a+b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]}{2 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1092: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 \left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 104 leaves, 5 steps) :

$$\frac{b x^2}{2 a \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{2 a x^2}-\frac{\sqrt{b} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 \sqrt{a} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 69 leaves) :

$$\frac{-2 \left(a+b x^4\right)+b x^4 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]}{4 a x^2 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1093: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^7 \left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$-\frac{b^2 x^2}{4 a^2 \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{6 a x^6}+\frac{b \left(a+b x^4\right)^{3/4}}{4 a^2 x^2}+\frac{b^{3/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{4 a^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 83 leaves):

$$\frac{-4 a^2+2 a b x^4+6 b^2 x^8-3 b^2 x^8 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]}{24 a^2 x^6 \left(a+b x^4\right)^{1/4}}$$

Problem 1094: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11} \left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\frac{7 b^3 x^2}{40 a^3 \left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{10 a x^{10}}+\frac{7 b \left(a+b x^4\right)^{3/4}}{60 a^2 x^6}-\frac{7 b^2 \left(a+b x^4\right)^{3/4}}{40 a^3 x^2}-\frac{7 b^{5/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{40 a^{5/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 94 leaves):

$$\frac{-24 a^3+4 a^2 b x^4-14 a b^2 x^8-42 b^3 x^{12}+21 b^3 x^{12} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]}{240 a^3 x^{10} \left(a+b x^4\right)^{1/4}}$$

Problem 1103: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{\left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$\frac{7 a^2 x^3}{40 b^2 \left(a+b x^4\right)^{1/4}}-\frac{7 a x^3 \left(a+b x^4\right)^{3/4}}{60 b^2}+\frac{x^7 \left(a+b x^4\right)^{3/4}}{10 b}+\frac{7 a^{5/2} \left(1+\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{40 b^{5/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{x^3 \left(-7 a^2-a b x^4+6 b^2 x^8+7 a^2 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{60 b^2 \left(a+b x^4\right)^{1/4}}$$

Problem 1104: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{a x^3}{4 b \left(a+b x^4\right)^{1/4}}+\frac{x^3 \left(a+b x^4\right)^{3/4}}{6 b}-\frac{a^{3/2} \left(1+\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{4 b^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^3 \left(a+b x^4-a \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{6 b \left(a+b x^4\right)^{1/4}}$$

Problem 1105: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$\frac{x^3}{2 \left(a+b x^4\right)^{1/4}}+\frac{\sqrt{a} \left(1+\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 \sqrt{b} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 52 leaves):

$$\frac{x^3 \left(\frac{a+b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{3 \left(a+b x^4\right)^{1/4}}$$

Problem 1106: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(a+b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 75 leaves, 5 steps):

$$-\frac{1}{x \left(a+b x^4\right)^{1/4}}+\frac{\sqrt{b} \left(1+\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{\sqrt{a} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 70 leaves):

$$\frac{-3\left(a+b x^4\right)+2 b x^4\left(1+\frac{b x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{3 a x\left(a+b x^4\right)^{1/4}}$$

- **Problem 1107: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^6\left(a+b x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$\frac{2 b}{5 a x\left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{5 a x^5}-\frac{2 b^{3/2}\left(1+\frac{a}{b x^4}\right)^{1/4} x\,\text{EllipticE}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{5 a^{3/2}\left(a+b x^4\right)^{1/4}}$$

Result (type 5, 83 leaves):

$$\frac{-3 a^2+3 a b x^4+6 b^2 x^8-4 b^2 x^8\left(1+\frac{b x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{15 a^2 x^5\left(a+b x^4\right)^{1/4}}$$

- **Problem 1108: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^{10}\left(a+b x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$-\frac{4 b^2}{15 a^2 x\left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{9 a x^9}+\frac{2 b\left(a+b x^4\right)^{3/4}}{15 a^2 x^5}+\frac{4 b^{5/2}\left(1+\frac{a}{b x^4}\right)^{1/4} x\,\text{EllipticE}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{15 a^{5/2}\left(a+b x^4\right)^{1/4}}$$

Result (type 5, 93 leaves):

$$\frac{-5 a^3+a^2 b x^4-6 a b^2 x^8-12 b^3 x^{12}+8 b^3 x^{12}\left(1+\frac{b x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{45 a^3 x^9\left(a+b x^4\right)^{1/4}}$$

- **Problem 1109: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^{14}\left(a+b x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 153 leaves, 8 steps):

$$\frac{8 b^3}{39 a^3 x\left(a+b x^4\right)^{1/4}}-\frac{\left(a+b x^4\right)^{3/4}}{13 a x^{13}}+\frac{10 b\left(a+b x^4\right)^{3/4}}{117 a^2 x^9}-\frac{4 b^2\left(a+b x^4\right)^{3/4}}{39 a^3 x^5}-\frac{8 b^{7/2}\left(1+\frac{a}{b x^4}\right)^{1/4} x\,\text{EllipticE}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{39 a^{7/2}\left(a+b x^4\right)^{1/4}}$$

Result (type 5, 104 leaves) :

$$\frac{1}{117\, a^4\, x^{13}\, \left(a+b\, x^4\right)^{1/4}}\left(-9\, a^4+a^3\, b\, x^4-2\, a^2\, b^2\, x^8+12\, a\, b^3\, x^{12}+24\, b^4\, x^{16}-16\, b^4\, x^{16}\, \left(1+\frac{b\, x^4}{a}\right)^{1/4}\, \text{Hypergeometric2F1}\left[\frac{1}{4},\, \frac{3}{4},\, \frac{7}{4},\, -\frac{b\, x^4}{a}\right]\right)$$

- **Problem 1115: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x\, \left(a+b\, x^4\right)^{3/4}}\, dx$$

Optimal (type 3, 55 leaves, 5 steps) :

$$-\frac{\text{ArcTan}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{2\, a^{3/4}}-\frac{\text{ArcTanh}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{2\, a^{3/4}}$$

Result (type 5, 48 leaves) :

$$-\frac{\left(1+\frac{a}{b\, x^4}\right)^{3/4}\, \text{Hypergeometric2F1}\left[\frac{3}{4},\, \frac{3}{4},\, \frac{7}{4},\, -\frac{a}{b\, x^4}\right]}{3\, \left(a+b\, x^4\right)^{3/4}}$$

- **Problem 1116: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^5\, \left(a+b\, x^4\right)^{3/4}}\, dx$$

Optimal (type 3, 78 leaves, 6 steps) :

$$-\frac{\left(a+b\, x^4\right)^{1/4}}{4\, a\, x^4}+\frac{3\, b\, \text{ArcTan}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{8\, a^{7/4}}+\frac{3\, b\, \text{ArcTanh}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{8\, a^{7/4}}$$

Result (type 5, 69 leaves) :

$$\frac{-a-b\, x^4+b\, \left(1+\frac{a}{b\, x^4}\right)^{3/4}\, x^4\, \text{Hypergeometric2F1}\left[\frac{3}{4},\, \frac{3}{4},\, \frac{7}{4},\, -\frac{a}{b\, x^4}\right]}{4\, a\, x^4\, \left(a+b\, x^4\right)^{3/4}}$$

- **Problem 1117: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^9\, \left(a+b\, x^4\right)^{3/4}}\, dx$$

Optimal (type 3, 104 leaves, 7 steps) :

$$-\frac{\left(a+b\, x^4\right)^{1/4}}{8\, a\, x^8}+\frac{7\, b\, \left(a+b\, x^4\right)^{1/4}}{32\, a^2\, x^4}-\frac{21\, b^2\, \text{ArcTan}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{64\, a^{11/4}}-\frac{21\, b^2\, \text{ArcTanh}\left[\frac{\left(a+b\, x^4\right)^{1/4}}{a^{1/4}}\right]}{64\, a^{11/4}}$$

Result (type 5, 83 leaves) :

$$\frac{-4\,a^2+3\,a\,b\,x^4+7\,b^2\,x^8-7\,b^2\left(1+\frac{a}{b\,x^4}\right)^{3/4}x^8\,\text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{a}{b\,x^4}\right]}{32\,a^2\,x^8\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1118: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{13}}{\left(a+b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$\frac{20\,a^2\,x^2\,\left(a+b\,x^4\right)^{1/4}}{77\,b^3}-\frac{10\,a\,x^6\,\left(a+b\,x^4\right)^{1/4}}{77\,b^2}+\frac{x^{10}\,\left(a+b\,x^4\right)^{1/4}}{11\,b}-\frac{40\,a^{7/2}\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{77\,b^{7/2}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{x^2\left(20\,a^3+10\,a^2\,b\,x^4-3\,a\,b^2\,x^8+7\,b^3\,x^{12}-20\,a^3\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b\,x^4}{a}\right]\right)}{77\,b^3\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1119: Result unnecessarily involves higher level functions.**

$$\int \frac{x^9}{\left(a+b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{2\,a\,x^2\,\left(a+b\,x^4\right)^{1/4}}{7\,b^2}+\frac{x^6\,\left(a+b\,x^4\right)^{1/4}}{7\,b}+\frac{4\,a^{5/2}\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{7\,b^{5/2}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{x^2\left(-2\,a^2-a\,b\,x^4+b^2\,x^8+2\,a^2\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b\,x^4}{a}\right]\right)}{7\,b^2\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1120: Result unnecessarily involves higher level functions.**

$$\int \frac{x^5}{\left(a+b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$\frac{x^2\,\left(a+b\,x^4\right)^{1/4}}{3\,b}-\frac{2\,a^{3/2}\left(1+\frac{b\,x^4}{a}\right)^{3/4}\text{EllipticF}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{3\,b^{3/2}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(a+b x^4-a \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{3 b \left(a+b x^4\right)^{3/4}}$$

Problem 1121: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$\frac{\sqrt{a} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{\sqrt{b} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 52 leaves):

$$\frac{x^2 \left(\frac{a+b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{2 \left(a+b x^4\right)^{3/4}}$$

Problem 1122: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{2 a x^2}-\frac{\sqrt{b} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 \sqrt{a} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{-2 \left(a+b x^4\right)-b x^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{4 a x^2 \left(a+b x^4\right)^{3/4}}$$

Problem 1123: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 \left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{6 a x^6}+\frac{5 b \left(a+b x^4\right)^{1/4}}{12 a^2 x^2}+\frac{5 b^{3/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{12 a^{3/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 83 leaves) :

$$\frac{-4 a^2+6 a b x^4+10 b^2 x^8+5 b^2 x^8 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{24 a^2 x^6 \left(a+b x^4\right)^{3/4}}$$

Problem 1124: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11} \left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 128 leaves, 6 steps) :

$$-\frac{\left(a+b x^4\right)^{1/4}}{10 a x^{10}}+\frac{3 b \left(a+b x^4\right)^{1/4}}{20 a^2 x^6}-\frac{3 b^2 \left(a+b x^4\right)^{1/4}}{8 a^3 x^2}-\frac{3 b^{5/2} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{8 a^{5/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 94 leaves) :

$$\frac{-8 a^3+4 a^2 b x^4-18 a b^2 x^8-30 b^3 x^{12}-15 b^3 x^{12} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{b x^4}{a}\right]}{80 a^3 x^{10} \left(a+b x^4\right)^{3/4}}$$

Problem 1125: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{\left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 3, 106 leaves, 6 steps) :

$$-\frac{7 a x^3 \left(a+b x^4\right)^{1/4}}{32 b^2}+\frac{x^7 \left(a+b x^4\right)^{1/4}}{8 b}-\frac{21 a^2 \text{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{64 b^{11/4}}+\frac{21 a^2 \text{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{64 b^{11/4}}$$

Result (type 5, 80 leaves) :

$$\frac{x^3 \left(-7 a^2-3 a b x^4+4 b^2 x^8+7 a^2 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{32 b^2 \left(a+b x^4\right)^{3/4}}$$

Problem 1126: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 3, 80 leaves, 5 steps) :

$$\frac{x^3 \left(a+b x^4\right)^{1/4}}{4 b}+\frac{3 a \operatorname{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{8 b^{7/4}}-\frac{3 a \operatorname{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{8 b^{7/4}}$$

Result (type 5, 64 leaves) :

$$\frac{x^3 \left(a+b x^4-a \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]\right)}{4 b \left(a+b x^4\right)^{3/4}}$$

- **Problem 1127: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2}{\left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 3, 57 leaves, 4 steps) :

$$-\frac{\operatorname{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{2 b^{3/4}}+\frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{2 b^{3/4}}$$

Result (type 5, 52 leaves) :

$$\frac{x^3 \left(\frac{a+b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{b x^4}{a}\right]}{3 \left(a+b x^4\right)^{3/4}}$$

- **Problem 1132: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{12}}{\left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 129 leaves, 7 steps) :

$$\frac{3 a^2 x \left(a+b x^4\right)^{1/4}}{8 b^3}-\frac{3 a x^5 \left(a+b x^4\right)^{1/4}}{20 b^2}+\frac{x^9 \left(a+b x^4\right)^{1/4}}{10 b}+\frac{3 a^{5/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{8 b^{5/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 90 leaves) :

$$\frac{15 a^3 x+9 a^2 b x^5-2 a b^2 x^9+4 b^3 x^{13}-15 a^3 x \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{b x^4}{a}\right]}{40 b^3 \left(a+b x^4\right)^{3/4}}$$

- **Problem 1133: Result unnecessarily involves higher level functions.**

$$\int \frac{x^8}{\left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 105 leaves, 6 steps) :

$$-\frac{5\,a\,x\,\left(a+b\,x^4\right)^{1/4}}{12\,b^2}+\frac{x^5\,\left(a+b\,x^4\right)^{1/4}}{6\,b}-\frac{5\,a^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{12\,b^{3/2}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 79 leaves) :

$$\frac{-5\,a^2\,x-3\,a\,b\,x^5+2\,b^2\,x^9+5\,a^2\,x\,\left(1+\frac{b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]}{12\,b^2\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1134: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4}{\left(a+b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 83 leaves, 5 steps) :

$$\frac{x\,\left(a+b\,x^4\right)^{1/4}}{2\,b}+\frac{\sqrt{a}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{2\,\sqrt{b}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 62 leaves) :

$$\frac{x\,\left(a+b\,x^4-a\,\left(1+\frac{b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]\right)}{2\,b\,\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1135: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(a+b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 61 leaves, 4 steps) :

$$-\frac{\sqrt{b}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{\sqrt{a}\,\left(a+b\,x^4\right)^{3/4}}$$

Result (type 5, 47 leaves) :

$$\frac{x\,\left(\frac{a+b\,x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{5}{4},\,-\frac{b\,x^4}{a}\right]}{\left(a+b\,x^4\right)^{3/4}}$$

- **Problem 1136: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4\,\left(a+b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 85 leaves, 5 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{3 a x^3}+\frac{2 b^{3/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{3 a^{3/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a-b x^4-2 b x^4 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{3 a x^3 \left(a+b x^4\right)^{3/4}}$$

Problem 1137: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^8 \left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 107 leaves, 6 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{7 a x^7}+\frac{2 b \left(a+b x^4\right)^{1/4}}{7 a^2 x^3}-\frac{4 b^{5/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{7 a^{5/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 82 leaves):

$$\frac{-a^2+a b x^4+2 b^2 x^8+4 b^2 x^8 \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{7 a^2 x^7 \left(a+b x^4\right)^{3/4}}$$

Problem 1138: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{12} \left(a+b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 131 leaves, 7 steps):

$$-\frac{\left(a+b x^4\right)^{1/4}}{11 a x^{11}}+\frac{10 b \left(a+b x^4\right)^{1/4}}{77 a^2 x^7}-\frac{20 b^2 \left(a+b x^4\right)^{1/4}}{77 a^3 x^3}+\frac{40 b^{7/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{77 a^{7/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 94 leaves):

$$\frac{-7 a^3+3 a^2 b x^4-10 a b^2 x^8-20 b^3 x^{12}-40 b^3 x^{12} \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{77 a^3 x^{11} \left(a+b x^4\right)^{3/4}}$$

Problem 1144: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\,(a+b\,x^4)^{5/4}}\,dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{1}{a\,(a+b\,x^4)^{1/4}}+\frac{\text{ArcTan}\left[\frac{(a+b\,x^4)^{1/4}}{a^{1/4}}\right]}{2\,a^{5/4}}-\frac{\text{ArcTanh}\left[\frac{(a+b\,x^4)^{1/4}}{a^{1/4}}\right]}{2\,a^{5/4}}$$

Result (type 5, 52 leaves):

$$\frac{1-\left(1+\frac{a}{b\,x^4}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b\,x^4}\right]}{a\,(a+b\,x^4)^{1/4}}$$

Problem 1145: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5\,(a+b\,x^4)^{5/4}}\,dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{5\,b}{4\,a^2\,(a+b\,x^4)^{1/4}}-\frac{1}{4\,a\,x^4\,(a+b\,x^4)^{1/4}}-\frac{5\,b\,\text{ArcTan}\left[\frac{(a+b\,x^4)^{1/4}}{a^{1/4}}\right]}{8\,a^{9/4}}+\frac{5\,b\,\text{ArcTanh}\left[\frac{(a+b\,x^4)^{1/4}}{a^{1/4}}\right]}{8\,a^{9/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a-5\,b\,x^4+5\,b\left(1+\frac{a}{b\,x^4}\right)^{1/4}x^4\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b\,x^4}\right]}{4\,a^2\,x^4\,(a+b\,x^4)^{1/4}}$$

Problem 1146: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^9\,(a+b\,x^4)^{5/4}}\,dx$$

Optimal (type 3, 125 leaves, 8 steps):

$$\frac{45\,b^2}{32\,a^3\,(a+b\,x^4)^{1/4}}-\frac{1}{8\,a\,x^8\,(a+b\,x^4)^{1/4}}+\frac{9\,b}{32\,a^2\,x^4\,(a+b\,x^4)^{1/4}}+\frac{45\,b^2\,\text{ArcTan}\left[\frac{(a+b\,x^4)^{1/4}}{a^{1/4}}\right]}{64\,a^{13/4}}-\frac{45\,b^2\,\text{ArcTanh}\left[\frac{(a+b\,x^4)^{1/4}}{a^{1/4}}\right]}{64\,a^{13/4}}$$

Result (type 5, 83 leaves):

$$\frac{-4\,a^2+9\,a\,b\,x^4+45\,b^2\,x^8-45\,b^2\left(1+\frac{a}{b\,x^4}\right)^{1/4}x^8\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{a}{b\,x^4}\right]}{32\,a^3\,x^8\,(a+b\,x^4)^{1/4}}$$

Problem 1147: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{\left(a+b x^4\right)^{5/4}} \, dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$\frac{4 a^2 x^2}{3 b^3 \left(a+b x^4\right)^{1/4}}-\frac{2 a x^6}{9 b^2 \left(a+b x^4\right)^{1/4}}+\frac{x^{10}}{9 b \left(a+b x^4\right)^{1/4}}-\frac{8 a^{5/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{3 b^{7/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 79 leaves):

$$\frac{x^2 \left(-12 a^2-2 a b x^4+b^2 x^8+12 a^2 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{9 b^3 \left(a+b x^4\right)^{1/4}}$$

Problem 1148: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{\left(a+b x^4\right)^{5/4}} \, dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{6 a x^2}{5 b^2 \left(a+b x^4\right)^{1/4}}+\frac{x^6}{5 b \left(a+b x^4\right)^{1/4}}+\frac{12 a^{3/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{5 b^{5/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^2 \left(6 a+b x^4-6 a \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{5 b^2 \left(a+b x^4\right)^{1/4}}$$

Problem 1149: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(a+b x^4\right)^{5/4}} \, dx$$

Optimal (type 4, 77 leaves, 4 steps):

$$\frac{x^2}{b \left(a+b x^4\right)^{1/4}}-\frac{2 \sqrt{a} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{b^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 54 leaves):

$$\frac{x^2 \left(-1+\left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{b \left(a+b x^4\right)^{1/4}}$$

- **Problem 1150: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{\left(a+b x^4\right)^{5/4}} \, dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$\frac{\left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{\sqrt{a} \sqrt{b} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 57 leaves):

$$-\frac{x^2 \left(-2+\left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]\right)}{2 a \left(a+b x^4\right)^{1/4}}$$

- **Problem 1151: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 \left(a+b x^4\right)^{5/4}} \, dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{1}{2 a x^2 \left(a+b x^4\right)^{1/4}}-\frac{3 \sqrt{b} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 a^{3/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 71 leaves):

$$\frac{-2 \left(a+3 b x^4\right)+3 b x^4 \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b x^4}{a}\right]}{4 a^2 x^2 \left(a+b x^4\right)^{1/4}}$$

- **Problem 1152: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^7 \left(a+b x^4\right)^{5/4}} \, dx$$

Optimal (type 4, 104 leaves, 5 steps):

$$-\frac{1}{6 a x^6 \left(a+b x^4\right)^{1/4}}+\frac{7 b}{12 a^2 x^2 \left(a+b x^4\right)^{1/4}}+\frac{7 b^{3/2} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{4 a^{5/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 83 leaves):

$$\frac{-4\,a^2+14\,a\,b\,x^4+42\,b^2\,x^8-21\,b^2\,x^8\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{3}{2},\,-\frac{b\,x^4}{a}\right]}{24\,a^3\,x^6\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1153: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11}\,\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$-\frac{1}{10\,a\,x^{10}\,\left(a+b\,x^4\right)^{1/4}}+\frac{11\,b}{60\,a^2\,x^6\,\left(a+b\,x^4\right)^{1/4}}-\frac{77\,b^2}{120\,a^3\,x^2\,\left(a+b\,x^4\right)^{1/4}}-\frac{77\,b^{5/2}\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{40\,a^{7/2}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 94 leaves):

$$\frac{-24\,a^3+44\,a^2\,b\,x^4-154\,a\,b^2\,x^8-462\,b^3\,x^{12}+231\,b^3\,x^{12}\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{3}{2},\,-\frac{b\,x^4}{a}\right]}{240\,a^4\,x^{10}\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1162: Result unnecessarily involves higher level functions.

$$\int \frac{x^{14}}{\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$\frac{77\,a^2\,x^3}{120\,b^3\,\left(a+b\,x^4\right)^{1/4}}-\frac{11\,a\,x^7}{60\,b^2\,\left(a+b\,x^4\right)^{1/4}}+\frac{x^{11}}{10\,b\,\left(a+b\,x^4\right)^{1/4}}+\frac{77\,a^{5/2}\,\left(1+\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{40\,b^{7/2}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{x^3\,\left(-77\,a^2-11\,a\,b\,x^4+6\,b^2\,x^8+77\,a^2\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{b\,x^4}{a}\right]\right)}{60\,b^3\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1163: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{7\,a\,x^3}{12\,b^2\,\left(a+b\,x^4\right)^{1/4}}+\frac{x^7}{6\,b\,\left(a+b\,x^4\right)^{1/4}}-\frac{7\,a^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{4\,b^{5/2}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 66 leaves) :

$$\frac{x^3\,\left(7\,a+b\,x^4-7\,a\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{b\,x^4}{a}\right]\right)}{6\,b^2\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1164: Result unnecessarily involves higher level functions.

$$\int\frac{x^6}{\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 83 leaves, 5 steps) :

$$\frac{x^3}{2\,b\,\left(a+b\,x^4\right)^{1/4}}+\frac{3\,\sqrt{a}\,\left(1+\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{2\,b^{3/2}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 54 leaves) :

$$\frac{x^3\,\left(-1+\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{b\,x^4}{a}\right]\right)}{b\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1165: Result unnecessarily involves higher level functions.

$$\int\frac{x^2}{\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 59 leaves, 4 steps) :

$$-\frac{\left(1+\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{\sqrt{a}\,\sqrt{b}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 58 leaves) :

$$-\frac{x^3\,\left(-3+2\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{b\,x^4}{a}\right]\right)}{3\,a\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1166: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^2\,\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 79 leaves, 5 steps):

$$-\frac{1}{a\,x\,\left(a+b\,x^4\right)^{1/4}}+\frac{2\,\sqrt{b}\,\left(1+\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{a^{3/2}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 71 leaves):

$$\frac{-3\,\left(a+2\,b\,x^4\right)+4\,b\,x^4\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{b\,x^4}{a}\right]}{3\,a^2\,x\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1167: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^6\,\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{1}{5\,a\,x^5\,\left(a+b\,x^4\right)^{1/4}}+\frac{6\,b}{5\,a^2\,x\,\left(a+b\,x^4\right)^{1/4}}-\frac{12\,b^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{5\,a^{5/2}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 83 leaves):

$$\frac{-a^2+6\,a\,b\,x^4+12\,b^2\,x^8-8\,b^2\,x^8\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{b\,x^4}{a}\right]}{5\,a^3\,x^5\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1168: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^{10}\,\left(a+b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 129 leaves, 7 steps):

$$-\frac{1}{9\,a\,x^9\,\left(a+b\,x^4\right)^{1/4}}+\frac{2\,b}{9\,a^2\,x^5\,\left(a+b\,x^4\right)^{1/4}}-\frac{4\,b^2}{3\,a^3\,x\,\left(a+b\,x^4\right)^{1/4}}+\frac{8\,b^{5/2}\,\left(1+\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{3\,a^{7/2}\,\left(a+b\,x^4\right)^{1/4}}$$

Result (type 5, 94 leaves):

$$\frac{-a^3+2\,a^2\,b\,x^4-12\,a\,b^2\,x^8-24\,b^3\,x^{12}+16\,b^3\,x^{12}\,\left(1+\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{3}{4},\,\frac{7}{4},\,-\frac{b\,x^4}{a}\right]}{9\,a^4\,x^9\,\left(a+b\,x^4\right)^{1/4}}$$

Problem 1169: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{14} \left(a+b x^4\right)^{5/4}} \, dx$$

Optimal (type 4, 153 leaves, 8 steps):

$$-\frac{1}{13 a x^{13} \left(a+b x^4\right)^{1/4}}+\frac{14 b}{117 a^2 x^9 \left(a+b x^4\right)^{1/4}}-\frac{28 b^2}{117 a^3 x^5 \left(a+b x^4\right)^{1/4}}+\frac{56 b^3}{39 a^4 x \left(a+b x^4\right)^{1/4}}-\frac{112 b^{7/2} \left(1+\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right], 2\right]}{39 a^{9/2} \left(a+b x^4\right)^{1/4}}$$

Result (type 5, 105 leaves):

$$\frac{1}{117 a^5 x^{13} \left(a+b x^4\right)^{1/4}}\left(-9 a^4+14 a^3 b x^4-28 a^2 b^2 x^8+168 a b^3 x^{12}+336 b^4 x^{16}-224 b^4 x^{16} \left(1+\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{b x^4}{a}\right]\right)$$

Problem 1170: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{7/4}} \, dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$\frac{x}{3 a \left(a+b x^4\right)^{3/4}}-\frac{2 \sqrt{b} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \,\text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right], 2\right]}{3 a^{3/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 56 leaves):

$$\frac{x+2 x \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{3 a \left(a+b x^4\right)^{3/4}}$$

Problem 1172: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{11/4}} \, dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{x}{7 a \left(a+b x^4\right)^{7/4}}+\frac{2 x}{7 a^2 \left(a+b x^4\right)^{3/4}}-\frac{4 \sqrt{b} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \,\text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right], 2\right]}{7 a^{5/2} \left(a+b x^4\right)^{3/4}}$$

Result (type 5, 72 leaves):

$$\frac{3 a x+2 b x^5+4 x \left(a+b x^4\right) \left(1+\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{b x^4}{a}\right]}{7 a^2 \left(a+b x^4\right)^{7/4}}$$

Problem 1180: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x}\,dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$\left(a-b x^4\right)^{1/4}-\frac{1}{2} a^{1/4} \text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]-\frac{1}{2} a^{1/4} \text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]$$

Result (type 5, 63 leaves):

$$\left(a-b x^4\right)^{1/4}-\frac{a \left(1-\frac{a}{b x^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{a}{b x^4}\right]}{3 \left(a-b x^4\right)^{3/4}}$$

Problem 1181: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^5}\,dx$$

Optimal (type 3, 78 leaves, 6 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{4 x^4}+\frac{b \text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{3/4}}+\frac{b \text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{3/4}}$$

Result (type 5, 67 leaves):

$$\frac{-3 a+3 b x^4+b \left(1-\frac{a}{b x^4}\right)^{3/4} x^4 \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{a}{b x^4}\right]}{12 x^4 \left(a-b x^4\right)^{3/4}}$$

Problem 1182: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^9}\,dx$$

Optimal (type 3, 105 leaves, 7 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{8 x^8}+\frac{b \left(a-b x^4\right)^{1/4}}{32 a x^4}+\frac{3 b^2 \text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{7/4}}+\frac{3 b^2 \text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{7/4}}$$

Result (type 5, 83 leaves):

$$\frac{-4 a^2+5 a b x^4-b^2 x^8+b^2 \left(1-\frac{a}{b x^4}\right)^{3/4} x^8 \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{a}{b x^4}\right]}{32 a x^8 \left(a-b x^4\right)^{3/4}}$$

Problem 1183: Result unnecessarily involves higher level functions.

$$\int x^9 \left(a-b x^4\right)^{1/4} \, dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$-\frac{2 a^2 x^2 \left(a-b x^4\right)^{1/4}}{77 b^2}-\frac{a x^6 \left(a-b x^4\right)^{1/4}}{77 b}+\frac{1}{11} x^{10} \left(a-b x^4\right)^{1/4}+\frac{4 a^{7/2} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{77 b^{5/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{x^2 \left(-2 a^3+a^2 b x^4+8 a b^2 x^8-7 b^3 x^{12}+2 a^3 \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]\right)}{77 b^2 \left(a-b x^4\right)^{3/4}}$$

Problem 1184: Result unnecessarily involves higher level functions.

$$\int x^5 \left(a-b x^4\right)^{1/4} \, dx$$

Optimal (type 4, 105 leaves, 5 steps):

$$-\frac{a x^2 \left(a-b x^4\right)^{1/4}}{21 b}+\frac{1}{7} x^6 \left(a-b x^4\right)^{1/4}+\frac{2 a^{5/2} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{21 b^{3/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 80 leaves):

$$\frac{x^2 \left(-a^2+4 a b x^4-3 b^2 x^8+a^2 \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]\right)}{21 b \left(a-b x^4\right)^{3/4}}$$

Problem 1185: Result unnecessarily involves higher level functions.

$$\int x \left(a-b x^4\right)^{1/4} \, dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$\frac{1}{3} x^2 \left(a-b x^4\right)^{1/4}+\frac{a^{3/2} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{3 \sqrt{b} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^2 \left(2 a-2 b x^4+a \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]\right)}{6 \left(a-b x^4\right)^{3/4}}$$

Problem 1186: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^3}\,dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{2 x^2}-\frac{\sqrt{a}\,\sqrt{b}\,\left(1-\frac{b x^4}{a}\right)^{3/4}\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcSin}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{2\,\left(a-b x^4\right)^{3/4}}$$

Result (type 5, 68 leaves):

$$\frac{-2 a+2 b x^4-b x^4\,\left(1-\frac{b x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]}{4 x^2\,\left(a-b x^4\right)^{3/4}}$$

Problem 1187: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^7}\,dx$$

Optimal (type 4, 105 leaves, 5 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{6 x^6}+\frac{b\,\left(a-b x^4\right)^{1/4}}{12 a x^2}-\frac{b^{3/2}\,\left(1-\frac{b x^4}{a}\right)^{3/4}\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcSin}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{12\,\sqrt{a}\,\left(a-b x^4\right)^{3/4}}$$

Result (type 5, 84 leaves):

$$\frac{-4 a^2+6 a b x^4-2 b^2 x^8-b^2 x^8\,\left(1-\frac{b x^4}{a}\right)^{3/4}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]}{24 a x^6\,\left(a-b x^4\right)^{3/4}}$$

Problem 1188: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^{11}}\,dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{10 x^{10}}+\frac{b\,\left(a-b x^4\right)^{1/4}}{60 a x^6}+\frac{b^2\,\left(a-b x^4\right)^{1/4}}{24 a^2 x^2}-\frac{b^{5/2}\,\left(1-\frac{b x^4}{a}\right)^{3/4}\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcSin}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{24 a^{3/2}\,\left(a-b x^4\right)^{3/4}}$$

Result (type 5, 95 leaves):

$$\frac{-24 a^3+28 a^2 b x^4+6 a b^2 x^8-10 b^3 x^{12}-5 b^3 x^{12} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]}{240 a^2 x^{10} \left(a-b x^4\right)^{3/4}}$$

Problem 1189: Result unnecessarily involves higher level functions.

$$\int x^6 \left(a-b x^4\right)^{1/4} \, dx$$

Optimal (type 3, 263 leaves, 12 steps):

$$-\frac{a x^3 \left(a-b x^4\right)^{1/4}}{32 b}+\frac{1}{8} x^7 \left(a-b x^4\right)^{1/4}-\frac{3 a^2 \text{ArcTan}\left[1-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{64 \sqrt{2} b^{7/4}}+$$

$$\frac{3 a^2 \text{ArcTan}\left[1+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{64 \sqrt{2} b^{7/4}}+\frac{3 a^2 \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{128 \sqrt{2} b^{7/4}}-\frac{3 a^2 \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{128 \sqrt{2} b^{7/4}}$$

Result (type 5, 80 leaves):

$$\frac{x^3 \left(-a^2+5 a b x^4-4 b^2 x^8+a^2 \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]\right)}{32 b \left(a-b x^4\right)^{3/4}}$$

Problem 1190: Result unnecessarily involves higher level functions.

$$\int x^2 \left(a-b x^4\right)^{1/4} \, dx$$

Optimal (type 3, 232 leaves, 11 steps):

$$\frac{1}{4} x^3 \left(a-b x^4\right)^{1/4}-\frac{a \text{ArcTan}\left[1-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{8 \sqrt{2} b^{3/4}}+\frac{a \text{ArcTan}\left[1+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{8 \sqrt{2} b^{3/4}}+\frac{a \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{16 \sqrt{2} b^{3/4}}-\frac{a \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{16 \sqrt{2} b^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x^3 \left(3 a-3 b x^4+a \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]\right)}{12 \left(a-b x^4\right)^{3/4}}$$

Problem 1191: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^2} \, dx$$

Optimal (type 3, 226 leaves, 11 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{x}+\frac{b^{1/4} \text{ArcTan}\left[1-\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{2 \sqrt{2}}-\frac{b^{1/4} \text{ArcTan}\left[1+\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{2 \sqrt{2}}-\frac{b^{1/4} \text{Log}\left[1+\frac{\sqrt{b}\, x^2}{\sqrt{a-b x^4}}-\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{4 \sqrt{2}}+\frac{b^{1/4} \text{Log}\left[1+\frac{\sqrt{b}\, x^2}{\sqrt{a-b x^4}}+\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{4 \sqrt{2}}$$

Result (type 5, 68 leaves) :

$$\frac{-3 a+3 b x^4-b x^4 \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]}{3 x \left(a-b x^4\right)^{3/4}}$$

Problem 1196: Result unnecessarily involves higher level functions.

$$\int x^{12} \left(a-b x^4\right)^{1/4} dx$$

Optimal (type 4, 156 leaves, 8 steps) :

$$-\frac{3 a^3 x \left(a-b x^4\right)^{1/4}}{112 b^3}-\frac{3 a^2 x^5 \left(a-b x^4\right)^{1/4}}{280 b^2}-\frac{a x^9 \left(a-b x^4\right)^{1/4}}{140 b}+\frac{1}{14} x^{13} \left(a-b x^4\right)^{1/4}-\frac{3 a^{7/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{112 b^{5/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 102 leaves) :

$$\frac{1}{560 b^3 \left(a-b x^4\right)^{3/4}}\left(-15 a^4 x+9 a^3 b x^5+2 a^2 b^2 x^9+44 a b^3 x^{13}-40 b^4 x^{17}+15 a^4 x \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]\right)$$

Problem 1197: Result unnecessarily involves higher level functions.

$$\int x^{8} \left(a-b x^4\right)^{1/4} dx$$

Optimal (type 4, 131 leaves, 7 steps) :

$$-\frac{a^2 x \left(a-b x^4\right)^{1/4}}{24 b^2}-\frac{a x^5 \left(a-b x^4\right)^{1/4}}{60 b}+\frac{1}{10} x^9 \left(a-b x^4\right)^{1/4}-\frac{a^{5/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{24 b^{3/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 91 leaves) :

$$\frac{-5 a^3 x+3 a^2 b x^5+14 a b^2 x^9-12 b^3 x^{13}+5 a^3 x \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]}{120 b^2 \left(a-b x^4\right)^{3/4}}$$

Problem 1198: Result unnecessarily involves higher level functions.

$$\int x^{4} \left(a-b x^4\right)^{1/4} dx$$

Optimal (type 4, 106 leaves, 6 steps) :

$$-\frac{a x \left(a-b x^4\right)^{1/4}}{12 b}+\frac{1}{6} x^5 \left(a-b x^4\right)^{1/4}-\frac{a^{3/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{12 \sqrt{b} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 79 leaves) :

$$\frac{-a^2 x+3 a b x^5-2 b^2 x^9+a^2 x \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]}{12 b \left(a-b x^4\right)^{3/4}}$$

Problem 1199: Result unnecessarily involves higher level functions.

$$\int \left(a-b x^4\right)^{1/4} \, dx$$

Optimal (type 4, 83 leaves, 5 steps) :

$$\frac{1}{2} x \left(a-b x^4\right)^{1/4}-\frac{\sqrt{a} \sqrt{b} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{2 \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 62 leaves) :

$$\frac{a x-b x^5+a x \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]}{2 \left(a-b x^4\right)^{3/4}}$$

Problem 1200: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^4} \, dx$$

Optimal (type 4, 85 leaves, 5 steps) :

$$-\frac{\left(a-b x^4\right)^{1/4}}{3 x^3}+\frac{b^{3/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{3 \sqrt{a} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 67 leaves) :

$$\frac{-a+b x^4-b x^4 \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]}{3 x^3 \left(a-b x^4\right)^{3/4}}$$

Problem 1201: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^8} \, dx$$

Optimal (type 4, 108 leaves, 6 steps) :

$$-\frac{\left(a-b x^4\right)^{1/4}}{7 x^7}+\frac{b \left(a-b x^4\right)^{1/4}}{21 a x^3}+\frac{2 b^{5/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{21 a^{3/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 84 leaves) :

$$\frac{-3 a^2+4 a b x^4-b^2 x^8-2 b^2 x^8 \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]}{21 a x^7 \left(a-b x^4\right)^{3/4}}$$

Problem 1202: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^{12}} \, dx$$

Optimal (type 4, 133 leaves, 7 steps) :

$$-\frac{\left(a-b x^4\right)^{1/4}}{11 x^{11}}+\frac{b \left(a-b x^4\right)^{1/4}}{77 a x^7}+\frac{2 b^2 \left(a-b x^4\right)^{1/4}}{77 a^2 x^3}+\frac{4 b^{7/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{77 a^{5/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 94 leaves) :

$$\frac{-7 a^3+8 a^2 b x^4+a b^2 x^8-2 b^3 x^{12}-4 b^3 x^{12} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]}{77 a^2 x^{11} \left(a-b x^4\right)^{3/4}}$$

Problem 1203: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{1/4}}{x^{16}} \, dx$$

Optimal (type 4, 158 leaves, 8 steps) :

$$-\frac{\left(a-b x^4\right)^{1/4}}{15 x^{15}}+\frac{b \left(a-b x^4\right)^{1/4}}{165 a x^{11}}+\frac{2 b^2 \left(a-b x^4\right)^{1/4}}{231 a^2 x^7}+\frac{4 b^3 \left(a-b x^4\right)^{1/4}}{231 a^3 x^3}+\frac{8 b^{9/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{231 a^{7/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 106 leaves) :

$$\frac{1}{1155 a^3 x^{15} \left(a-b x^4\right)^{3/4}}\left(-77 a^4+84 a^3 b x^4+3 a^2 b^2 x^8+10 a b^3 x^{12}-20 b^4 x^{16}-40 b^4 x^{16} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b x^4}{a}\right]\right)$$

Problem 1209: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x \left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 3, 57 leaves, 5 steps) :

$$\frac{\text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{2 a^{1/4}}-\frac{\text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{2 a^{1/4}}$$

Result (type 5, 47 leaves) :

$$-\frac{\left(1-\frac{a}{b x^4}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{a}{b x^4}\right]}{\left(a-b x^4\right)^{1/4}}$$

- **Problem 1210: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^5 \left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 3, 81 leaves, 6 steps) :

$$-\frac{\left(a-b x^4\right)^{3/4}}{4 a x^4}+\frac{b \text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{5/4}}-\frac{b \text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{5/4}}$$

Result (type 5, 70 leaves) :

$$\frac{-a+b x^4-b \left(1-\frac{a}{b x^4}\right)^{1/4} x^4 \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{a}{b x^4}\right]}{4 a x^4 \left(a-b x^4\right)^{1/4}}$$

- **Problem 1211: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^9 \left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 3, 108 leaves, 7 steps) :

$$-\frac{\left(a-b x^4\right)^{3/4}}{8 a x^8}-\frac{5 b \left(a-b x^4\right)^{3/4}}{32 a^2 x^4}+\frac{5 b^2 \text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{9/4}}-\frac{5 b^2 \text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{9/4}}$$

Result (type 5, 84 leaves) :

$$\frac{-4 a^2-a b x^4+5 b^2 x^8-5 b^2 \left(1-\frac{a}{b x^4}\right)^{1/4} x^8 \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{a}{b x^4}\right]}{32 a^2 x^8 \left(a-b x^4\right)^{1/4}}$$

- **Problem 1212: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{13}}{\left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 133 leaves, 6 steps) :

$$-\frac{4\,a^2\,x^2\,\left(a-b\,x^4\right)^{3/4}}{39\,b^3}-\frac{10\,a\,x^6\,\left(a-b\,x^4\right)^{3/4}}{117\,b^2}-\frac{x^{10}\,\left(a-b\,x^4\right)^{3/4}}{13\,b}+\frac{8\,a^{7/2}\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{39\,b^{7/2}\,\left(a-b\,x^4\right)^{1/4}}$$

Result (type 5, 91 leaves) :

$$\frac{x^2\,\left(-12\,a^3+2\,a^2\,b\,x^4+a\,b^2\,x^8+9\,b^3\,x^{12}+12\,a^3\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{b\,x^4}{a}\right]\right)}{117\,b^3\,\left(a-b\,x^4\right)^{1/4}}$$

Problem 1213: Result unnecessarily involves higher level functions.

$$\int\frac{x^9}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 108 leaves, 5 steps) :

$$-\frac{2\,a\,x^2\,\left(a-b\,x^4\right)^{3/4}}{15\,b^2}-\frac{x^6\,\left(a-b\,x^4\right)^{3/4}}{9\,b}+\frac{4\,a^{5/2}\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{15\,b^{5/2}\,\left(a-b\,x^4\right)^{1/4}}$$

Result (type 5, 80 leaves) :

$$\frac{x^2\,\left(-6\,a^2+a\,b\,x^4+5\,b^2\,x^8+6\,a^2\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{b\,x^4}{a}\right]\right)}{45\,b^2\,\left(a-b\,x^4\right)^{1/4}}$$

Problem 1214: Result unnecessarily involves higher level functions.

$$\int\frac{x^5}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 85 leaves, 4 steps) :

$$-\frac{x^2\,\left(a-b\,x^4\right)^{3/4}}{5\,b}+\frac{2\,a^{3/2}\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcSin}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{5\,b^{3/2}\,\left(a-b\,x^4\right)^{1/4}}$$

Result (type 5, 66 leaves) :

$$\frac{x^2\,\left(-a+b\,x^4+a\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\,\frac{1}{2},\,\frac{3}{2},\,\frac{b\,x^4}{a}\right]\right)}{5\,b\,\left(a-b\,x^4\right)^{1/4}}$$

Problem 1215: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 59 leaves, 3 steps):

$$\frac{\sqrt{a}\left(1-\frac{b x^4}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcSin}\left[\frac{\sqrt{b}\ x^2}{\sqrt{a}}\right],2\right]}{\sqrt{b}\left(a-b x^4\right)^{1/4}}$$

Result (type 5, 53 leaves):

$$\frac{x^2\left(\frac{a-b x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{b x^4}{a}\right]}{2\left(a-b x^4\right)^{1/4}}$$

- **Problem 1216: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^3\left(a-b x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{\left(a-b x^4\right)^{3/4}}{2 a x^2}-\frac{\sqrt{b}\left(1-\frac{b x^4}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcSin}\left[\frac{\sqrt{b}\ x^2}{\sqrt{a}}\right],2\right]}{2\sqrt{a}\left(a-b x^4\right)^{1/4}}$$

Result (type 5, 71 leaves):

$$\frac{-2 a+2 b x^4-b x^4\left(1-\frac{b x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{b x^4}{a}\right]}{4 a x^2\left(a-b x^4\right)^{1/4}}$$

- **Problem 1217: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^7\left(a-b x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 108 leaves, 5 steps):

$$-\frac{\left(a-b x^4\right)^{3/4}}{6 a x^6}-\frac{b\left(a-b x^4\right)^{3/4}}{4 a^2 x^2}-\frac{b^{3/2}\left(1-\frac{b x^4}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcSin}\left[\frac{\sqrt{b}\ x^2}{\sqrt{a}}\right],2\right]}{4 a^{3/2}\left(a-b x^4\right)^{1/4}}$$

Result (type 5, 84 leaves):

$$\frac{-4 a^2-2 a b x^4+6 b^2 x^8-3 b^2 x^8\left(1-\frac{b x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{b x^4}{a}\right]}{24 a^2 x^6\left(a-b x^4\right)^{1/4}}$$

Problem 1218: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^{11} \left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$-\frac{\left(a-b x^4\right)^{3/4}}{10 a x^{10}}-\frac{7 b \left(a-b x^4\right)^{3/4}}{60 a^2 x^6}-\frac{7 b^2 \left(a-b x^4\right)^{3/4}}{40 a^3 x^2}-\frac{7 b^{5/2} \left(1-\frac{b x^4}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{40 a^{5/2} \left(a-b x^4\right)^{1/4}}$$

Result (type 5, 95 leaves):

$$\frac{-24 a^3-4 a^2 b x^4-14 a b^2 x^8+42 b^3 x^{12}-21 b^3 x^{12} \left(1-\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{b x^4}{a}\right]}{240 a^3 x^{10} \left(a-b x^4\right)^{1/4}}$$

Problem 1227: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{\left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$-\frac{7 a^2 \left(a-b x^4\right)^{3/4}}{40 b^3 x}-\frac{7 a x^3 \left(a-b x^4\right)^{3/4}}{60 b^2}-\frac{x^7 \left(a-b x^4\right)^{3/4}}{10 b}+\frac{7 a^{5/2} \left(1-\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{40 b^{5/2} \left(a-b x^4\right)^{1/4}}$$

Result (type 5, 80 leaves):

$$\frac{x^3 \left(-7 a^2+a b x^4+6 b^2 x^8+7 a^2 \left(1-\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]\right)}{60 b^2 \left(a-b x^4\right)^{1/4}}$$

Problem 1228: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 109 leaves, 6 steps):

$$-\frac{a \left(a-b x^4\right)^{3/4}}{4 b^2 x}-\frac{x^3 \left(a-b x^4\right)^{3/4}}{6 b}+\frac{a^{3/2} \left(1-\frac{a}{b x^4}\right)^{1/4} x \,\text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{4 b^{3/2} \left(a-b x^4\right)^{1/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^3 \left(-a+b x^4+a \left(1-\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]\right)}{6 b \left(a-b x^4\right)^{1/4}}$$

- **Problem 1229: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2}{\left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$-\frac{\left(a-b x^4\right)^{3/4}}{2 b x}+\frac{\sqrt{a} \left(1-\frac{a}{b x^4}\right)^{1/4} x \, \text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{2 \sqrt{b} \left(a-b x^4\right)^{1/4}}$$

Result (type 5, 53 leaves):

$$\frac{x^3 \left(\frac{a-b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]}{3 \left(a-b x^4\right)^{1/4}}$$

- **Problem 1230: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 \left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 61 leaves, 4 steps):

$$-\frac{\sqrt{b} \left(1-\frac{a}{b x^4}\right)^{1/4} x \, \text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{\sqrt{a} \left(a-b x^4\right)^{1/4}}$$

Result (type 5, 71 leaves):

$$\frac{-3 a+3 b x^4-2 b x^4 \left(1-\frac{b x^4}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]}{3 a x \left(a-b x^4\right)^{1/4}}$$

- **Problem 1231: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^6 \left(a-b x^4\right)^{1/4}} \, dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$-\frac{\left(a-b x^4\right)^{3/4}}{5 a x^5}-\frac{2 b^{3/2} \left(1-\frac{a}{b x^4}\right)^{1/4} x \, \text{EllipticE}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{5 a^{3/2} \left(a-b x^4\right)^{1/4}}$$

Result (type 5, 84 leaves):

$$\frac{-3\left(a^2+a\,b\,x^4-2\,b^2\,x^8\right)-4\,b^2\,x^8\left(1-\frac{b\,x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b\,x^4}{a}\right]}{15\,a^2\,x^5\left(a-b\,x^4\right)^{1/4}}$$

Problem 1232: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^{10}\left(a-b\,x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 109 leaves, 6 steps):

$$-\frac{\left(a-b\,x^4\right)^{3/4}}{9\,a\,x^9}-\frac{2\,b\left(a-b\,x^4\right)^{3/4}}{15\,a^2\,x^5}-\frac{4\,b^{5/2}\left(1-\frac{a}{b\,x^4}\right)^{1/4}x\,\text{EllipticE}\left[\frac{1}{2}\text{ArcCsc}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{15\,a^{5/2}\left(a-b\,x^4\right)^{1/4}}$$

Result (type 5, 95 leaves):

$$\frac{-5\,a^3-a^2\,b\,x^4-6\,a\,b^2\,x^8+12\,b^3\,x^{12}-8\,b^3\,x^{12}\left(1-\frac{b\,x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b\,x^4}{a}\right]}{45\,a^3\,x^9\left(a-b\,x^4\right)^{1/4}}$$

Problem 1233: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^{14}\left(a-b\,x^4\right)^{1/4}}\,dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$-\frac{\left(a-b\,x^4\right)^{3/4}}{13\,a\,x^{13}}-\frac{10\,b\left(a-b\,x^4\right)^{3/4}}{117\,a^2\,x^9}-\frac{4\,b^2\left(a-b\,x^4\right)^{3/4}}{39\,a^3\,x^5}-\frac{8\,b^{7/2}\left(1-\frac{a}{b\,x^4}\right)^{1/4}x\,\text{EllipticE}\left[\frac{1}{2}\text{ArcCsc}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{39\,a^{7/2}\left(a-b\,x^4\right)^{1/4}}$$

Result (type 5, 106 leaves):

$$\frac{1}{117\,a^4\,x^{13}\left(a-b\,x^4\right)^{1/4}}\left(-9\,a^4-a^3\,b\,x^4-2\,a^2\,b^2\,x^8-12\,a\,b^3\,x^{12}+24\,b^4\,x^{16}-16\,b^4\,x^{16}\left(1-\frac{b\,x^4}{a}\right)^{1/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b\,x^4}{a}\right]\right)$$

Problem 1239: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x\left(a-b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 3, 57 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right]}{2\,a^{3/4}}-\frac{\text{ArcTanh}\left[\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right]}{2\,a^{3/4}}$$

Result (type 5, 49 leaves):

$$-\frac{\left(1-\frac{a}{b x^4}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{a}{b x^4}\right]}{3 \left(a-b x^4\right)^{3/4}}$$

- **Problem 1240: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^5 \left(a-b x^4\right)^{3/4}} \, dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{4 a x^4}-\frac{3 b \text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{7/4}}-\frac{3 b \text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{8 a^{7/4}}$$

Result (type 5, 70 leaves):

$$\frac{-a+b x^4-b \left(1-\frac{a}{b x^4}\right)^{3/4} x^4 \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{a}{b x^4}\right]}{4 a x^4 \left(a-b x^4\right)^{3/4}}$$

- **Problem 1241: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^9 \left(a-b x^4\right)^{3/4}} \, dx$$

Optimal (type 3, 108 leaves, 7 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{8 a x^8}-\frac{7 b \left(a-b x^4\right)^{1/4}}{32 a^2 x^4}-\frac{21 b^2 \text{ArcTan}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{11/4}}-\frac{21 b^2 \text{ArcTanh}\left[\frac{\left(a-b x^4\right)^{1/4}}{a^{1/4}}\right]}{64 a^{11/4}}$$

Result (type 5, 84 leaves):

$$\frac{-4 a^2-3 a b x^4+7 b^2 x^8-7 b^2 \left(1-\frac{a}{b x^4}\right)^{3/4} x^8 \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{a}{b x^4}\right]}{32 a^2 x^8 \left(a-b x^4\right)^{3/4}}$$

- **Problem 1242: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{13}}{\left(a-b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$-\frac{20 a^2 x^2 \left(a-b x^4\right)^{1/4}}{77 b^3}-\frac{10 a x^6 \left(a-b x^4\right)^{1/4}}{77 b^2}-\frac{x^{10} \left(a-b x^4\right)^{1/4}}{11 b}+\frac{40 a^{7/2} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{77 b^{7/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 92 leaves):

$$\frac{x^2\left(-20 a^3+10 a^2 b x^4+3 a b^2 x^8+7 b^3 x^{12}+20 a^3\left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]\right)}{77 b^3\left(a-b x^4\right)^{3/4}}$$

- **Problem 1243: Result unnecessarily involves higher level functions.**

$$\int \frac{x^9}{\left(a-b x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 108 leaves, 5 steps):

$$-\frac{2 a x^2\left(a-b x^4\right)^{1/4}}{7 b^2}-\frac{x^6\left(a-b x^4\right)^{1/4}}{7 b}+\frac{4 a^{5/2}\left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{7 b^{5/2}\left(a-b x^4\right)^{3/4}}$$

Result (type 5, 79 leaves):

$$\frac{x^2\left(-2 a^2+a b x^4+b^2 x^8+2 a^2\left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]\right)}{7 b^2\left(a-b x^4\right)^{3/4}}$$

- **Problem 1244: Result unnecessarily involves higher level functions.**

$$\int \frac{x^5}{\left(a-b x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{x^2\left(a-b x^4\right)^{1/4}}{3 b}+\frac{2 a^{3/2}\left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{3 b^{3/2}\left(a-b x^4\right)^{3/4}}$$

Result (type 5, 66 leaves):

$$\frac{x^2\left(-a+b x^4+a\left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]\right)}{3 b\left(a-b x^4\right)^{3/4}}$$

- **Problem 1245: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{\left(a-b x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 59 leaves, 3 steps):

$$\frac{\sqrt{a}\left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{\sqrt{b}\left(a-b x^4\right)^{3/4}}$$

Result (type 5, 53 leaves):

$$\frac{x^2 \left(\frac{a-b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]}{2 \left(a - b x^4\right)^{3/4}}$$

- **Problem 1246: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 \left(a - b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$-\frac{\left(a - b x^4\right)^{1/4}}{2 a x^2} + \frac{\sqrt{b} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{2 \sqrt{a} \left(a - b x^4\right)^{3/4}}$$

Result (type 5, 70 leaves):

$$\frac{-2 a + 2 b x^4 + b x^4 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]}{4 a x^2 \left(a - b x^4\right)^{3/4}}$$

- **Problem 1247: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^7 \left(a - b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 108 leaves, 5 steps):

$$-\frac{\left(a - b x^4\right)^{1/4}}{6 a x^6} - \frac{5 b \left(a - b x^4\right)^{1/4}}{12 a^2 x^2} + \frac{5 b^{3/2} \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{12 a^{3/2} \left(a - b x^4\right)^{3/4}}$$

Result (type 5, 84 leaves):

$$\frac{-4 a^2 - 6 a b x^4 + 10 b^2 x^8 + 5 b^2 x^8 \left(1 - \frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{b x^4}{a}\right]}{24 a^2 x^6 \left(a - b x^4\right)^{3/4}}$$

- **Problem 1248: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^{11} \left(a - b x^4\right)^{3/4}} \, dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$-\frac{\left(a-b x^4\right)^{1/4}}{10 a x^{10}}-\frac{3 b \left(a-b x^4\right)^{1/4}}{20 a^2 x^6}-\frac{3 b^2 \left(a-b x^4\right)^{1/4}}{8 a^3 x^2}+\frac{3 b^{5/2} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left[\frac{1}{2} \text{ArcSin}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{8 a^{5/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 95 leaves) :

$$\frac{-8 a^3-4 a^2 b x^4-18 a b^2 x^8+30 b^3 x^{12}+15 b^3 x^{12} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{b x^4}{a}\right]}{80 a^3 x^{10} \left(a-b x^4\right)^{3/4}}$$

Problem 1249: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{\left(a-b x^4\right)^{3/4}} \, dx$$

Optimal (type 3, 266 leaves, 12 steps) :

$$-\frac{7 a x^3 \left(a-b x^4\right)^{1/4}}{32 b^2}-\frac{x^7 \left(a-b x^4\right)^{1/4}}{8 b}-\frac{21 a^2 \text{ArcTan}\left[1-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{64 \sqrt{2} b^{11/4}}+$$
$$\frac{21 a^2 \text{ArcTan}\left[1+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{64 \sqrt{2} b^{11/4}}+\frac{21 a^2 \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{128 \sqrt{2} b^{11/4}}-\frac{21 a^2 \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{128 \sqrt{2} b^{11/4}}$$

Result (type 5, 81 leaves) :

$$\frac{x^3 \left(-7 a^2+3 a b x^4+4 b^2 x^8+7 a^2 \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]\right)}{32 b^2 \left(a-b x^4\right)^{3/4}}$$

Problem 1250: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a-b x^4\right)^{3/4}} \, dx$$

Optimal (type 3, 235 leaves, 11 steps) :

$$-\frac{x^3 \left(a-b x^4\right)^{1/4}}{4 b}-\frac{3 a \text{ArcTan}\left[1-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{8 \sqrt{2} b^{7/4}}+\frac{3 a \text{ArcTan}\left[1+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{8 \sqrt{2} b^{7/4}}+\frac{3 a \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}-\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{16 \sqrt{2} b^{7/4}}-\frac{3 a \text{Log}\left[1+\frac{\sqrt{b} x^2}{\sqrt{a-b x^4}}+\frac{\sqrt{2} b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{16 \sqrt{2} b^{7/4}}$$

Result (type 5, 66 leaves) :

$$\frac{x^3 \left(-a+b x^4+a \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{b x^4}{a}\right]\right)}{4 b \left(a-b x^4\right)^{3/4}}$$

Problem 1251: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a-b x^4\right)^{3/4}}\, dx$$

Optimal (type 3, 209 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[1-\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{2 \sqrt{2}\, b^{3/4}}+\frac{\text{ArcTan}\left[1+\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{2 \sqrt{2}\, b^{3/4}}+\frac{\text{Log}\left[1+\frac{\sqrt{b}\, x^2}{\sqrt{a-b x^4}}-\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{4 \sqrt{2}\, b^{3/4}}-\frac{\text{Log}\left[1+\frac{\sqrt{b}\, x^2}{\sqrt{a-b x^4}}+\frac{\sqrt{2}\, b^{1/4} x}{\left(a-b x^4\right)^{1/4}}\right]}{4 \sqrt{2}\, b^{3/4}}$$

Result (type 5, 53 leaves):

$$\frac{x^3 \left(\frac{a-b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^4}{a}\right]}{3 \left(a-b x^4\right)^{3/4}}$$

Problem 1256: Result unnecessarily involves higher level functions.

$$\int \frac{x^{12}}{\left(a-b x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$-\frac{3 a^2 x \left(a-b x^4\right)^{1/4}}{8 b^3}-\frac{3 a x^5 \left(a-b x^4\right)^{1/4}}{20 b^2}-\frac{x^9 \left(a-b x^4\right)^{1/4}}{10 b}-\frac{3 a^{5/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right], 2\right]}{8 b^{5/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 91 leaves):

$$\frac{-15 a^3 x+9 a^2 b x^5+2 a b^2 x^9+4 b^3 x^{13}+15 a^3 x \left(1-\frac{b x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b x^4}{a}\right]}{40 b^3 \left(a-b x^4\right)^{3/4}}$$

Problem 1257: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{\left(a-b x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 109 leaves, 6 steps):

$$-\frac{5 a x \left(a-b x^4\right)^{1/4}}{12 b^2}-\frac{x^5 \left(a-b x^4\right)^{1/4}}{6 b}-\frac{5 a^{3/2} \left(1-\frac{a}{b x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2} \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right], 2\right]}{12 b^{3/2} \left(a-b x^4\right)^{3/4}}$$

Result (type 5, 80 leaves):

$$\frac{-5\, a^2\, x+3\, a\, b\, x^5+2\, b^2\, x^9+5\, a^2\, x\, \left(1-\frac{b\, x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\, \frac{3}{4},\, \frac{5}{4},\, \frac{b\, x^4}{a}\right]}{12\, b^2\, \left(a-b\, x^4\right)^{3/4}}$$

- **Problem 1258: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4}{\left(a-b\, x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$-\frac{x\, \left(a-b\, x^4\right)^{1/4}}{2\, b}-\frac{\sqrt{a}\, \left(1-\frac{a}{b\, x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2}\, \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],\, 2\right]}{2\, \sqrt{b}\, \left(a-b\, x^4\right)^{3/4}}$$

Result (type 5, 64 leaves):

$$\frac{x\, \left(-a+b\, x^4+a\, \left(1-\frac{b\, x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\, \frac{3}{4},\, \frac{5}{4},\, \frac{b\, x^4}{a}\right]\right)}{2\, b\, \left(a-b\, x^4\right)^{3/4}}$$

- **Problem 1259: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(a-b\, x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 63 leaves, 4 steps):

$$-\frac{\sqrt{b}\, \left(1-\frac{a}{b\, x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2}\, \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],\, 2\right]}{\sqrt{a}\, \left(a-b\, x^4\right)^{3/4}}$$

Result (type 5, 48 leaves):

$$\frac{x\, \left(\frac{a-b\, x^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4},\, \frac{3}{4},\, \frac{5}{4},\, \frac{b\, x^4}{a}\right]}{\left(a-b\, x^4\right)^{3/4}}$$

- **Problem 1260: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4\, \left(a-b\, x^4\right)^{3/4}}\, dx$$

Optimal (type 4, 88 leaves, 5 steps):

$$-\frac{\left(a-b\, x^4\right)^{1/4}}{3\, a\, x^3}-\frac{2\, b^{3/2}\, \left(1-\frac{a}{b\, x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2}\, \text{ArcCsc}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],\, 2\right]}{3\, a^{3/2}\, \left(a-b\, x^4\right)^{3/4}}$$

Result (type 5, 70 leaves) :

$$\frac{-a+b\,x^4+2\,b\,x^4\left(1-\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b\,x^4}{a}\right]}{3\,a\,x^3\left(a-b\,x^4\right)^{3/4}}$$

Problem 1261: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^8\left(a-b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 111 leaves, 6 steps) :

$$-\frac{\left(a-b\,x^4\right)^{1/4}}{7\,a\,x^7}-\frac{2\,b\left(a-b\,x^4\right)^{1/4}}{7\,a^2\,x^3}-\frac{4\,b^{5/2}\left(1-\frac{a}{b\,x^4}\right)^{3/4}x^3\,\text{EllipticF}\left[\frac{1}{2}\text{ArcCsc}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{7\,a^{5/2}\left(a-b\,x^4\right)^{3/4}}$$

Result (type 5, 84 leaves) :

$$\frac{-a^2-a\,b\,x^4+2\,b^2\,x^8+4\,b^2\,x^8\left(1-\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b\,x^4}{a}\right]}{7\,a^2\,x^7\left(a-b\,x^4\right)^{3/4}}$$

Problem 1262: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^{12}\left(a-b\,x^4\right)^{3/4}}\,dx$$

Optimal (type 4, 136 leaves, 7 steps) :

$$-\frac{\left(a-b\,x^4\right)^{1/4}}{11\,a\,x^{11}}-\frac{10\,b\left(a-b\,x^4\right)^{1/4}}{77\,a^2\,x^7}-\frac{20\,b^2\left(a-b\,x^4\right)^{1/4}}{77\,a^3\,x^3}-\frac{40\,b^{7/2}\left(1-\frac{a}{b\,x^4}\right)^{3/4}x^3\,\text{EllipticF}\left[\frac{1}{2}\text{ArcCsc}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{77\,a^{7/2}\left(a-b\,x^4\right)^{3/4}}$$

Result (type 5, 95 leaves) :

$$\frac{-7\,a^3-3\,a^2\,b\,x^4-10\,a\,b^2\,x^8+20\,b^3\,x^{12}+40\,b^3\,x^{12}\left(1-\frac{b\,x^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{b\,x^4}{a}\right]}{77\,a^3\,x^{11}\left(a-b\,x^4\right)^{3/4}}$$

Problem 1263: Result unnecessarily involves higher level functions.

$$\int\frac{x^2}{\left(a-b\,x^4\right)^{5/4}}\,dx$$

Optimal (type 4, 81 leaves, 5 steps) :

$$\frac{1}{b\,x\,\left(a-b\,x^4\right)^{1/4}}-\frac{\left(1-\frac{a}{b\,x^4}\right)^{1/4}\,x\,\text{EllipticE}\left[\frac{1}{2}\,\text{ArcCsc}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{\sqrt{a}\,\sqrt{b}\,\left(a-b\,x^4\right)^{1/4}}$$

Result (type 5, 59 leaves) :

$$-\frac{x^3\,\left(-3+2\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{b\,x^4}{a}\right]\right)}{3\,a\,\left(a-b\,x^4\right)^{1/4}}$$

Problem 1330: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^4\,\left(a+b\,x^6\right)}\,dx$$

Optimal (type 3, 40 leaves, 3 steps) :

$$-\frac{1}{3\,a\,x^3}-\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right]}{3\,a^{3/2}}$$

Result (type 3, 101 leaves) :

$$\frac{-\sqrt{a}+\sqrt{b}\,x^3\,\text{ArcTan}\left[\frac{b^{1/6}\,x}{a^{1/6}}\right]+\sqrt{b}\,x^3\,\text{ArcTan}\left[\sqrt{3}-\frac{2\,b^{1/6}\,x}{a^{1/6}}\right]-\sqrt{b}\,x^3\,\text{ArcTan}\left[\sqrt{3}+\frac{2\,b^{1/6}\,x}{a^{1/6}}\right]}{3\,a^{3/2}\,x^3}$$

Problem 1350: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2}{1-x^6}\,dx$$

Optimal (type 3, 8 leaves, 2 steps) :

$$\frac{\text{ArcTanh}\left[x^3\right]}{3}$$

Result (type 3, 23 leaves) :

$$-\frac{1}{6}\,\text{Log}\left[1-x^3\right]+\frac{1}{6}\,\text{Log}\left[1+x^3\right]$$

Problem 1396: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^7}{\sqrt{2+x^6}}\,dx$$

Optimal (type 4, 186 leaves, 3 steps) :

$$\frac{1}{5}x^2\sqrt{2+x^6}-\frac{2\times 2^{5/6}\sqrt{2+\sqrt{3}}\,\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{5\times 3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Result (type 4, 133 leaves) :

$$\frac{1}{15\sqrt{2+x^6}}\left(3x^2\left(2+x^6\right)-4(-1)^{1/6}2^{1/3}3^{3/4}\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\,\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{2/3}x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 1397: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x}{\sqrt{2+x^6}}\,dx$$

Optimal (type 4, 166 leaves, 2 steps) :

$$\frac{\sqrt{2+\sqrt{3}}\,\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{2^{1/6}\,3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Result (type 4, 116 leaves) :

$$\frac{1}{3^{1/4}\sqrt{2+x^6}}(-1)^{1/6}2^{1/3}\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\,\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 1398: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{x^5\sqrt{2+x^6}}\,dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$-\frac{\sqrt{2+x^6}}{8x^4}-\frac{\sqrt{2+\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{8\times 2^{1/6}\,3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Result (type 4, 136 leaves):

$$-\frac{\sqrt{2+x^6}}{8x^4}-\frac{1}{4\times 2^{2/3}\,3^{1/4}\sqrt{2+x^6}}$$

$$(-1)^{1/6}\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

- **Problem 1402: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^9}{\sqrt{2+x^6}}\,dx$$

Optimal (type 4, 378 leaves, 5 steps):

$$\frac{1}{7}x^4\sqrt{2+x^6}-\frac{8\sqrt{2+x^6}}{7\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)}+\frac{4\times 2^{1/6}\,3^{1/4}\sqrt{2-\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{7\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}-$$

$$\frac{8\times 2^{2/3}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{7\times 3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Result (type 4, 189 leaves):

$$\frac{1}{7} x^4 \sqrt{2+x^6} + \frac{1}{7 \times 3^{1/4} \sqrt{2+x^6}} 8 \, i \, 2^{2/3} \sqrt{(-1)^{5/6} \left(-1+\left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1+\left(-\frac{1}{2}\right)^{1/3} x^2+\left(-\frac{1}{2}\right)^{2/3} x^4}$$
$$\left(-i \sqrt{3} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{1/3} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 1403: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3}{\sqrt{2+x^6}} \, dx$$

Optimal (type 4, 354 leaves, 4 steps):

$$\frac{\sqrt{2+x^6}}{2^{1/3}\left(1+\sqrt{3}\right)+x^2} - \frac{3^{1/4} \sqrt{2-\sqrt{3}} \left(2^{1/3}+x^2\right) \sqrt{\frac{2^{2/3}-2^{1/3} x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right], -7-4\sqrt{3}\right]}{2^{5/6} \sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}} \sqrt{2+x^6}} +$$
$$\frac{2^{2/3} \left(2^{1/3}+x^2\right) \sqrt{\frac{2^{2/3}-2^{1/3} x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}} \sqrt{2+x^6}}$$

Result (type 4, 170 leaves):

$$-\frac{1}{3^{1/4} \sqrt{2+x^6}} i \, 2^{2/3} \sqrt{(-1)^{5/6} \left(-1+\left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1+\left(-\frac{1}{2}\right)^{1/3} x^2+\left(-\frac{1}{2}\right)^{2/3} x^4}$$
$$\left(-i \sqrt{3} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{1/3} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6} x^2}{2^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 1404: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x^3 \sqrt{2+x^6}} \, dx$$

Optimal (type 4, 378 leaves, 5 steps):

$$-\frac{\sqrt{2+x^6}}{4x^2}+\frac{\sqrt{2+x^6}}{4\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{4\times 2^{5/6}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}+$$

$$\frac{\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{2\times 2^{1/3}\,3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Result (type 4, 189 leaves):

$$-\frac{\sqrt{2+x^6}}{4x^2}-\frac{1}{2\times 2^{1/3}\,3^{1/4}\sqrt{2+x^6}}\,\mathbb{i}\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}$$

$$\left(-\mathbb{i}\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 1420: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^{13}}{\left(2+x^6\right)^{3/2}} \, dx$$

Optimal (type 4, 202 leaves, 4 steps):

$$-\frac{x^8}{3\sqrt{2+x^6}}+\frac{8}{15}x^2\sqrt{2+x^6}-\frac{16\times 2^{5/6}\sqrt{2+\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{15\times 3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Result (type 4, 144 leaves) :

$$\frac{1}{45\sqrt{2+x^6}}\left(48x^2+9x^8-16(-1)^{1/6}2^{1/3}3^{3/4}\sqrt{-(-1)^{1/6}\left(2(-1)^{2/3}+2^{2/3}x^2\right)}\right.$$
$$\left.\sqrt{2+(-1)^{1/3}2^{2/3}x^2+(-1)^{2/3}2^{1/3}x^4}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{2/3}x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 1421: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^7}{\left(2+x^6\right)^{3/2}}\,dx$$

Optimal (type 4, 186 leaves, 3 steps) :

$$-\frac{x^2}{3\sqrt{2+x^6}}+\frac{2^{5/6}\sqrt{2+\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Result (type 4, 136 leaves) :

$$-\frac{x^2}{3\sqrt{2+x^6}}+\frac{1}{3\times 3^{1/4}\sqrt{2+x^6}}$$
$$2(-1)^{1/6}2^{1/3}\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 1422: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(2+x^6\right)^{3/2}}\,dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$\frac{x^2}{6\sqrt{2+x^6}}+\frac{\sqrt{2+\sqrt{3}}\,\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{6\times 2^{1/6}\,3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Result (type 4, 136 leaves):

$$\frac{x^2}{6\sqrt{2+x^6}}+\frac{1}{3\times 2^{2/3}\,3^{1/4}\sqrt{2+x^6}}$$

$$(-1)^{1/6}\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\,\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

Problem 1423: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^5\left(2+x^6\right)^{3/2}}\,dx$$

Optimal (type 4, 202 leaves, 4 steps):

$$\frac{1}{6x^4\sqrt{2+x^6}}-\frac{7\sqrt{2+x^6}}{48x^4}-\frac{7\sqrt{2+\sqrt{3}}\,\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{48\times 2^{1/6}\,3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Result (type 4, 146 leaves):

$$-\frac{1}{288\,x^4\,\sqrt{2+x^6}}\left(36+42\,x^6+7\,(-1)^{1/6}\,2^{1/3}\,3^{3/4}\,x^4\,\sqrt{-(-1)^{1/6}\left(2\,(-1)^{2/3}+2^{2/3}\,x^2\right)}\right.$$
$$\left.\sqrt{2+(-1)^{1/3}\,2^{2/3}\,x^2+(-1)^{2/3}\,2^{1/3}\,x^4}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{2/3}\,x^2\right)}}{2\times 3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 1428: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{15}}{\left(2+x^6\right)^{3/2}}\,dx$$

Optimal (type 4, 394 leaves, 6 steps):

$$-\frac{x^{10}}{3\,\sqrt{2+x^6}}+\frac{10}{21}\,x^4\,\sqrt{2+x^6}-\frac{80\,\sqrt{2+x^6}}{21\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)}+$$
$$\frac{40\times 2^{1/6}\,\sqrt{2-\sqrt{3}}\,\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}\,x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],\,-7-4\,\sqrt{3}\right]}{7\times 3^{3/4}\,\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}-$$
$$\frac{80\times 2^{2/3}\,\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}\,x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],\,-7-4\,\sqrt{3}\right]}{21\times 3^{1/4}\,\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Result (type 4, 195 leaves):

$$\frac{1}{63\sqrt{2+x^6}}\left(3x^4\left(20+3x^6\right)+40\times 2^{2/3}\,3^{3/4}\sqrt{-(-1)^{1/6}\left(2(-1)^{2/3}+2^{2/3}x^2\right)}\sqrt{2+(-1)^{1/3}2^{2/3}x^2+(-1)^{2/3}2^{1/3}x^4}\right.$$
$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{2/3}x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{2/3}x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 1429: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^9}{\left(2+x^6\right)^{3/2}}\,dx$$

Optimal (type 4, 376 leaves, 5 steps):

$$-\frac{x^4}{3\sqrt{2+x^6}}+\frac{4\sqrt{2+x^6}}{3\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)}-\frac{2\times 2^{1/6}\sqrt{2-\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{3^{3/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}+$$
$$\frac{4\times 2^{2/3}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Result (type 4, 177 leaves):

$$\frac{1}{9\sqrt{2+x^6}}\left(-3x^4-4\times 2^{2/3}\,3^{3/4}\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}\right.$$

$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{2/3}x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(2+2^{2/3}x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 1430: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{\left(2+x^6\right)^{3/2}}\,dx$$

Optimal (type 4, 378 leaves, 5 steps) :

$$\frac{x^4}{6\sqrt{2+x^6}}-\frac{\sqrt{2+x^6}}{6\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)}+\frac{\sqrt{2-\sqrt{3}}\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{2\times 2^{5/6}\,3^{3/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}-$$

$$\frac{\left(2^{1/3}+x^2\right)\sqrt{\frac{2^{2/3}-2^{1/3}x^2+x^4}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3}\left(1-\sqrt{3}\right)+x^2}{2^{1/3}\left(1+\sqrt{3}\right)+x^2}\right],-7-4\sqrt{3}\right]}{3\times 2^{1/3}\,3^{1/4}\sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Result (type 4, 189 leaves) :

$$\frac{x^4}{6\sqrt{2+x^6}}+\frac{1}{3\times 2^{1/3}\,3^{1/4}\sqrt{2+x^6}}\,i\sqrt{(-1)^{5/6}\left(-1+\left(-\frac{1}{2}\right)^{1/3}x^2\right)}\sqrt{1+\left(-\frac{1}{2}\right)^{1/3}x^2+\left(-\frac{1}{2}\right)^{2/3}x^4}$$

$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{(-1)^{5/6}x^2}{2^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 1431: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3 \left(2+x^6\right)^{3/2}} \, dx$$

Optimal (type 4, 394 leaves, 6 steps):

$$\frac{1}{6 x^2 \sqrt{2+x^6}}-\frac{5 \sqrt{2+x^6}}{24 x^2}+\frac{5 \sqrt{2+x^6}}{24 \left(2^{1/3} \left(1+\sqrt{3}\right)+x^2\right)}-\frac{5 \sqrt{2-\sqrt{3}} \left(2^{1/3}+x^2\right) \sqrt{\frac{2^{2/3}-2^{1/3} x^2+x^4}{\left(2^{1/3} \left(1+\sqrt{3}\right)+x^2\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{2^{1/3} \left(1-\sqrt{3}\right)+x^2}{2^{1/3} \left(1+\sqrt{3}\right)+x^2}\right],-7-4 \sqrt{3}\right]}{8\times 2^{5/6} 3^{3/4} \sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3} \left(1+\sqrt{3}\right)+x^2\right)^2}} \sqrt{2+x^6}}+$$

$$\frac{5 \left(2^{1/3}+x^2\right) \sqrt{\frac{2^{2/3}-2^{1/3} x^2+x^4}{\left(2^{1/3} \left(1+\sqrt{3}\right)+x^2\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{1/3} \left(1-\sqrt{3}\right)+x^2}{2^{1/3} \left(1+\sqrt{3}\right)+x^2}\right],-7-4 \sqrt{3}\right]}{12\times 2^{1/3} 3^{1/4} \sqrt{\frac{2^{1/3}+x^2}{\left(2^{1/3} \left(1+\sqrt{3}\right)+x^2\right)^2}} \sqrt{2+x^6}}$$

Result (type 4, 198 leaves):

$$\frac{1}{72 x^2 \sqrt{2+x^6}} i \left(6 i x^6+9 i \left(2+x^6\right)+5 i\, 2^{2/3} 3^{3/4} x^2 \sqrt{(-1)^{5/6} \left(-1+\left(-\frac{1}{2}\right)^{1/3} x^2\right)} \sqrt{1+\left(-\frac{1}{2}\right)^{1/3} x^2+\left(-\frac{1}{2}\right)^{2/3} x^4}\right.$$

$$\left.\left(\sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right) \left(2+2^{2/3} x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-i+\sqrt{3}\right) \left(2+2^{2/3} x^2\right)}}{2\times 3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

Problem 1458: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^8\right)} \, dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$-\frac{1}{4 a x^4}-\frac{\sqrt{b} \,\text{ArcTan}\left[\frac{\sqrt{b} x^4}{\sqrt{a}}\right]}{4 a^{3/2}}$$

Result (type 3, 164 leaves):

$$\frac{1}{4 a^{3/2} x^4}\left(-\sqrt{a}+\sqrt{b}\, x^4 \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{\pi}{8}\right]-\frac{b^{1/8} x \operatorname{Csc}\left[\frac{\pi}{8}\right]}{a^{1/8}}\right]+\sqrt{b}\, x^4 \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{\pi}{8}\right]+\frac{b^{1/8} x \operatorname{Csc}\left[\frac{\pi}{8}\right]}{a^{1/8}}\right]+\sqrt{b}\, x^4 \operatorname{ArcTan}\left[\frac{b^{1/8} x \operatorname{Sec}\left[\frac{\pi}{8}\right]}{a^{1/8}}-\operatorname{Tan}\left[\frac{\pi}{8}\right]\right]-\sqrt{b}\, x^4 \operatorname{ArcTan}\left[\frac{b^{1/8} x \operatorname{Sec}\left[\frac{\pi}{8}\right]}{a^{1/8}}+\operatorname{Tan}\left[\frac{\pi}{8}\right]\right]\right)$$

- **Problem 1474: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{1-x^8}\, dx$$

Optimal (type 3, 8 leaves, 2 steps) :

$$\frac{\operatorname{ArcTanh}\left[x^4\right]}{4}$$

Result (type 3, 23 leaves) :

$$-\frac{1}{8} \operatorname{Log}\left[1-x^4\right]+\frac{1}{8} \operatorname{Log}\left[1+x^4\right]$$

- **Problem 1496: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^3\left(1+x^8\right)}\, dx$$

Optimal (type 3, 100 leaves, 11 steps) :

$$-\frac{1}{2 x^2}+\frac{\operatorname{ArcTan}\left[1-\sqrt{2}\, x^2\right]}{4 \sqrt{2}}-\frac{\operatorname{ArcTan}\left[1+\sqrt{2}\, x^2\right]}{4 \sqrt{2}}-\frac{\operatorname{Log}\left[1-\sqrt{2}\, x^2+x^4\right]}{8 \sqrt{2}}+\frac{\operatorname{Log}\left[1+\sqrt{2}\, x^2+x^4\right]}{8 \sqrt{2}}$$

Result (type 3, 208 leaves) :

$$-\frac{1}{2 x^2}-\frac{\operatorname{ArcTan}\left[\left(x-\operatorname{Cos}\left[\frac{\pi}{8}\right]\right) \operatorname{Csc}\left[\frac{\pi}{8}\right]\right]}{4 \sqrt{2}}+\frac{\operatorname{ArcTan}\left[\left(x+\operatorname{Cos}\left[\frac{\pi}{8}\right]\right) \operatorname{Csc}\left[\frac{\pi}{8}\right]\right]}{4 \sqrt{2}}-\frac{\operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{\pi}{8}\right]\left(x-\operatorname{Sin}\left[\frac{\pi}{8}\right]\right)\right]}{4 \sqrt{2}}+\frac{\operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{\pi}{8}\right]\left(x+\operatorname{Sin}\left[\frac{\pi}{8}\right]\right)\right]}{4 \sqrt{2}}-\frac{\operatorname{Log}\left[1+x^2-2 x \operatorname{Cos}\left[\frac{\pi}{8}\right]\right]}{8 \sqrt{2}}-\frac{\operatorname{Log}\left[1+x^2+2 x \operatorname{Cos}\left[\frac{\pi}{8}\right]\right]}{8 \sqrt{2}}+\frac{\operatorname{Log}\left[1+x^2-2 x \operatorname{Sin}\left[\frac{\pi}{8}\right]\right]}{8 \sqrt{2}}+\frac{\operatorname{Log}\left[1+x^2+2 x \operatorname{Sin}\left[\frac{\pi}{8}\right]\right]}{8 \sqrt{2}}$$

- **Problem 1510: Result unnecessarily involves higher level functions.**

$$\int x \sqrt{1+x^8}\, dx$$

Optimal (type 4, 62 leaves, 3 steps) :

$$\frac{1}{6} x^2 \sqrt{1+x^8}+\frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{6 \sqrt{1+x^8}}$$

Result (type 5, 34 leaves):

$$\frac{1}{6} x^2 \left(\sqrt{1+x^8}+2\, \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right]\right)$$

- **Problem 1512: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{1+x^8}}{x^3} \, dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$-\frac{\sqrt{1+x^8}}{2 x^2}+\frac{x^2 \sqrt{1+x^8}}{1+x^4}-\frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{\sqrt{1+x^8}}+\frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{2 \sqrt{1+x^8}}$$

Result (type 5, 39 leaves):

$$-\frac{\sqrt{1+x^8}}{2 x^2}+\frac{1}{3} x^6\, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]$$

- **Problem 1524: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{13}}{\sqrt{1+x^8}} \, dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$\frac{1}{10} x^6 \sqrt{1+x^8}-\frac{3 x^2 \sqrt{1+x^8}}{10 \left(1+x^4\right)}+\frac{3 \left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{10 \sqrt{1+x^8}}-\frac{3 \left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{20 \sqrt{1+x^8}}$$

Result (type 5, 34 leaves):

$$\frac{1}{10} x^6 \left(\sqrt{1+x^8}-\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]\right)$$

- **Problem 1525: Result unnecessarily involves higher level functions.**

$$\int \frac{x^9}{\sqrt{1+x^8}} \, dx$$

Optimal (type 4, 62 leaves, 3 steps):

$$\frac{1}{6} x^2 \sqrt{1+x^8} - \frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}} \text{ EllipticF}\left[2 \text{ ArcTan}\left[x^2\right], \frac{1}{2}\right]}{12 \sqrt{1+x^8}}$$

Result (type 5, 34 leaves):

$$\frac{1}{6} x^2 \left(\sqrt{1+x^8} - \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right]\right)$$

- **Problem 1526: Result unnecessarily involves higher level functions.**

$$\int \frac{x^5}{\sqrt{1+x^8}} \, dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$\frac{x^2 \sqrt{1+x^8}}{2 \left(1+x^4\right)} - \frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}} \text{ EllipticE}\left[2 \text{ ArcTan}\left[x^2\right], \frac{1}{2}\right]}{2 \sqrt{1+x^8}} + \frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}} \text{ EllipticF}\left[2 \text{ ArcTan}\left[x^2\right], \frac{1}{2}\right]}{4 \sqrt{1+x^8}}$$

Result (type 5, 22 leaves):

$$\frac{1}{6} x^6 \text{ Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]$$

- **Problem 1527: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{\sqrt{1+x^8}} \, dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}} \text{ EllipticF}\left[2 \text{ ArcTan}\left[x^2\right], \frac{1}{2}\right]}{4 \sqrt{1+x^8}}$$

Result (type 5, 22 leaves):

$$\frac{1}{2} x^2 \text{ Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right]$$

- **Problem 1528: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 \sqrt{1+x^8}} \, dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{\sqrt{1+x^8}}{2 x^2}+\frac{x^2 \sqrt{1+x^8}}{2 \left(1+x^4\right)}-\frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}} \, \text{EllipticE}\left[2 \, \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{2 \sqrt{1+x^8}}+\frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{4 \sqrt{1+x^8}}$$

Result (type 5, 39 leaves):

$$-\frac{\sqrt{1+x^8}}{2 x^2}+\frac{1}{6} x^6 \, \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^8\right]$$

- **Problem 1529: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^7 \sqrt{1+x^8}} \, dx$$

Optimal (type 4, 62 leaves, 3 steps):

$$-\frac{\sqrt{1+x^8}}{6 x^6}-\frac{\left(1+x^4\right) \sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[x^2\right], \frac{1}{2}\right]}{12 \sqrt{1+x^8}}$$

Result (type 5, 36 leaves):

$$-\frac{\sqrt{1+x^8}+x^8 \, \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^8\right]}{6 x^6}$$

- **Problem 1542: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^4}{\sqrt{-2+x^{10}}} \, dx$$

Optimal (type 3, 18 leaves, 3 steps):

$$\frac{1}{5} \, \text{ArcTanh}\left[\frac{x^5}{\sqrt{-2+x^{10}}}\right]$$

Result (type 3, 42 leaves):

$$-\frac{1}{10}\,\text{Log}\left[1-\frac{x^5}{\sqrt{-2+x^{10}}}\right]+\frac{1}{10}\,\text{Log}\left[1+\frac{x^5}{\sqrt{-2+x^{10}}}\right]$$

- **Problem 1578: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx$$

Optimal (type 1, 16 leaves, 1 step):

$$-\frac{\left(a+\frac{b}{x}\right)^4}{4\,b}$$

Result (type 1, 39 leaves):

$$-\frac{b^3}{4\,x^4}-\frac{a\,b^2}{x^3}-\frac{3\,a^2\,b}{2\,x^2}-\frac{a^3}{x}$$

- **Problem 1588: Result more than twice size of optimal antiderivative.**

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx$$

Optimal (type 1, 47 leaves, 3 steps):

$$\frac{b^2\,(b+a\,x)^9}{9\,a^3}-\frac{b\,(b+a\,x)^{10}}{5\,a^3}+\frac{(b+a\,x)^{11}}{11\,a^3}$$

Result (type 1, 102 leaves):

$$\frac{b^8\,x^3}{3}+2\,a\,b^7\,x^4+\frac{28}{5}\,a^2\,b^6\,x^5+\frac{28}{3}\,a^3\,b^5\,x^6+10\,a^4\,b^4\,x^7+7\,a^5\,b^3\,x^8+\frac{28}{9}\,a^6\,b^2\,x^9+\frac{4}{5}\,a^7\,b\,x^{10}+\frac{a^8\,x^{11}}{11}$$

- **Problem 1589: Result more than twice size of optimal antiderivative.**

$$\int \left(a+\frac{b}{x}\right)^8 x^9\,dx$$

Optimal (type 1, 30 leaves, 3 steps):

$$-\frac{b\,(b+a\,x)^9}{9\,a^2}+\frac{(b+a\,x)^{10}}{10\,a^2}$$

Result (type 1, 104 leaves):

$$\frac{b^8\,x^2}{2}+\frac{8}{3}\,a\,b^7\,x^3+7\,a^2\,b^6\,x^4+\frac{56}{5}\,a^3\,b^5\,x^5+\frac{35}{3}\,a^4\,b^4\,x^6+8\,a^5\,b^3\,x^7+\frac{7}{2}\,a^6\,b^2\,x^8+\frac{8}{9}\,a^7\,b\,x^9+\frac{a^8\,x^{10}}{10}$$

Problem 1600: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx$$

Optimal (type 1, 16 leaves, 1 step):

$$-\frac{\left(a+\frac{b}{x}\right)^9}{9\,b}$$

Result (type 1, 96 leaves):

$$-\frac{b^8}{9\,x^9}-\frac{a\,b^7}{x^8}-\frac{4\,a^2\,b^6}{x^7}-\frac{28\,a^3\,b^5}{3\,x^6}-\frac{14\,a^4\,b^4}{x^5}-\frac{14\,a^5\,b^3}{x^4}-\frac{28\,a^6\,b^2}{3\,x^3}-\frac{4\,a^7\,b}{x^2}-\frac{a^8}{x}$$

Problem 1601: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx$$

Optimal (type 1, 36 leaves, 3 steps):

$$-\frac{(b+a\,x)^9}{10\,b\,x^{10}}+\frac{a\,(b+a\,x)^9}{90\,b^2\,x^9}$$

Result (type 1, 104 leaves):

$$-\frac{b^8}{10\,x^{10}}-\frac{8\,a\,b^7}{9\,x^9}-\frac{7\,a^2\,b^6}{2\,x^8}-\frac{8\,a^3\,b^5}{x^7}-\frac{35\,a^4\,b^4}{3\,x^6}-\frac{56\,a^5\,b^3}{5\,x^5}-\frac{7\,a^6\,b^2}{x^4}-\frac{8\,a^7\,b}{3\,x^3}-\frac{a^8}{2\,x^2}$$

Problem 1839: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx$$

Optimal (type 1, 16 leaves, 1 step):

$$-\frac{\left(a+\frac{b}{x^2}\right)^4}{8\,b}$$

Result (type 1, 43 leaves):

$$-\frac{b^3}{8\,x^8}-\frac{a\,b^2}{2\,x^6}-\frac{3\,a^2\,b}{4\,x^4}-\frac{a^3}{2\,x^2}$$

- **Problem 1917: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\ x}\,dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 50 leaves):

$$\frac{\sqrt{b+a\,x^2}\ \text{ArcTanh}\left[\frac{\sqrt{a}\ x}{\sqrt{b+a\,x^2}}\right]}{\sqrt{a}\ \sqrt{a+\frac{b}{x^2}}\ x}$$

- **Problem 1927: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\ x}\,dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-a+\frac{b}{x^2}}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 56 leaves):

$$\frac{\sqrt{-b+a\,x^2}\ \text{ArcTanh}\left[\frac{\sqrt{a}\ x}{\sqrt{-b+a\,x^2}}\right]}{\sqrt{a}\ \sqrt{-a+\frac{b}{x^2}}\ x}$$

- **Problem 1928: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}\ x^2}\,dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-\frac{\text{ArcCsch}\left[\frac{\sqrt{2}\,x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 56 leaves) :

$$\frac{\sqrt{b+2\,x^2}\,\left(\text{Log}[x]-\text{Log}\left[b+\sqrt{b}\,\sqrt{b+2\,x^2}\,\right]\right)}{\sqrt{b}\,\sqrt{2+\frac{b}{x^2}}\,x}$$

Problem 1929: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx$$

Optimal (type 3, 20 leaves, 2 steps) :

$$-\frac{\text{ArcCsc}\left[\frac{\sqrt{2}\,x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 64 leaves) :

$$-\frac{i\,\sqrt{2-\frac{b}{x^2}}\,x\,\text{Log}\left[\frac{2\left(-i\,\sqrt{b}+\sqrt{-b+2\,x^2}\right)}{x}\right]}{\sqrt{b}\,\sqrt{-b+2\,x^2}}$$

Problem 1959: Result more than twice size of optimal antiderivative.

$$\int\left(1+\frac{b}{x^2}\right)^{3/2}(c\,x)^m\,dx$$

Optimal (type 5, 44 leaves, 2 steps) :

$$\frac{(c\,x)^{1+m}\,\text{Hypergeometric2F1}\left[-\frac{3}{2},\,\frac{1}{2}\,(-1-m),\,\frac{1-m}{2},\,-\frac{b}{x^2}\right]}{c\,(1+m)}$$

Result (type 5, 100 leaves) :

$$\frac{1}{(-2+m)\,m\,x\,\sqrt{\frac{b+x^2}{b}}}\sqrt{1+\frac{b}{x^2}}\,(c\,x)^m\left(b\,m\,\text{Hypergeometric2F1}\left[-\frac{1}{2},\,-1+\frac{m}{2},\,\frac{m}{2},\,-\frac{x^2}{b}\right]+(-2+m)\,x^2\,\text{Hypergeometric2F1}\left[-\frac{1}{2},\,\frac{m}{2},\,1+\frac{m}{2},\,-\frac{x^2}{b}\right]\right)$$

- **Problem 1962: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c\,x)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx$$

Optimal (type 5, 44 leaves, 2 steps):

$$\frac{(c\,x)^{1+m}\,\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{b}{x^2}\right]}{c\,(1+m)}$$

Result (type 5, 91 leaves):

$$\frac{x\,(c\,x)^m\sqrt{\frac{b+x^2}{b}}\left(\text{Hypergeometric2F1}\left[\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},-\frac{x^2}{b}\right]-\text{Hypergeometric2F1}\left[\frac{3}{2},1+\frac{m}{2},2+\frac{m}{2},-\frac{x^2}{b}\right]\right)}{(2+m)\sqrt{1+\frac{b}{x^2}}}$$

- **Problem 1998: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+\frac{b}{x^3}}\,x^7\,dx$$

Optimal (type 4, 291 leaves, 5 steps):

$$-\frac{21\,b^2\sqrt{a+\frac{b}{x^3}}\,x^2}{320\,a^2}+\frac{3\,b\sqrt{a+\frac{b}{x^3}}\,x^5}{80\,a}+\frac{1}{8}\sqrt{a+\frac{b}{x^3}}\,x^8-$$

$$\left(7\times 3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(320\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)$$

Result (type 4, 207 leaves):

$$\frac{1}{320\, a^2\, (-b)^{1/3}\, (b + a\, x^3)}\sqrt{a + \frac{b}{x^3}}\; x^2 \left((-b)^{1/3} \left(-21\, b^3 - 9\, a\, b^2\, x^3 + 52\, a^2\, b\, x^6 + 40\, a^3\, x^9\right) - \right.$$

$$\left. 7\, i\, 3^{3/4}\, a^{1/3}\, b^3 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}\, x}\right)}\; x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3}\, x}{a^{1/3}} + x^2}{x^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i\, (-b)^{1/3}}{a^{1/3}\, x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 1999: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}}\; x^4\, dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$\frac{3\, b \sqrt{a + \frac{b}{x^3}}\; x^2}{20\, a} + \frac{1}{5}\sqrt{a + \frac{b}{x^3}}\; x^5 + \frac{3^{3/4}\sqrt{2 + \sqrt{3}}\; b^{5/3} \left(a^{1/3} + \frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3} + \frac{b^{2/3}}{x^2} - \frac{a^{1/3}\, b^{1/3}}{x}}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + \frac{b^{1/3}}{x}\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + \frac{b^{1/3}}{x}}{\left(1 + \sqrt{3}\right) a^{1/3} + \frac{b^{1/3}}{x}}\right], -7 - 4\sqrt{3}\right]}{20\, a \sqrt{a + \frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3} + \frac{b^{1/3}}{x}\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + \frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 196 leaves):

$$\frac{1}{20\, a\, (-b)^{1/3}\, (b + a\, x^3)}\sqrt{a + \frac{b}{x^3}}\; x^2 \left((-b)^{1/3} \left(3\, b^2 + 7\, a\, b\, x^3 + 4\, a^2\, x^6\right) + \right.$$

$$\left. i\, 3^{3/4}\, a^{1/3}\, b^2 \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3}}{a^{1/3}\, x}\right)}\; x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}} + \frac{(-b)^{1/3}\, x}{a^{1/3}} + x^2}{x^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i\, (-b)^{1/3}}{a^{1/3}\, x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 2000: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + \frac{b}{x^3}}\; x\, dx$$

Optimal (type 4, 242 leaves, 3 steps):

$$\frac{1}{2}\sqrt{a+\frac{b}{x^3}}\,x^2-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\;b^{2/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 162 leaves):

$$\frac{1}{2}\sqrt{a+\frac{b}{x^3}}\,x^2\left(1+\frac{1}{b+a\,x^3}\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,(-b)^{2/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\;x\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}x}{a^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 2001: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$$

Optimal (type 4, 243 leaves, 3 steps):

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{5\,x}-\frac{2\times3^{3/4}\sqrt{2+\sqrt{3}}\;a\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{5\,b^{1/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 164 leaves):

$$\frac{1}{5\,x}2\sqrt{a+\frac{b}{x^3}}$$

$$\left(-1-1\Big/\left(\left(-b\right)^{1/3}\left(b+a\,x^3\right)\right)\,\mathrm{i}\,3^{3/4}\,a^{4/3}\sqrt{\left(-1\right)^{5/6}\left(-1+\frac{\left(-b\right)^{1/3}}{a^{1/3}\,x}\right)}\,x^4\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\,x}{a^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]\right)$$

- **Problem 2002: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{11\,x^4}-\frac{6\,a\sqrt{a+\frac{b}{x^3}}}{55\,b\,x}+\left(4\times3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(55\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)$$

Result (type 4, 192 leaves):

$$\frac{1}{55\,\left(-b\right)^{4/3}x^4\left(b+a\,x^3\right)}2\sqrt{a+\frac{b}{x^3}}\left(\left(-b\right)^{1/3}\left(5\,b^2+8\,a\,b\,x^3+3\,a^2\,x^6\right)-\right.$$

$$\left.2\,\mathrm{i}\,3^{3/4}\,a^{7/3}\sqrt{\left(-1\right)^{5/6}\left(-1+\frac{\left(-b\right)^{1/3}}{a^{1/3}\,x}\right)}\,x^7\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\,x}{a^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]\right)$$

Problem 2003: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$$

Optimal (type 4, 291 leaves, 5 steps):

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7}-\frac{6a\sqrt{a+\frac{b}{x^3}}}{187bx^4}+\frac{48a^2\sqrt{a+\frac{b}{x^3}}}{935b^2x}-$$

$$\left(32\times 3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(935\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)$$

Result (type 4, 203 leaves):

$$\frac{1}{935\,(-b)^{7/3}x^7\left(b+a x^3\right)}2\sqrt{a+\frac{b}{x^3}}\left((-b)^{1/3}\left(-55b^3-70ab^2x^3+9a^2bx^6+24a^3x^9\right)-\right.$$

$$\left.16\,i\,3^{3/4}a^{10/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}x}\right)}\,x^{10}\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}x}{a^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 2004: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+\frac{b}{x^3}}\,x^6\,dx$$

Optimal (type 4, 563 leaves, 7 steps):

$$\frac{15\, b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112\, a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{15\, b^2\sqrt{a+\frac{b}{x^3}}\; x}{112\, a^2}+\frac{3\, b\sqrt{a+\frac{b}{x^3}}\; x^4}{56\, a}+\frac{1}{7}\sqrt{a+\frac{b}{x^3}}\; x^7-\left(15\times 3^{1/4}\sqrt{2-\sqrt{3}}\; b^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(224\, a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)+\frac{5\times 3^{3/4}\, b^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{56\sqrt{2}\; a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 375 leaves) :

$$\frac{1}{112\, a^2}\sqrt{a+\frac{b}{x^3}}\; x\left(-\frac{15\, a^{1/3}b^2 x}{b^{1/3}+a^{1/3}x}+2\, a\, x^3\left(3\, b+8\, a\, x^3\right)-\left(15\,(-1)^{2/3}\, b^{7/3}\left(b^{1/3}+a^{1/3}x\right)\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}x\left(b^{1/3}-(-1)^{1/3}a^{1/3}x\right)}{\left(b^{1/3}+a^{1/3}x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}a^{1/3}x}{b^{1/3}+a^{1/3}x}}\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)\Big/\left(2\left(-1+(-1)^{2/3}\right)\left(b^{2/3}-a^{1/3}b^{1/3}x+a^{2/3}x^2\right)\right)\right)$$

- **Problem 2005: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{a+\frac{b}{x^3}}\; x^3\, dx$$

Optimal (type 4, 539 leaves, 6 steps) :

$$-\frac{3\,b^{4/3}\sqrt{a+\frac{b}{x^3}}}{8\,a\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{3\,b\sqrt{a+\frac{b}{x^3}}\;x}{8\,a}+\frac{1}{4}\sqrt{a+\frac{b}{x^3}}\;x^4+$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;b^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(16\,a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)-\frac{3^{3/4}\,b^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{4\sqrt{2}\,a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 359 leaves) :

$$\frac{1}{8}\sqrt{a+\frac{b}{x^3}}\;x\left(2\,x^3+\frac{3\,b\,x}{a^{2/3}\,b^{1/3}+a\,x}+\left(3\,(-1)^{2/3}\,b^{4/3}\left(b^{1/3}+a^{1/3}\,x\right)\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}\right.\right.$$

$$\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)\Big/$$

$$\left.\left(2\left(-1+(-1)^{2/3}\right)a\left(b^{2/3}-a^{1/3}\,b^{1/3}\,x+a^{2/3}\,x^2\right)\right)\right)$$

- **Problem 2006: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{a+\frac{b}{x^3}}\;dx$$

Optimal (type 4, 507 leaves, 5 steps) :

$$-\frac{3\,b^{1/3}\sqrt{a+\frac{b}{x^3}}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}+\sqrt{a+\frac{b}{x^3}}\;x+$$

$$\left(3\times3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\,b^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)-\frac{\sqrt{2}\;3^{3/4}\,a^{1/3}\,b^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 351 leaves) :

$$\sqrt{a+\frac{b}{x^3}}\;x\left(-2+\frac{3\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}+\left(3\,(-1)^{2/3}\,b^{1/3}\left(b^{1/3}+a^{1/3}\,x\right)\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}\right.\right.$$

$$\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\Bigg)\Bigg/$$

$$\left.\left.\left(2\left(-1+(-1)^{2/3}\right)\left(b^{2/3}-a^{1/3}\,b^{1/3}\,x+a^{2/3}\,x^2\right)\right)\right)$$

- **Problem 2007: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

Optimal (type 4, 517 leaves, 5 steps) :

$$-\frac{6\,a\sqrt{a+\frac{b}{x^3}}}{7\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2\sqrt{a+\frac{b}{x^3}}}{7\,x^2}+$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(7\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)-\frac{2\sqrt{2}\;3^{3/4}\,a^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{7\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 366 leaves) :

$$\frac{1}{7\,b}2\sqrt{a+\frac{b}{x^3}}\;x\left(-3\,a-\frac{b}{x^3}+\frac{3\,a^{4/3}\,x}{b^{1/3}+a^{1/3}\,x}+\left(3\,(-1)^{2/3}\,a\,b^{1/3}\left(b^{1/3}+a^{1/3}\,x\right)\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}\right.\right.$$

$$\left.\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)\Bigg/\right.$$

$$\left.\left(2\left(-1+(-1)^{2/3}\right)\left(b^{2/3}-a^{1/3}\,b^{1/3}\,x+a^{2/3}\,x^2\right)\right)\right)$$

- **Problem 2008: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

Optimal (type 4, 541 leaves, 6 steps) :

$$\frac{24 a^2 \sqrt{a+\frac{b}{x^3}}}{91 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2 \sqrt{a+\frac{b}{x^3}}}{13 x^5}-\frac{6 a \sqrt{a+\frac{b}{x^3}}}{91 b x^2}-$$

$$\left(12\times 3^{1/4} \sqrt{2-\sqrt{3}}\ a^{7/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3} b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{5/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)+\frac{8 \sqrt{2}\ 3^{3/4} a^{7/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3} b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{91 b^{5/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 377 leaves) :

$$\frac{1}{91 b^2} 2 \sqrt{a+\frac{b}{x^3}}\ x$$

$$\left(12 a^2-\frac{7 b^2}{x^6}-\frac{3 a b}{x^3}-\frac{12 a^{7/3} x}{b^{1/3}+a^{1/3} x}-\left(6 (-1)^{2/3} a^2 b^{1/3} \left(b^{1/3}+a^{1/3} x\right) \sqrt{\frac{\left(1+(-1)^{1/3}\right) a^{1/3} x \left(b^{1/3}-(-1)^{1/3} a^{1/3} x\right)}{\left(b^{1/3}+a^{1/3} x\right)^2}} \sqrt{\frac{b^{1/3}+(-1)^{2/3} a^{1/3} x}{b^{1/3}+a^{1/3} x}}\right.\right.$$

$$\left.\left.\left(\left(-3-i\sqrt{3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right) a^{1/3} x}{b^{1/3}+a^{1/3} x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right) a^{1/3} x}{b^{1/3}+a^{1/3} x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)\Big/\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right) \left(b^{2/3}-a^{1/3} b^{1/3} x+a^{2/3} x^2\right)\right)\right)$$

Problem 2009: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

Optimal (type 4, 565 leaves, 7 steps):

$$-\frac{240\,a^3\sqrt{a+\frac{b}{x^3}}}{1729\,b^{8/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19\,x^8}-\frac{6\,a\sqrt{a+\frac{b}{x^3}}}{247\,b\,x^5}+\frac{60\,a^2\sqrt{a+\frac{b}{x^3}}}{1729\,b^2\,x^2}+$$

$$\left(120\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{10/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(1729\,b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)-\frac{80\sqrt{2}\,3^{3/4}\,a^{10/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{1729\,b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 388 leaves):

$$\frac{1}{1729\,b^3}2\sqrt{a+\frac{b}{x^3}}\,x\left(-120\,a^3-\frac{91\,b^3}{x^9}-\frac{21\,a\,b^2}{x^6}+\frac{30\,a^2\,b}{x^3}+\frac{120\,a^{10/3}\,x}{b^{1/3}+a^{1/3}\,x}+\left(60\,(-1)^{2/3}\,a^3\,b^{1/3}\,\left(b^{1/3}+a^{1/3}\,x\right)\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\right.\right.$$

$$\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}\left(\left(-3-\mathrm{i}\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]+\right.$$

$$\left.\left.\left.\left(1+\mathrm{i}\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]\right)\right)\Big/\left(\left(-1+(-1)^{2/3}\right)\left(b^{2/3}-a^{1/3}\,b^{1/3}\,x+a^{2/3}\,x^2\right)\right)\right)$$

- **Problem 2019: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right]}{3\sqrt{a}}$$

Result (type 3, 59 leaves):

$$\frac{2\sqrt{b+a\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b+a\,x^3}}\right]}{3\sqrt{a}\,\sqrt{a+\frac{b}{x^3}}\,x^{3/2}}$$

- **Problem 2024: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Optimal (type 4, 294 leaves, 5 steps) :

$$\frac{91\,b^2\sqrt{a+\frac{b}{x^3}}\,x^2}{320\,a^3}-\frac{13\,b\sqrt{a+\frac{b}{x^3}}\,x^5}{80\,a^2}+\frac{\sqrt{a+\frac{b}{x^3}}\,x^8}{8\,a}+\frac{91\sqrt{2+\sqrt{3}}\,b^{8/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{320\times 3^{1/4}\,a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 199 leaves) :

$$\frac{1}{960\,a^3\,(-b)^{1/3}\sqrt{a+\frac{b}{x^3}}\,x}\left(3\,(-b)^{1/3}\left(91\,b^3+39\,a\,b^2\,x^3-12\,a^2\,b\,x^6+40\,a^3\,x^9\right)+\right.$$
$$\left.91\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,b^3\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\,x\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}x}{a^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 2025: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Optimal (type 4, 270 leaves, 4 steps) :

$$-\frac{7\,b\sqrt{a+\frac{b}{x^3}}\,x^2}{20\,a^2}+\frac{\sqrt{a+\frac{b}{x^3}}\,x^5}{5\,a}-\frac{7\sqrt{2+\sqrt{3}}\,b^{5/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{20\times 3^{1/4}\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 188 leaves) :

$$\frac{1}{60\,a^2\,(-b)^{1/3}\sqrt{a+\frac{b}{x^3}}\,x}\left(-3\,(-b)^{1/3}\left(7\,b^2+3\,a\,b\,x^3-4\,a^2\,x^6\right)-\right.$$
$$\left.7\,i\,3^{3/4}\,a^{1/3}\,b^2\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\;x\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\,x}{a^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

■ **Problem 2026: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Optimal (type 4, 248 leaves, 3 steps):

$$\frac{\sqrt{a+\frac{b}{x^3}}\;x^2}{2\,a}+\frac{\sqrt{2+\sqrt{3}}\;b^{2/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{2\times 3^{1/4}\,a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 174 leaves):

$$\frac{b+a\,x^3}{2\,a\sqrt{a+\frac{b}{x^3}}\;x}+\frac{i\,b\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\sqrt{1+\frac{(-b)^{2/3}}{a^{2/3}\,x^2}+\frac{(-b)^{1/3}}{a^{1/3}\,x}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]}{2\times 3^{1/4}\,a^{2/3}\,(-b)^{1/3}\sqrt{a+\frac{b}{x^3}}}$$

■ **Problem 2027: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{a+\frac{b}{x^3}}\;x^2}\,dx$$

Optimal (type 4, 221 leaves, 2 steps):

$$-\frac{2\sqrt{2+\sqrt{3}}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{3^{1/4}\,b^{1/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 142 leaves) :

$$-\frac{2\,i\,a^{1/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}x}\right)}\sqrt{1+\frac{(-b)^{2/3}}{a^{2/3}x^2}+\frac{(-b)^{1/3}}{a^{1/3}x}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]}{3^{1/4}\,(-b)^{1/3}\sqrt{a+\frac{b}{x^3}}}$$

Problem 2028: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{a+\frac{b}{x^3}}\ x^5}\,dx$$

Optimal (type 4, 246 leaves, 3 steps) :

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{5\,b\,x}+\frac{4\sqrt{2+\sqrt{3}}\ a\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{5\times3^{1/4}\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 170 leaves) :

$$-\frac{1}{15\,(-b)^{4/3}\sqrt{a+\frac{b}{x^3}}\ x^4}$$

$$\left(-6\,(-b)^{1/3}\left(b+a\,x^3\right)+4\,i\,3^{3/4}\,a^{4/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}x}\right)}\ x^4\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}x}{a^{1/3}}+x^2}{x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}}{a^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 2029: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\, x^8}\, dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{11\, b\, x^4}+\frac{16\, a\sqrt{a+\frac{b}{x^3}}}{55\, b^2\, x}-\frac{32\sqrt{2+\sqrt{3}}\, a^2\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{55\times 3^{1/4}\, b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 184 leaves):

$$\frac{1}{165\,(-b)^{7/3}\sqrt{a+\frac{b}{x^3}}\, x^7}\left(6\,(-b)^{1/3}\left(-5\, b^2+3\, a\, b\, x^3+8\, a^2\, x^6\right)-\right.$$
$$\left.32\, i\, 3^{3/4}\, a^{7/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\, x}\right)}\, x^7\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\, x}{a^{1/3}}+x^2}{x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}}{a^{1/3}\, x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 2030: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\, dx$$

Optimal (type 4, 566 leaves, 7 steps):

$$-\frac{55\,b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112\,a^3\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{55\,b^2\sqrt{a+\frac{b}{x^3}}\;x}{112\,a^3}-\frac{11\,b\sqrt{a+\frac{b}{x^3}}\;x^4}{56\,a^2}+\frac{\sqrt{a+\frac{b}{x^3}}\;x^7}{7\,a}+$$

$$\left(55\times 3^{1/4}\sqrt{2-\sqrt{3}}\;b^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(224\,a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)-\frac{55\,b^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{56\sqrt{2}\;3^{1/4}\,a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 372 leaves) :

$$\frac{1}{112\,a^3\sqrt{a+\frac{b}{x^3}}\;x^2}\left(55\left(a^{1/3}\,b^{8/3}\,x-a^{2/3}\,b^{7/3}\,x^2+a\,b^2\,x^3\right)+2\,a\,x^3\left(-11\,b^2-3\,a\,b\,x^3+8\,a^2\,x^6\right)+\right.$$

$$1\Big/\left(2\left(-1+(-1)^{2/3}\right)\right)55\,(-1)^{2/3}\,b^{7/3}\left(b^{1/3}+a^{1/3}\,x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)$$

- **Problem 2031: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Optimal (type 4, 542 leaves, 6 steps) :

$$\frac{5\,b^{4/3}\sqrt{a+\frac{b}{x^3}}}{8\,a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{5\,b\sqrt{a+\frac{b}{x^3}}\;x}{8\,a^2}+\frac{\sqrt{a+\frac{b}{x^3}}\;x^4}{4\,a}-$$

$$\left(5\times3^{1/4}\sqrt{2-\sqrt{3}}\;b^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(16\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)+\frac{5\,b^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{4\sqrt{2}\;3^{1/4}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 356 leaves) :

$$\frac{1}{8\,a\sqrt{a+\frac{b}{x^3}}\;x^2}\left(5\,b\,x\left(-\frac{b^{2/3}}{a^{2/3}}+\frac{b^{1/3}\,x}{a^{1/3}}-x^2\right)+2\,x^3\left(b+a\,x^3\right)-\right.$$

$$1\Big/\left(2\left(-1+(-1)^{2/3}\right)a\right)5\,(-1)^{2/3}\,b^{4/3}\left(b^{1/3}+a^{1/3}\,x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)$$

■ **Problem 2032: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Optimal (type 4, 513 leaves, 5 steps) :

$$-\frac{b^{1/3}\sqrt{a+\frac{b}{x^3}}}{a\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{\sqrt{a+\frac{b}{x^3}}\,x}{a}+\frac{3^{1/4}\sqrt{2-\sqrt{3}}\,b^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{2\,a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}-$$

$$\frac{\sqrt{2}\,b^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{3^{1/4}\,a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 334 leaves) :

$$\frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^2}$$

$$\left(x\left(\frac{b^{2/3}}{a^{2/3}}-\frac{b^{1/3}x}{a^{1/3}}+x^2\right)+1\Big/\left(2\left(-1+(-1)^{2/3}\right)a\right)(-1)^{2/3}b^{1/3}\left(b^{1/3}+a^{1/3}x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}x\left(b^{1/3}-(-1)^{1/3}a^{1/3}x\right)}{\left(b^{1/3}+a^{1/3}x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}a^{1/3}x}{b^{1/3}+a^{1/3}x}}\right.$$

$$\left.\left(\left(-3-\mathrm{i}\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]+\left(1+\mathrm{i}\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]\right)\right)$$

- **Problem 2033: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^3}\,dx$$

Optimal (type 4, 491 leaves, 4 steps) :

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}-$$

$$\frac{2\sqrt{2}\;a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 335 leaves) :

$$\frac{1}{b\sqrt{a+\frac{b}{x^3}}\;x^2}2\left(-b+a^{1/3}b^{2/3}x-a^{2/3}b^{1/3}x^2+\right.$$

$$1\Big/\left(2\left(-1+(-1)^{2/3}\right)\right)(-1)^{2/3}b^{1/3}\left(b^{1/3}+a^{1/3}x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}x\left(b^{1/3}-(-1)^{1/3}a^{1/3}x\right)}{\left(b^{1/3}+a^{1/3}x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}a^{1/3}x}{b^{1/3}+a^{1/3}x}}$$

$$\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)$$

- **Problem 2034: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{a+\frac{b}{x^3}}\;x^6}\,dx$$

Optimal (type 4, 520 leaves, 5 steps) :

$$\frac{8\,a\sqrt{a+\frac{b}{x^3}}}{7\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2\sqrt{a+\frac{b}{x^3}}}{7\,b\,x^2}-$$

$$\left(4\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7\,b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)+\frac{8\sqrt{2}\;a^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{7\times 3^{1/4}\,b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 363 leaves) :

$$\frac{1}{7\,b^2\sqrt{a+\frac{b}{x^3}}\;x^2}2\left(-4\,a^{4/3}\,x\left(b^{2/3}-a^{1/3}\,b^{1/3}\,x+a^{2/3}\,x^2\right)+\frac{\left(b+a\,x^3\right)\left(-b+4\,a\,x^3\right)}{x^3}-\right.$$

$$1\Big/\left(-1+(-1)^{2/3}\right)2\,(-1)^{2/3}\,a\,b^{1/3}\left(b^{1/3}+a^{1/3}\,x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left.\left(\left(-3-\mathrm{i}\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]+\left(1+\mathrm{i}\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]\right)\right)$$

- **Problem 2035: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\;x^9}\,dx$$

Optimal (type 4, 544 leaves, 6 steps) :

$$-\frac{80 a^2 \sqrt{a+\frac{b}{x^3}}}{91 b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2 \sqrt{a+\frac{b}{x^3}}}{13 b x^5}+\frac{20 a \sqrt{a+\frac{b}{x^3}}}{91 b^2 x^2}+$$

$$\left(40\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{7/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3} b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{8/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)-\frac{80 \sqrt{2}\, a^{7/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3} b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4 \sqrt{3}\right]}{91\times 3^{1/4} b^{8/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 377 leaves) :

$$\frac{1}{91 b^3 \sqrt{a+\frac{b}{x^3}}\, x^2} 2 \left(40 a^{7/3} x \left(b^{2/3}-a^{1/3} b^{1/3} x+a^{2/3} x^2\right)-\frac{\left(b+a x^3\right) \left(7 b^2-10 a b x^3+40 a^2 x^6\right)}{x^6}+\right.$$

$$1\Big/\left(\left(-1+(-1)^{2/3}\right) 20 (-1)^{2/3} a^2 b^{1/3} \left(b^{1/3}+a^{1/3} x\right)^2 \sqrt{\frac{\left(1+(-1)^{1/3}\right) a^{1/3} x \left(b^{1/3}-(-1)^{1/3} a^{1/3} x\right)}{\left(b^{1/3}+a^{1/3} x\right)^2}} \sqrt{\frac{b^{1/3}+(-1)^{2/3} a^{1/3} x}{b^{1/3}+a^{1/3} x}}\right.$$

$$\left.\left.\left(\left(-3-i \sqrt{3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i \sqrt{3}\right) a^{1/3} x}{b^{1/3}+a^{1/3} x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i \sqrt{3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i \sqrt{3}\right) a^{1/3} x}{b^{1/3}+a^{1/3} x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)\right)$$

- **Problem 2036: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\, x^{12}}\, dx$$

Optimal (type 4, 568 leaves, 7 steps) :

$$\frac{1280\,a^3\sqrt{a+\frac{b}{x^3}}}{1729\,b^{11/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19\,b\,x^8}+\frac{32\,a\sqrt{a+\frac{b}{x^3}}}{247\,b^2\,x^5}-\frac{320\,a^2\sqrt{a+\frac{b}{x^3}}}{1729\,b^3\,x^2}-$$

$$\left(640\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{10/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(1729\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\right)+\frac{1280\sqrt{2}\;a^{10/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{1729\times 3^{1/4}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 387 leaves) :

$$\frac{1}{1729\,b^4\sqrt{a+\frac{b}{x^3}}\;x^2}2\left(-640\,a^{10/3}\,x\left(b^{2/3}-a^{1/3}b^{1/3}x+a^{2/3}x^2\right)+\frac{\left(b+a\,x^3\right)\left(-91\,b^3+112\,a\,b^2x^3-160\,a^2b\,x^6+640\,a^3x^9\right)}{x^9}-\right.$$

$$1\Big/\left(-1+(-1)^{2/3}\right)320\,(-1)^{2/3}a^3b^{1/3}\left(b^{1/3}+a^{1/3}x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}x\left(b^{1/3}-(-1)^{1/3}a^{1/3}x\right)}{\left(b^{1/3}+a^{1/3}x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}a^{1/3}x}{b^{1/3}+a^{1/3}x}}$$

$$\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)$$

- **Problem 2044: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Optimal (type 4, 315 leaves, 6 steps) :

$$\frac{1729\,b^2\sqrt{a+\frac{b}{x^3}}\,x^2}{960\,a^4}-\frac{247\,b\sqrt{a+\frac{b}{x^3}}\,x^5}{240\,a^3}-\frac{2\,x^8}{3\,a\sqrt{a+\frac{b}{x^3}}}+\frac{19\sqrt{a+\frac{b}{x^3}}\,x^8}{24\,a^2}+$$

$$\frac{1729\sqrt{2+\sqrt{3}}\,b^{8/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{960\times3^{1/4}\,a^4\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 199 leaves) :

$$\frac{1}{2880\,a^4\,(-b)^{1/3}\sqrt{a+\frac{b}{x^3}}\,x}\left(3\,(-b)^{1/3}\left(1729\,b^3+741\,a\,b^2\,x^3-228\,a^2\,b\,x^6+120\,a^3\,x^9\right)+\right.$$

$$\left.1729\,i\,3^{3/4}\,a^{1/3}\,b^3\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\,x\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\,x}{a^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 2045: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Optimal (type 4, 291 leaves, 5 steps) :

$$-\frac{91\,b\sqrt{a+\frac{b}{x^3}}\,x^2}{60\,a^3}-\frac{2\,x^5}{3\,a\sqrt{a+\frac{b}{x^3}}}+\frac{13\sqrt{a+\frac{b}{x^3}}\,x^5}{15\,a^2}-\frac{91\sqrt{2+\sqrt{3}}\,b^{5/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{60\times3^{1/4}\,a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 188 leaves) :

$$\frac{1}{180\, a^3\, (-b)^{1/3} \sqrt{a+\frac{b}{x^3}}\; x}\left(-3\, (-b)^{1/3} \left(91\, b^2+39\, a\, b\, x^3-12\, a^2\, x^6\right)-\right.$$

$$\left.91\, i\, 3^{3/4}\, a^{1/3}\, b^2 \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\, x}\right)}\; x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\, x}{a^{1/3}}+x^2}{x^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, (-b)^{1/3}}{a^{1/3}\, x}}}{3^{1/4}}\right],\, (-1)^{1/3}\right]\right)$$

Problem 2046: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\, dx$$

Optimal (type 4, 269 leaves, 4 steps):

$$-\frac{2\, x^2}{3\, a \sqrt{a+\frac{b}{x^3}}}+\frac{7 \sqrt{a+\frac{b}{x^3}}\; x^2}{6\, a^2}+\frac{7 \sqrt{2+\sqrt{3}}\; b^{2/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\, b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}\right],\, -7-4\sqrt{3}\right]}{6\times 3^{1/4}\, a^2 \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 175 leaves):

$$\frac{1}{18\, a^2\, (-b)^{1/3} \sqrt{a+\frac{b}{x^3}}\; x}$$

$$\left(3\, (-b)^{1/3} \left(7\, b+3\, a\, x^3\right)+7\, i\, 3^{3/4}\, a^{1/3}\, b \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\, x}\right)}\; x \sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\, x}{a^{1/3}}+x^2}{x^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, (-b)^{1/3}}{a^{1/3}\, x}}}{3^{1/4}}\right],\, (-1)^{1/3}\right]\right)$$

Problem 2047: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\, dx$$

Optimal (type 4, 248 leaves, 3 steps):

$$-\frac{2}{3\,a\,\sqrt{a+\frac{b}{x^3}}\;x}-\frac{2\,\sqrt{2+\sqrt{3}}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\,\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\,\sqrt{3}\right]}{3\times 3^{1/4}\,a\,b^{1/3}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 164 leaves) :

$$\frac{1}{9\,a\,(-b)^{1/3}\,\sqrt{a+\frac{b}{x^3}}\;x}\left(-6\,(-b)^{1/3}-2\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\;x\,\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\,x}{a^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 2048: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^5}\,dx$$

Optimal (type 4, 245 leaves, 3 steps) :

$$\frac{2}{3\,b\,\sqrt{a+\frac{b}{x^3}}\;x}-\frac{4\,\sqrt{2+\sqrt{3}}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\,\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\,\sqrt{3}\right]}{3\times 3^{1/4}\,b^{4/3}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 161 leaves) :

$$-\frac{1}{9\,(-b)^{4/3}\sqrt{a+\frac{b}{x^3}}\;x}\left(6\,(-b)^{1/3}-4\,\mathrm{i}\,3^{3/4}\,a^{1/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\;x\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\,x}{a^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 2049: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^8}\,dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$\frac{2}{3\,b\sqrt{a+\frac{b}{x^3}}\;x^4}-\frac{16\sqrt{a+\frac{b}{x^3}}}{15\,b^2\,x}+\frac{32\sqrt{2+\sqrt{3}}\;a\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{15\times 3^{1/4}\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 173 leaves):

$$\frac{1}{45\,(-b)^{7/3}\sqrt{a+\frac{b}{x^3}}\;x^4}\left(-6\,(-b)^{1/3}\left(3\,b+8\,a\,x^3\right)+32\,\mathrm{i}\,3^{3/4}\,a^{4/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}}{a^{1/3}\,x}\right)}\;x^4\sqrt{\frac{\frac{(-b)^{2/3}}{a^{2/3}}+\frac{(-b)^{1/3}\,x}{a^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}}{a^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 2050: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Optimal (type 4, 587 leaves, 8 steps):

$$-\frac{935\,b^{7/3}\sqrt{a+\frac{b}{x^3}}}{336\,a^4\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{935\,b^2\sqrt{a+\frac{b}{x^3}}\,x}{336\,a^4}-\frac{187\,b\sqrt{a+\frac{b}{x^3}}\,x^4}{168\,a^3}-\frac{2\,x^7}{3\,a\sqrt{a+\frac{b}{x^3}}}+\frac{17\sqrt{a+\frac{b}{x^3}}\,x^7}{21\,a^2}+$$

$$\frac{935\sqrt{2-\sqrt{3}}\,b^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{224\times 3^{3/4}\,a^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}-$$

$$\frac{935\,b^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{168\sqrt{2}\,3^{1/4}\,a^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 390 leaves):

$$\frac{1}{336\,a^4\left(a+\frac{b}{x^3}\right)^{3/2}x^5}\left(b+a\,x^3\right)\Bigg(-224\,a\,b^2\,x^3-150\,a\,b\,x^3\left(b+a\,x^3\right)+48\,a^2\,x^6\left(b+a\,x^3\right)+935\left(a^{1/3}\,b^{8/3}\,x-a^{2/3}\,b^{7/3}\,x^2+a\,b^2\,x^3\right)+$$

$$1\Big/\left(2\left(-1+(-1)^{2/3}\right)\right)935\,(-1)^{2/3}\,b^{7/3}\left(b^{1/3}+a^{1/3}\,x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\Bigg)$$

■ **Problem 2051: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Optimal (type 4, 563 leaves, 7 steps) :

$$\frac{55\,b^{4/3}\sqrt{a+\frac{b}{x^3}}}{24\,a^3\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{55\,b\sqrt{a+\frac{b}{x^3}}\,x}{24\,a^3}-\frac{2\,x^4}{3\,a\sqrt{a+\frac{b}{x^3}}}+\frac{11\sqrt{a+\frac{b}{x^3}}\,x^4}{12\,a^2}-$$

$$\frac{55\sqrt{2-\sqrt{3}}\;b^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{16\times 3^{3/4}\,a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}+$$

$$\frac{55\,b^{4/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{12\sqrt{2}\;3^{1/4}\,a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 370 leaves) :

$$\frac{1}{24\,a^3\left(a+\frac{b}{x^3}\right)^{3/2}x^5}\left(b+a\,x^3\right)\Bigg(16\,a\,b\,x^3+6\,a\,x^3\left(b+a\,x^3\right)-55\left(a^{1/3}b^{5/3}x-a^{2/3}b^{4/3}x^2+a\,b\,x^3\right)-$$

$$1\Big/\left(2\left(-1+(-1)^{2/3}\right)\right)55\,(-1)^{2/3}b^{4/3}\left(b^{1/3}+a^{1/3}x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}x\left(b^{1/3}-(-1)^{1/3}a^{1/3}x\right)}{\left(b^{1/3}+a^{1/3}x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}a^{1/3}x}{b^{1/3}+a^{1/3}x}}$$

$$\left(\left(-3-\mathrm{i}\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]+\left(1+\mathrm{i}\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}x}{b^{1/3}+a^{1/3}x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]\right)\Bigg)$$

■ Problem 2052: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Optimal (type 4, 539 leaves, 6 steps) :

$$-\frac{5\,b^{1/3}\sqrt{a+\frac{b}{x^3}}}{3\,a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2\,x}{3\,a\sqrt{a+\frac{b}{x^3}}}+\frac{5\sqrt{a+\frac{b}{x^3}}\;x}{3\,a^2}+$$

$$\frac{5\sqrt{2-\sqrt{3}}\;b^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{2\times 3^{3/4}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}-$$

$$\frac{5\sqrt{2}\;b^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\sqrt{3}\right]}{3\times 3^{1/4}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 353 leaves) :

$$\frac{1}{3\,a\,\left(a+\frac{b}{x^3}\right)^{3/2}x^5}\left(b+a\,x^3\right)\left(-2\,x^3+5\,x\left(\frac{b^{2/3}}{a^{2/3}}-\frac{b^{1/3}\,x}{a^{1/3}}+x^2\right)+\right.$$

$$1\Big/\left(2\left(-1+(-1)^{2/3}\right)a\right)5\,(-1)^{2/3}\,b^{1/3}\left(b^{1/3}+a^{1/3}\,x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)$$

- **Problem 2053: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^3}\,dx$$

Optimal (type 4, 520 leaves, 5 steps):

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,a\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}-\frac{2}{3\,a\sqrt{a+\frac{b}{x^3}}\,x^2}-\frac{\sqrt{2-\sqrt{3}}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{3^{3/4}\,a^{2/3}\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}+$$

$$\frac{2\sqrt{2}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4}\,a^{2/3}\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 352 leaves):

$$\frac{1}{3\,b\,\left(a+\frac{b}{x^3}\right)^{3/2}x^5}2\,\left(b+a\,x^3\right)\left(x^3+x\,\left(-\frac{b^{2/3}}{a^{2/3}}+\frac{b^{1/3}\,x}{a^{1/3}}-x^2\right)-\right.$$

$$1\Big/\left(2\,\left(-1+(-1)^{2/3}\right)\,a\right)\,(-1)^{2/3}\,b^{1/3}\,\left(b^{1/3}+a^{1/3}\,x\right)^2\,\sqrt{\frac{\left(1+(-1)^{1/3}\right)\,a^{1/3}\,x\,\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\,\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left.\left(\left(-3-i\,\sqrt{3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\,\sqrt{3}\right)\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\,\sqrt{3}\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\,\sqrt{3}\right)\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\,\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)$$

- **Problem 2054: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^6}\,dx$$

Optimal (type 4, 517 leaves, 5 steps):

$$-\frac{8\,\sqrt{a+\frac{b}{x^3}}}{3\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{2}{3\,b\,\sqrt{a+\frac{b}{x^3}}\,x^2}+\frac{4\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\,\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\,\sqrt{3}\right]}{3^{3/4}\,b^{5/3}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}-$$

$$\frac{8\,\sqrt{2}\,a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\,\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\,b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}}\right],\,-7-4\,\sqrt{3}\right]}{3\times 3^{1/4}\,b^{5/3}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 362 leaves):

$$\frac{1}{3\, b^2 \left(a+\frac{b}{x^3}\right)^{3/2} x^5} 2 \left(b+a\, x^3\right) \left(-a\, x^3-3 \left(b+a\, x^3\right)+4 \left(a^{1/3}\, b^{2/3}\, x-a^{2/3}\, b^{1/3}\, x^2+a\, x^3\right)+\right.$$

$$1\Big/\left(-1+(-1)^{2/3}\right) 2\, (-1)^{2/3}\, b^{1/3} \left(b^{1/3}+a^{1/3}\, x\right)^2 \sqrt{\frac{\left(1+(-1)^{1/3}\right) a^{1/3}\, x \left(b^{1/3}-(-1)^{1/3}\, a^{1/3}\, x\right)}{\left(b^{1/3}+a^{1/3}\, x\right)^2}}\, \sqrt{\frac{b^{1/3}+(-1)^{2/3}\, a^{1/3}\, x}{b^{1/3}+a^{1/3}\, x}}$$

$$\left.\left(\left(-3-i \sqrt{3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i \sqrt{3}\right) a^{1/3}\, x}{b^{1/3}+a^{1/3}\, x}}}{\sqrt{2}}\right], \frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i \sqrt{3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i \sqrt{3}\right) a^{1/3}\, x}{b^{1/3}+a^{1/3}\, x}}}{\sqrt{2}}\right], \frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)\right)$$

■ Problem 2055: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9}\, dx$$

Optimal (type 4, 541 leaves, 6 steps) :

$$\frac{80\, a \sqrt{a+\frac{b}{x^3}}}{21\, b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{2}{3\, b \sqrt{a+\frac{b}{x^3}}\, x^5}-\frac{20 \sqrt{a+\frac{b}{x^3}}}{21\, b^2\, x^2}-$$

$$\frac{40 \sqrt{2-\sqrt{3}}\, a^{4/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\, b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}\right], -7-4 \sqrt{3}\right]}{7\times 3^{3/4}\, b^{8/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}+$$

$$\frac{80 \sqrt{2}\, a^{4/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right) \sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}\, b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}}\right], -7-4 \sqrt{3}\right]}{21\times 3^{1/4}\, b^{8/3} \sqrt{a+\frac{b}{x^3}} \sqrt{\frac{a^{1/3} \left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 380 leaves) :

$$\frac{1}{21\,b^3\left(a+\frac{b}{x^3}\right)^{3/2}x^5}2\left(b+a\,x^3\right)\left(7\,a^2\,x^3-40\,a^{4/3}\,x\left(b^{2/3}-a^{1/3}\,b^{1/3}\,x+a^{2/3}\,x^2\right)+33\,a\left(b+a\,x^3\right)-\frac{3\,b\left(b+a\,x^3\right)}{x^3}-\right.$$

$$1\Big/\left(-1+(-1)^{2/3}\right)20\,(-1)^{2/3}\,a\,b^{1/3}\left(b^{1/3}+a^{1/3}\,x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left.\left(\left(-3-\mathrm{i}\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]+\left(1+\mathrm{i}\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+\mathrm{i}\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-\mathrm{i}+\sqrt{3}}{\mathrm{i}+\sqrt{3}}\right]\right)\right)$$

- **Problem 2056: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^{12}}\,dx$$

Optimal (type 4, 565 leaves, 7 steps) :

$$-\frac{1280\,a^2\sqrt{a+\frac{b}{x^3}}}{273\,b^{11/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)}+\frac{2}{3\,b\sqrt{a+\frac{b}{x^3}}\;x^8}-\frac{32\sqrt{a+\frac{b}{x^3}}}{39\,b^2\,x^5}+\frac{320\,a\sqrt{a+\frac{b}{x^3}}}{273\,b^3\,x^2}+$$

$$\frac{640\sqrt{2-\sqrt{3}}\;a^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{91\times 3^{3/4}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}-$$

$$\frac{1280\sqrt{2}\;a^{7/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{a^{1/3}b^{1/3}}{x}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}}\right],-7-4\sqrt{3}\right]}{273\times 3^{1/4}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}}{x}\right)^2}}}$$

Result (type 4, 400 leaves) :

$$\frac{1}{273\,b^4\left(a+\frac{b}{x^3}\right)^{3/2}x^5}2\left(b+a\,x^3\right)\left(-91\,a^3\,x^3+640\,a^{7/3}\,x\left(b^{2/3}-a^{1/3}\,b^{1/3}\,x+a^{2/3}\,x^2\right)-549\,a^2\left(b+a\,x^3\right)-\frac{21\,b^2\left(b+a\,x^3\right)}{x^6}+\frac{69\,a\,b\left(b+a\,x^3\right)}{x^3}+\right.$$

$$1\Big/\left(-1+(-1)^{2/3}\right)320\,(-1)^{2/3}\,a^2\,b^{1/3}\left(b^{1/3}+a^{1/3}\,x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)a^{1/3}\,x\left(b^{1/3}-(-1)^{1/3}\,a^{1/3}\,x\right)}{\left(b^{1/3}+a^{1/3}\,x\right)^2}}\sqrt{\frac{b^{1/3}+(-1)^{2/3}\,a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}$$

$$\left.\left(\left(-3-i\sqrt{3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]+\left(1+i\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(3+i\sqrt{3}\right)a^{1/3}\,x}{b^{1/3}+a^{1/3}\,x}}}{\sqrt{2}}\right],\frac{-i+\sqrt{3}}{i+\sqrt{3}}\right]\right)\right)$$

Problem 2062: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+\frac{b}{x^4}}\, x^2 \, dx$$

Optimal (type 4, 107 leaves, 3 steps):

$$\frac{1}{3}\sqrt{a+\frac{b}{x^4}}\, x^3-\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{3\,a^{1/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 93 leaves):

$$\frac{1}{3}\sqrt{a+\frac{b}{x^4}}\, x^2\left(x-\frac{2\,i\,b\sqrt{1+\frac{a\,x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(b+a\,x^4\right)}\right)$$

Problem 2063: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+\frac{b}{x^4}}\, dx$$

Optimal (type 4, 224 leaves, 5 steps):

$$-\frac{2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\sqrt{a+\frac{b}{x^4}}\, x+\frac{2\,a^{1/4}\,b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{a+\frac{b}{x^4}}}-$$

$$\frac{a^{1/4}\,b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 119 leaves):

$$\sqrt{a+\frac{b}{x^4}}\;x\left(-1+\frac{2\,i\,a\,x\sqrt{1+\frac{a x^4}{b}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]\right)}{\left(\frac{i\sqrt{a}}{\sqrt{b}}\right)^{3/2}\left(b+a\,x^4\right)}\right)$$

- **Problem 2064: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$$

Optimal (type 4, 107 leaves, 3 steps) :

$$-\frac{\sqrt{a+\frac{b}{x^4}}}{3\,x}-\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{3\,b^{1/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 96 leaves) :

$$\frac{\sqrt{a+\frac{b}{x^4}}\left(-1-\frac{2\,i\,a\,x^3\sqrt{1+\frac{a x^4}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(b+a\,x^4\right)}\right)}{3\,x}$$

- **Problem 2065: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$$

Optimal (type 4, 236 leaves, 5 steps) :

$$-\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}-\frac{2a\sqrt{a+\frac{b}{x^4}}}{5\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\frac{2a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}}-$$

$$\frac{a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 138 leaves) :

$$\frac{1}{5}\sqrt{a+\frac{b}{x^4}}\,x^2\left(-\frac{b+2ax^4}{bx^5}-\frac{1}{b+ax^4}2\,i\,a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{1+\frac{ax^4}{b}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)\right)$$

■ Problem 2070: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+\frac{b}{x^4}\right)^{3/2}x^2\,dx$$

Optimal (type 4, 126 leaves, 4 steps) :

$$-\frac{2b\sqrt{a+\frac{b}{x^4}}}{3x}+\frac{1}{3}\left(a+\frac{b}{x^4}\right)^{3/2}x^3-\frac{2a^{3/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{3\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 128 leaves) :

$$\frac{\sqrt{a+\frac{b}{x^4}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-b^2+a^2x^8\right)-4\,i\,a\,b\,x^3\sqrt{1+\frac{ax^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)}{3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\left(b+ax^4\right)}$$

■ Problem 2071: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+\frac{b}{x^4}\right)^{3/2}dx$$

Optimal (type 4, 250 leaves, 6 steps) :

$$-\frac{6\,b\,\sqrt{a+\frac{b}{x^4}}}{5\,x^3}-\frac{12\,a\,\sqrt{b}\,\sqrt{a+\frac{b}{x^4}}}{5\,\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\,x}+\left(a+\frac{b}{x^4}\right)^{3/2}\,x+\frac{12\,a^{5/4}\,b^{1/4}\,\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\,\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\,\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{5\,\sqrt{a+\frac{b}{x^4}}}-\frac{6\,a^{5/4}\,b^{1/4}\,\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\,\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\,\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{5\,\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 196 leaves) :

$$-\frac{1}{5\,\sqrt{\frac{i\,\sqrt{a}}{\sqrt{b}}}\,x^3\,\left(b+a\,x^4\right)}\sqrt{a+\frac{b}{x^4}}\,\left(\sqrt{\frac{i\,\sqrt{a}}{\sqrt{b}}}\,\left(b^2+8\,a\,b\,x^4+7\,a^2\,x^8\right)-12\,a^{3/2}\,\sqrt{b}\,x^5\,\sqrt{1+\frac{a\,x^4}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]+12\,a^{3/2}\,\sqrt{b}\,x^5\,\sqrt{1+\frac{a\,x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)$$

- **Problem 2072: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 126 leaves, 4 steps) :

$$-\frac{2\,a\,\sqrt{a+\frac{b}{x^4}}}{7\,x}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7\,x}-\frac{2\,a^{7/4}\,\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\,\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\,\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{7\,b^{1/4}\,\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 135 leaves) :

$$-\frac{\sqrt{a+\frac{b}{x^4}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(b^2+4\,a\,b\,x^4+3\,a^2\,x^8\right)+4\,i\,a^2\,x^7\sqrt{1+\frac{a\,x^4}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]\right)}{7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x^5\left(b+a\,x^4\right)}$$

- **Problem 2073: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 257 leaves, 6 steps):

$$-\frac{2\,a\sqrt{a+\frac{b}{x^4}}}{15\,x^3}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9\,x^3}-\frac{4\,a^2\sqrt{a+\frac{b}{x^4}}}{15\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+$$

$$\frac{4\,a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{15\,b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{2\,a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{15\,b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 213 leaves):

$$\frac{\left(a+\frac{b}{x^4}\right)^{3/2}\left(-\frac{b}{9\,x^9}-\frac{11\,a}{45\,x^5}-\frac{4\,a^2}{15\,b\,x}\right)x^6}{b+a\,x^4}+$$

$$\left(4\,a^{5/2}\left(a+\frac{b}{x^4}\right)^{3/2}x^6\sqrt{1-\frac{i\sqrt{a}\;x^2}{\sqrt{b}}}\sqrt{1+\frac{i\sqrt{a}\;x^2}{\sqrt{b}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]\right)\right)\Big/$$

$$\left(15\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{b}\left(b+a\,x^4\right)^2\right)$$

- **Problem 2078: Result unnecessarily involves imaginary or complex numbers.**

$$\int\left(a+\frac{b}{x^4}\right)^{5/2}x^2\,dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$-\frac{20\,a\,b\,\sqrt{a+\frac{b}{x^4}}}{21\,x}-\frac{10\,b\,\left(a+\frac{b}{x^4}\right)^{3/2}}{21\,x}+\frac{1}{3}\left(a+\frac{b}{x^4}\right)^{5/2}x^3-\frac{20\,a^{7/4}\,b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{21\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 149 leaves) :

$$\frac{1}{21\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x^5\left(b+a\,x^4\right)}\sqrt{a+\frac{b}{x^4}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-3\,b^3-19\,a\,b^2\,x^4-9\,a^2\,b\,x^8+7\,a^3\,x^{12}\right)-40\,i\,a^2\,b\,x^7\sqrt{1+\frac{a\,x^4}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]\right)$$

- **Problem 2079: Result unnecessarily involves imaginary or complex numbers.**

$$\int\left(a+\frac{b}{x^4}\right)^{5/2}dx$$

Optimal (type 4, 272 leaves, 7 steps) :

$$-\frac{4\,a\,b\,\sqrt{a+\frac{b}{x^4}}}{3\,x^3}-\frac{10\,b\,\left(a+\frac{b}{x^4}\right)^{3/2}}{9\,x^3}-\frac{8\,a^2\,\sqrt{b}\,\sqrt{a+\frac{b}{x^4}}}{3\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\left(a+\frac{b}{x^4}\right)^{5/2}x+$$

$$\frac{8\,a^{9/4}\,b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{3\sqrt{a+\frac{b}{x^4}}}-\frac{4\,a^{9/4}\,b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{3\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 207 leaves) :

$$-\frac{1}{9\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x^7\left(b+a\,x^4\right)}\sqrt{a+\frac{b}{x^4}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(b^3+5\,a\,b^2\,x^4+19\,a^2\,b\,x^8+15\,a^3\,x^{12}\right)-24\,a^{5/2}\,\sqrt{b}\,x^9\,\sqrt{1+\frac{a\,x^4}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]+24\,a^{5/2}\,\sqrt{b}\,x^9\,\sqrt{1+\frac{a\,x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)$$

- **Problem 2080: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx$$

Optimal (type 4, 147 leaves, 5 steps) :

$$-\frac{20\,a^2\,\sqrt{a+\frac{b}{x^4}}}{77\,x}-\frac{10\,a\,\left(a+\frac{b}{x^4}\right)^{3/2}}{77\,x}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11\,x}-\frac{20\,a^{11/4}\,\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\,\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\,\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{77\,b^{1/4}\,\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 148 leaves) :

$$-\frac{1}{77\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x^9\left(b+a\,x^4\right)}\sqrt{a+\frac{b}{x^4}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(7\,b^3+31\,a\,b^2\,x^4+61\,a^2\,b\,x^8+37\,a^3\,x^{12}\right)+40\,i\,a^3\,x^{11}\,\sqrt{1+\frac{a\,x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)$$

- **Problem 2081: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx$$

Optimal (type 4, 278 leaves, 7 steps) :

$$-\frac{4 a^2 \sqrt{a+\frac{b}{x^4}}}{39 x^3}-\frac{10 a \left(a+\frac{b}{x^4}\right)^{3/2}}{117 x^3}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13 x^3}-\frac{8 a^3 \sqrt{a+\frac{b}{x^4}}}{39 \sqrt{b} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) x}+$$

$$\frac{8 a^{13/4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{39 b^{3/4} \sqrt{a+\frac{b}{x^4}}}-\frac{4 a^{13/4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{39 b^{3/4} \sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 223 leaves) :

$$-\left(\sqrt{a+\frac{b}{x^4}} \left(\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} \left(9 b^4+37 a b^3 x^4+59 a^2 b^2 x^8+55 a^3 b x^{12}+24 a^4 x^{16}\right)-24 a^{7/2} \sqrt{b} x^{13} \sqrt{1+\frac{a x^4}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]+\right.\right.$$

$$\left.\left. 24 a^{7/2} \sqrt{b} x^{13} \sqrt{1+\frac{a x^4}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} x\right], -1\right]\right)\right) \Big/ \left(117 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}} b x^{11} \left(b+a x^4\right)\right)$$

Problem 2084: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}} x} \, dx$$

Optimal (type 3, 27 leaves, 3 steps) :

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right]}{2 \sqrt{a}}$$

Result (type 3, 55 leaves) :

$$\frac{\sqrt{b+a x^4} \text{ArcTanh}\left[\frac{\sqrt{a} x^2}{\sqrt{b+a x^4}}\right]}{2 \sqrt{a} \sqrt{a+\frac{b}{x^4}} x^2}$$

Problem 2086: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

Optimal (type 4, 110 leaves, 3 steps):

$$\frac{\sqrt{a+\frac{b}{x^4}}\;x^3}{3\,a}+\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{6\,a^{5/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 113 leaves):

$$\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\left(b+a\,x^4\right)+i\,b\sqrt{1+\frac{a\,x^4}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]}{3\,a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+\frac{b}{x^4}}\;x^2}$$

Problem 2087: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

Optimal (type 4, 231 leaves, 5 steps):

$$-\frac{\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{a\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\frac{\sqrt{a+\frac{b}{x^4}}\;x}{a}+\frac{b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\sqrt{a+\frac{b}{x^4}}}-$$

$$\frac{b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}\,x}{b^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 107 leaves):

$$\frac{i\sqrt{1+\frac{a x^4}{b}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]\right)}{\left(\frac{i\sqrt{a}}{\sqrt{b}}\right)^{3/2}\sqrt{a+\frac{b}{x^4}}\;x^2}$$

Problem 2088: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{a+\frac{b}{x^4}}\;x^2}\,dx$$

Optimal (type 4, 88 leaves, 2 steps):

$$-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,b^{1/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 77 leaves):

$$-\frac{i\sqrt{1+\frac{a x^4}{b}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x\right],-1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+\frac{b}{x^4}}\;x^2}$$

Problem 2089: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{a+\frac{b}{x^4}}\;x^4}\,dx$$

Optimal (type 4, 212 leaves, 4 steps):

$$-\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\frac{a^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{a^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{2\,b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 173 leaves):

$$-\frac{b+a x^4}{b\sqrt{a+\frac{b}{x^4}}\,x^3}+\frac{1}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{b}\,\sqrt{a+\frac{b}{x^4}}\,x^2}$$

$$\sqrt{a}\,\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)$$

- **Problem 2094: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$-\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}+\frac{5\sqrt{a+\frac{b}{x^4}}\,x^3}{6a^2}+\frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{12a^{9/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 116 leaves):

$$\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\left(5b+2ax^4\right)+5\,i\,b\sqrt{1+\frac{a x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]}{6a^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a+\frac{b}{x^4}}\,x^2}$$

- **Problem 2095: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$$

Optimal (type 4, 258 leaves, 6 steps):

$$-\frac{3\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{2a^2\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}-\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}+\frac{3\sqrt{a+\frac{b}{x^4}}\,x}{2a^2}+$$

$$\frac{3b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{2a^{7/4}\sqrt{a+\frac{b}{x^4}}}-\frac{3b^{1/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{4a^{7/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 166 leaves) :

$$\frac{1}{2a^{3/2}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+\frac{b}{x^4}}\,x^2}$$

$$\left(-\sqrt{a}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x^3+3\sqrt{b}\sqrt{1+\frac{a x^4}{b}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]-3\sqrt{b}\sqrt{1+\frac{a x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)$$

- **Problem 2096: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^2}\,dx$$

Optimal (type 4, 110 leaves, 3 steps) :

$$-\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x}-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{4a^{5/4}b^{1/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 105 leaves) :

$$-\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x+i\sqrt{1+\frac{a x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]}{2a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+\frac{b}{x^4}}\,x^2}$$

Problem 2097: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4} \, dx$$

Optimal (type 4, 241 leaves, 5 steps):

$$-\frac{1}{2 a \sqrt{a+\frac{b}{x^4}}\, x^3}+\frac{\sqrt{a+\frac{b}{x^4}}}{2 a \sqrt{b} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) x}-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} b^{3/4} \sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{4 a^{3/4} b^{3/4} \sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 166 leaves):

$$\left(i \left(\sqrt{a} \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, x^3-\sqrt{b} \sqrt{1+\frac{a x^4}{b}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, x\right], -1\right]+\sqrt{b} \sqrt{1+\frac{a x^4}{b}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, x\right], -1\right]\right)\right) \Big/ \left(2 \left(\frac{i \sqrt{a}}{\sqrt{b}}\right)^{3/2} b^{3/2} \sqrt{a+\frac{b}{x^4}}\, x^2\right)$$

Problem 2102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}} \, dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$-\frac{x^3}{6 a \left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{3 x^3}{4 a^2 \sqrt{a+\frac{b}{x^4}}}+\frac{5 \sqrt{a+\frac{b}{x^4}}\, x^3}{4 a^3}+\frac{5 b^{3/4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{8 a^{13/4} \sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 118 leaves):

$$\frac{\frac{15 b^2 x+21 a b x^5+4 a^2 x^9}{b+a x^4}+\frac{15 i b \sqrt{1+\frac{a x^4}{b}}\, \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, x\right],-1\right]}{\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}}{12 a^3 \sqrt{a+\frac{b}{x^4}}\, x^2}$$

- **Problem 2103: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}} \, dx$$

Optimal (type 4, 277 leaves, 7 steps):

$$-\frac{7 \sqrt{b} \sqrt{a+\frac{b}{x^4}}}{4 a^3 \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) x}-\frac{x}{6 a \left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{7 x}{12 a^2 \sqrt{a+\frac{b}{x^4}}}+\frac{7 \sqrt{a+\frac{b}{x^4}}\, x}{4 a^3}+$$

$$\frac{7 b^{1/4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) \text{EllipticE}\left[2 \,\text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{4 a^{11/4} \sqrt{a+\frac{b}{x^4}}}-\frac{7 b^{1/4} \sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}} \left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right) \text{EllipticF}\left[2 \,\text{ArcCot}\left[\frac{a^{1/4} x}{b^{1/4}}\right], \frac{1}{2}\right]}{8 a^{11/4} \sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 153 leaves):

$$\frac{\left(b+a x^4\right)^2 \left(-\frac{x^3 \left(7 b+9 a x^4\right)}{3 a^2 \left(b+a x^4\right)}+\frac{7 i \sqrt{1+\frac{a x^4}{b}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, x\right],-1\right]-\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, x\right],-1\right]\right)}{a^2 \left(\frac{i \sqrt{a}}{\sqrt{b}}\right)^{3/2}}\right)}{4 \left(a+\frac{b}{x^4}\right)^{5/2} x^{10}}$$

- **Problem 2104: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2} \, dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$-\frac{1}{6\,a\left(a+\frac{b}{x^4}\right)^{3/2}x}-\frac{5}{12\,a^2\sqrt{a+\frac{b}{x^4}}\,x}-\frac{5\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{24\,a^{9/4}\,b^{1/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 107 leaves) :

$$\frac{-\frac{5\,b\,x+7\,a\,x^5}{b+a\,x^4}-\frac{5\,i\sqrt{1+\frac{a\,x^4}{b}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}}{12\,a^2\sqrt{a+\frac{b}{x^4}}\,x^2}$$

Problem 2105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x^4}\,dx$$

Optimal (type 4, 262 leaves, 6 steps) :

$$-\frac{1}{6\,a\left(a+\frac{b}{x^4}\right)^{3/2}x^3}-\frac{1}{4\,a^2\sqrt{a+\frac{b}{x^4}}\,x^3}+\frac{\sqrt{a+\frac{b}{x^4}}}{4\,a^2\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}-$$

$$\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{7/4}\,b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}\left[\frac{a^{1/4}x}{b^{1/4}}\right],\frac{1}{2}\right]}{8\,a^{7/4}\,b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

Result (type 4, 155 leaves) :

$$\frac{\left(b+a\,x^4\right)^2\left(\frac{b\,x^3+3\,a\,x^7}{3\,a\,b^2+3\,a^2\,b\,x^4}+\frac{i\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{1+\frac{a\,x^4}{b}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right],-1\right]\right)}{a^2}\right)}{4\left(a+\frac{b}{x^4}\right)^{5/2}x^{10}}$$

- **Problem 2107: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\ x}\, dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right]}{5\sqrt{a}}$$

Result (type 8, 17 leaves):

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\ x}\, dx$$

- **Problem 2108: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\ x}\, dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-a+\frac{b}{x^5}}}{\sqrt{a}}\right]}{5\sqrt{a}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\ x}\, dx$$

- **Problem 2149: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\, dx$$

Optimal (type 2, 21 leaves, 1 step):

$$-\frac{\left(a+b\sqrt{x}\right)^6}{3\,a\,x^3}$$

Result (type 2, 63 leaves) :

$$-\frac{a^5+6 a^4 b \sqrt{x}+15 a^3 b^2 x+20 a^2 b^3 x^{3/2}+15 a b^4 x^2+6 b^5 x^{5/2}}{3 x^3}$$

Problem 2157: Result more than twice size of optimal antiderivative.

$$\int \left(a+b \sqrt{x}\right)^{10} \, dx$$

Optimal (type 2, 38 leaves, 3 steps) :

$$-\frac{2 a \left(a+b \sqrt{x}\right)^{11}}{11 b^2}+\frac{\left(a+b \sqrt{x}\right)^{12}}{6 b^2}$$

Result (type 2, 131 leaves) :

$$a^{10} x+\frac{20}{3} a^9 b x^{3/2}+\frac{45}{2} a^8 b^2 x^2+48 a^7 b^3 x^{5/2}+70 a^6 b^4 x^3+72 a^5 b^5 x^{7/2}+\frac{105}{2} a^4 b^6 x^4+\frac{80}{3} a^3 b^7 x^{9/2}+9 a^2 b^8 x^5+\frac{20}{11} a b^9 x^{11/2}+\frac{b^{10} x^6}{6}$$

Problem 2164: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b \sqrt{x}\right)^{10}}{x^7} \, dx$$

Optimal (type 2, 46 leaves, 3 steps) :

$$-\frac{\left(a+b \sqrt{x}\right)^{11}}{6 a x^6}+\frac{b \left(a+b \sqrt{x}\right)^{11}}{66 a^2 x^{11/2}}$$

Result (type 2, 124 leaves) :

$$-\frac{1}{66 x^6}\left(11 a^{10}+120 a^9 b \sqrt{x}+594 a^8 b^2 x+1760 a^7 b^3 x^{3/2}+3465 a^6 b^4 x^2+4752 a^5 b^5 x^{5/2}+4620 a^4 b^6 x^3+3168 a^3 b^7 x^{7/2}+1485 a^2 b^8 x^4+440 a b^9 x^{9/2}+66 b^{10} x^5\right)$$

Problem 2173: Result more than twice size of optimal antiderivative.

$$\int \left(a+b \sqrt{x}\right)^{15} x \, dx$$

Optimal (type 2, 80 leaves, 3 steps) :

$$-\frac{a^3 \left(a+b \sqrt{x}\right)^{16}}{8 b^4}+\frac{6 a^2 \left(a+b \sqrt{x}\right)^{17}}{17 b^4}-\frac{a \left(a+b \sqrt{x}\right)^{18}}{3 b^4}+\frac{2 \left(a+b \sqrt{x}\right)^{19}}{19 b^4}$$

Result (type 2, 199 leaves) :

$$\frac{a^{15}x^2}{2}+6\,a^{14}\,b\,x^{5/2}+35\,a^{13}\,b^2\,x^3+130\,a^{12}\,b^3\,x^{7/2}+\frac{1365}{4}\,a^{11}\,b^4\,x^4+\frac{2002}{3}\,a^{10}\,b^5\,x^{9/2}+1001\,a^9\,b^6\,x^5+1170\,a^8\,b^7\,x^{11/2}+\frac{2145}{2}\,a^7\,b^8\,x^6+770\,a^6\,b^9\,x^{13/2}+429\,a^5\,b^{10}\,x^7+182\,a^4\,b^{11}\,x^{15/2}+\frac{455}{8}\,a^3\,b^{12}\,x^8+\frac{210}{17}\,a^2\,b^{13}\,x^{17/2}+\frac{5}{3}\,a\,b^{14}\,x^9+\frac{2}{19}\,b^{15}\,x^{19/2}$$

- **Problem 2174: Result more than twice size of optimal antiderivative.**

$$\int\left(a+b\sqrt{x}\right)^{15}\,dx$$

Optimal (type 2, 38 leaves, 3 steps):

$$-\frac{a\left(a+b\sqrt{x}\right)^{16}}{8\,b^2}+\frac{2\left(a+b\sqrt{x}\right)^{17}}{17\,b^2}$$

Result (type 2, 190 leaves):

$$a^{15}\,x+10\,a^{14}\,b\,x^{3/2}+\frac{105}{2}\,a^{13}\,b^2\,x^2+182\,a^{12}\,b^3\,x^{5/2}+455\,a^{11}\,b^4\,x^3+858\,a^{10}\,b^5\,x^{7/2}+\frac{5005}{4}\,a^9\,b^6\,x^4+1430\,a^8\,b^7\,x^{9/2}+1287\,a^7\,b^8\,x^5+910\,a^6\,b^9\,x^{11/2}+\frac{1001}{2}\,a^5\,b^{10}\,x^6+210\,a^4\,b^{11}\,x^{13/2}+65\,a^3\,b^{12}\,x^7+14\,a^2\,b^{13}\,x^{15/2}+\frac{15}{8}\,a\,b^{14}\,x^8+\frac{2}{17}\,b^{15}\,x^{17/2}$$

- **Problem 2182: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx$$

Optimal (type 2, 21 leaves, 1 step):

$$-\frac{\left(a+b\sqrt{x}\right)^{16}}{8\,a\,x^8}$$

Result (type 2, 183 leaves):

$$-\frac{1}{8\,x^8}\left(a^{15}+16\,a^{14}\,b\sqrt{x}+120\,a^{13}\,b^2\,x+560\,a^{12}\,b^3\,x^{3/2}+1820\,a^{11}\,b^4\,x^2+4368\,a^{10}\,b^5\,x^{5/2}+8008\,a^9\,b^6\,x^3+11\,440\,a^8\,b^7\,x^{7/2}+12\,870\,a^7\,b^8\,x^4+11\,440\,a^6\,b^9\,x^{9/2}+8008\,a^5\,b^{10}\,x^5+4368\,a^4\,b^{11}\,x^{11/2}+1820\,a^3\,b^{12}\,x^6+560\,a^2\,b^{13}\,x^{13/2}+120\,a\,b^{14}\,x^7+16\,b^{15}\,x^{15/2}\right)$$

- **Problem 2183: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx$$

Optimal (type 2, 70 leaves, 4 steps):

$$-\frac{\left(a+b\sqrt{x}\right)^{16}}{9\,a\,x^9}+\frac{2\,b\left(a+b\sqrt{x}\right)^{16}}{153\,a^2\,x^{17/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{16}}{1224\,a^3\,x^8}$$

Result (type 2, 185 leaves):

$$-\frac{1}{1224 x^9}\left(136 a^{15}+2160 a^{14} b \sqrt{x}+16065 a^{13} b^2 x+74256 a^{12} b^3 x^{3/2}+238680 a^{11} b^4 x^2+565488 a^{10} b^5 x^{5/2}+1021020 a^9 b^6 x^3+1432080 a^8 b^7 x^{7/2}+1575288 a^7 b^8 x^4+1361360 a^6 b^9 x^{9/2}+918918 a^5 b^{10} x^5+477360 a^4 b^{11} x^{11/2}+185640 a^3 b^{12} x^6+51408 a^2 b^{13} x^{13/2}+9180 a b^{14} x^7+816 b^{15} x^{15/2}\right)$$

- **Problem 2218: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx$$

Optimal (type 2, 21 leaves, 1 step):

$$\frac{x^2}{2 a \left(a+b\sqrt{x}\right)^4}$$

Result (type 2, 50 leaves):

$$-\frac{a^3+4 a^2 b \sqrt{x}+6 a b^2 x+4 b^3 x^{3/2}}{2 b^4 \left(a+b\sqrt{x}\right)^4}$$

- **Problem 2271: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3}{\left(a+b x^{3/2}\right)^{2/3}}\,dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$-\frac{5 a x \left(a+b x^{3/2}\right)^{1/3}}{9 b^2}+\frac{x^{5/2} \left(a+b x^{3/2}\right)^{1/3}}{3 b}-\frac{10 a^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} \sqrt{x}}{\left(a+b x^{3/2}\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{8/3}}-\frac{5 a^2 \operatorname{Log}\left[b^{1/3} \sqrt{x}-\left(a+b x^{3/2}\right)^{1/3}\right]}{9 b^{8/3}}$$

Result (type 5, 87 leaves):

$$\frac{-5 a^2 x-2 a b x^{5/2}+3 b^2 x^4+5 a^2 x \left(1+\frac{b x^{3/2}}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^{3/2}}{a}\right]}{9 b^2 \left(a+b x^{3/2}\right)^{2/3}}$$

- **Problem 2272: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(a+b x^{3/2}\right)^{2/3}}\,dx$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3}\sqrt{x}}{\left(a+b x^{3/2}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3}}-\frac{\text{Log}\left[b^{1/3}\sqrt{x}-\left(a+b x^{3/2}\right)^{1/3}\right]}{b^{2/3}}$$

Result (type 5, 53 leaves) :

$$\frac{x\left(\frac{a+b x^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^{3/2}}{a}\right]}{\left(a+b x^{3/2}\right)^{2/3}}$$

■ **Problem 2279: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x\left(a+b x^{3/2}\right)^{2/3}}\,dx$$

Optimal (type 3, 85 leaves, 5 steps) :

$$-\frac{2\,\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b x^{3/2}\right)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{\sqrt{3}\, a^{2/3}}-\frac{\text{Log}[x]}{2\, a^{2/3}}+\frac{\text{Log}\left[a^{1/3}-\left(a+b x^{3/2}\right)^{1/3}\right]}{a^{2/3}}$$

Result (type 5, 52 leaves) :

$$-\frac{\left(1+\frac{a}{b x^{3/2}}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{a}{b x^{3/2}}\right]}{\left(a+b x^{3/2}\right)^{2/3}}$$

■ **Problem 2280: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4\left(a+b x^{3/2}\right)^{2/3}}\,dx$$

Optimal (type 3, 148 leaves, 7 steps) :

$$-\frac{\left(a+b x^{3/2}\right)^{1/3}}{3\, a\, x^3}+\frac{5\, b\left(a+b x^{3/2}\right)^{1/3}}{9\, a^2\, x^{3/2}}-\frac{10\, b^2\,\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b x^{3/2}\right)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{9\sqrt{3}\, a^{8/3}}-\frac{5\, b^2\,\text{Log}[x]}{18\, a^{8/3}}+\frac{5\, b^2\,\text{Log}\left[a^{1/3}-\left(a+b x^{3/2}\right)^{1/3}\right]}{9\, a^{8/3}}$$

Result (type 5, 91 leaves) :

$$\frac{-3 a^2+2 a b x^{3/2}+5 b^2 x^3-5 b^2\left(1+\frac{a}{b x^{3/2}}\right)^{2/3} x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{a}{b x^{3/2}}\right]}{9\, a^2\, x^3\left(a+b x^{3/2}\right)^{2/3}}$$

■ **Problem 2281: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^4}{\left(a+b x^{3/2}\right)^{2/3}}\,dx$$

Optimal (type 5, 42 leaves, 3 steps):

$$\frac{x^5 \left(a+b x^{3/2}\right)^{1/3} \text{Hypergeometric2F1}\left[1, \frac{11}{3}, \frac{13}{3}, -\frac{b x^{3/2}}{a}\right]}{5 a}$$

Result (type 5, 103 leaves):

$$\frac{\sqrt{x} \left(14 a^3+7 a^2 b x^{3/2}-2 a b^2 x^3+5 b^3 x^{9/2}-14 a^3 \left(1+\frac{b x^{3/2}}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^{3/2}}{a}\right]\right)}{20 b^3 \left(a+b x^{3/2}\right)^{2/3}}$$

- **Problem 2319: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^{1/3}\right)^5}{x^3} \, dx$$

Optimal (type 2, 21 leaves, 1 step):

$$-\frac{\left(a+b x^{1/3}\right)^6}{2 a x^2}$$

Result (type 2, 65 leaves):

$$-\frac{a^5+6 a^4 b x^{1/3}+15 a^3 b^2 x^{2/3}+20 a^2 b^3 x+15 a b^4 x^{4/3}+6 b^5 x^{5/3}}{2 x^2}$$

- **Problem 2328: Result more than twice size of optimal antiderivative.**

$$\int \left(a+b x^{1/3}\right)^{10} \, dx$$

Optimal (type 2, 59 leaves, 3 steps):

$$\frac{3 a^2 \left(a+b x^{1/3}\right)^{11}}{11 b^3}-\frac{a \left(a+b x^{1/3}\right)^{12}}{2 b^3}+\frac{3 \left(a+b x^{1/3}\right)^{13}}{13 b^3}$$

Result (type 2, 133 leaves):

$$a^{10} x+\frac{15}{2} a^9 b x^{4/3}+27 a^8 b^2 x^{5/3}+60 a^7 b^3 x^2+90 a^6 b^4 x^{7/3}+\frac{189}{2} a^5 b^5 x^{8/3}+70 a^4 b^6 x^3+36 a^3 b^7 x^{10/3}+\frac{135}{11} a^2 b^8 x^{11/3}+\frac{5}{2} a b^9 x^4+\frac{3}{13} b^{10} x^{13/3}$$

- **Problem 2333: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^{1/3}\right)^{10}}{x^5} \, dx$$

Optimal (type 2, 46 leaves, 3 steps):

$$-\frac{\left(a+b x^{1/3}\right)^{11}}{4 a x^4}+\frac{b \left(a+b x^{1/3}\right)^{11}}{44 a^2 x^{11/3}}$$

Result (type 2, 128 leaves) :

$$-\frac{1}{44 x^4}\left(11 a^{10}+120 a^9 b x^{1/3}+594 a^8 b^2 x^{2/3}+1760 a^7 b^3 x+3465 a^6 b^4 x^{4/3}+4752 a^5 b^5 x^{5/3}+4620 a^4 b^6 x^2+3168 a^3 b^7 x^{7/3}+1485 a^2 b^8 x^{8/3}+440 a b^9 x^3+66 b^{10} x^{10/3}\right)$$

Problem 2344: Result more than twice size of optimal antiderivative.

$$\int \left(a+b x^{1/3}\right)^{15} dx$$

Optimal (type 2, 59 leaves, 3 steps) :

$$\frac{3 a^2 \left(a+b x^{1/3}\right)^{16}}{16 b^3}-\frac{6 a \left(a+b x^{1/3}\right)^{17}}{17 b^3}+\frac{\left(a+b x^{1/3}\right)^{18}}{6 b^3}$$

Result (type 2, 204 leaves) :

$$a^{15} x+\frac{45}{4} a^{14} b x^{4/3}+63 a^{13} b^2 x^{5/3}+\frac{455}{2} a^{12} b^3 x^2+585 a^{11} b^4 x^{7/3}+\frac{9009}{8} a^{10} b^5 x^{8/3}+\frac{5005}{3} a^9 b^6 x^3+\frac{3861}{2} a^8 b^7 x^{10/3}+1755 a^7 b^8 x^{11/3}+\frac{5005}{4} a^6 b^9 x^4+693 a^5 b^{10} x^{13/3}+\frac{585}{2} a^4 b^{11} x^{14/3}+91 a^3 b^{12} x^5+\frac{315}{16} a^2 b^{13} x^{16/3}+\frac{45}{17} a b^{14} x^{17/3}+\frac{b^{15} x^6}{6}$$

Problem 2350: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^{1/3}\right)^{15}}{x^7} dx$$

Optimal (type 2, 72 leaves, 4 steps) :

$$-\frac{\left(a+b x^{1/3}\right)^{16}}{6 a x^6}+\frac{b \left(a+b x^{1/3}\right)^{16}}{51 a^2 x^{17/3}}-\frac{b^2 \left(a+b x^{1/3}\right)^{16}}{816 a^3 x^{16/3}}$$

Result (type 2, 189 leaves) :

$$-\frac{1}{816 x^6}\left(136 a^{15}+2160 a^{14} b x^{1/3}+16065 a^{13} b^2 x^{2/3}+74256 a^{12} b^3 x+238680 a^{11} b^4 x^{4/3}+565488 a^{10} b^5 x^{5/3}+1021020 a^9 b^6 x^2+1432080 a^8 b^7 x^{7/3}+1575288 a^7 b^8 x^{8/3}+1361360 a^6 b^9 x^3+918918 a^5 b^{10} x^{10/3}+477360 a^4 b^{11} x^{11/3}+185640 a^3 b^{12} x^4+51408 a^2 b^{13} x^{13/3}+9180 a b^{14} x^{14/3}+816 b^{15} x^5\right)$$

Problem 2393: Result unnecessarily involves higher level functions.

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx$$

Optimal (type 3, 95 leaves, 4 steps) :

$$\left(a+\frac{b}{x^{3/2}}\right)^{2/3} x-\frac{2 b^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3}}{\left(a+\frac{b}{x^{3/2}}\right)^{1/3} \sqrt{x}}}{\sqrt{3}}\right]}{\sqrt{3}}+b^{2/3} \operatorname{Log}\left[\left(a+\frac{b}{x^{3/2}}\right)^{1/3}-\frac{b^{1/3}}{\sqrt{x}}\right]$$

Result (type 5, 53 leaves) :

$$\frac{\left(a+\frac{b}{x^{3/2}}\right)^{2/3} x\,\text{Hypergeometric2F1}\left[-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\frac{b}{a x^{3/2}}\right]}{\left(\frac{a+\frac{b}{x^{3/2}}}{a}\right)^{2/3}}$$

- **Problem 2478: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b x^n)^2}\,dx$$

Optimal (type 5, 24 leaves, 1 step) :

$$\frac{x\,\text{Hypergeometric2F1}\left[2,\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]}{a^2}$$

Result (type 5, 49 leaves) :

$$\frac{x\left(a+(-1+n)\,(a+b x^n)\,\text{Hypergeometric2F1}\left[1,\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]\right)}{a^2 n\,(a+b x^n)}$$

- **Problem 2482: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{(a+b x^n)^3}\,dx$$

Optimal (type 5, 33 leaves, 1 step) :

$$\frac{x^2\,\text{Hypergeometric2F1}\left[3,\frac{2}{n},\frac{2+n}{n},-\frac{b x^n}{a}\right]}{2 a^3}$$

Result (type 5, 74 leaves) :

$$\frac{x^2\left(\frac{a\,(a\,(-2+3 n)+2 b\,(-1+n)\,x^n)}{(a+b x^n)^2}+\left(2-3 n+n^2\right)\text{Hypergeometric2F1}\left[1,\frac{2}{n},\frac{2+n}{n},-\frac{b x^n}{a}\right]\right)}{2 a^3 n^2}$$

- **Problem 2483: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b x^n)^3}\,dx$$

Optimal (type 5, 24 leaves, 1 step) :

$$\frac{x\,\text{Hypergeometric2F1}\left[3,\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]}{a^3}$$

Result (type 5, 71 leaves) :

$$\frac{x\left(\frac{a\,(a\,(-1+3\,n)+b\,(-1+2\,n)\,x^n)}{(a+b\,x^n)^2}+\left(1-3\,n+2\,n^2\right)\text{Hypergeometric2F1}\left[1,\frac{1}{n},1+\frac{1}{n},-\frac{b\,x^n}{a}\right]\right)}{2\,a^3\,n^2}$$

Problem 2485: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\,(a+b\,x^n)^3}\,dx$$

Optimal (type 5, 34 leaves, 1 step):

$$-\frac{\text{Hypergeometric2F1}\left[3,-\frac{1}{n},-\frac{1-n}{n},-\frac{b\,x^n}{a}\right]}{a^3\,x}$$

Result (type 5, 76 leaves):

$$\frac{\frac{a\,(a+3\,a\,n+b\,(1+2\,n)\,x^n)}{(a+b\,x^n)^2}-\left(1+3\,n+2\,n^2\right)\text{Hypergeometric2F1}\left[1,-\frac{1}{n},\frac{-1+n}{n},-\frac{b\,x^n}{a}\right]}{2\,a^3\,n^2\,x}$$

Problem 2486: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\,(a+b\,x^n)^3}\,dx$$

Optimal (type 5, 36 leaves, 1 step):

$$-\frac{\text{Hypergeometric2F1}\left[3,-\frac{2}{n},-\frac{2-n}{n},-\frac{b\,x^n}{a}\right]}{2\,a^3\,x^2}$$

Result (type 5, 75 leaves):

$$\frac{\frac{a\,(a\,(2+3\,n)+2\,b\,(1+n)\,x^n)}{(a+b\,x^n)^2}-\left(2+3\,n+n^2\right)\text{Hypergeometric2F1}\left[1,-\frac{2}{n},\frac{-2+n}{n},-\frac{b\,x^n}{a}\right]}{2\,a^3\,n^2\,x^2}$$

Problem 2492: Result more than twice size of optimal antiderivative.

$$\int x\,(a+b\,x^n)^{3/2}\,dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{x^2\,(a+b\,x^n)^{5/2}\,\text{Hypergeometric2F1}\left[1,\frac{5}{2}+\frac{2}{n},\frac{2+n}{n},-\frac{b\,x^n}{a}\right]}{2\,a}$$

Result (type 5, 102 leaves):

$$\frac{x^2\left(4\,(a+b\,x^n)\,(4\,a\,(1+n)+b\,(4+n)\,x^n)+3\,a^2\,n^2\,\sqrt{1+\frac{b\,x^n}{a}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{n},\frac{2+n}{n},-\frac{b\,x^n}{a}\right]\right)}{2\,(4+n)\,(4+3\,n)\,\sqrt{a+b\,x^n}}$$

Problem 2493: Result more than twice size of optimal antiderivative.

$$\int (a+b x^n)^{3/2}\, dx$$

Optimal (type 5, 39 leaves, 2 steps):

$$\frac{x\,(a+b x^n)^{5/2}\,\text{Hypergeometric2F1}\left[1,\ \frac{5}{2}+\frac{1}{n},\ 1+\frac{1}{n},\ -\frac{b x^n}{a}\right]}{a}$$

Result (type 5, 94 leaves):

$$\frac{x\left(2\,(a+b x^n)\,\left(a\,(2+4 n)+b\,(2+n)\,x^n\right)+3 a^2 n^2 \sqrt{1+\frac{b x^n}{a}}\ \text{Hypergeometric2F1}\left[\frac{1}{2},\ \frac{1}{n},\ 1+\frac{1}{n},\ -\frac{b x^n}{a}\right]\right)}{(2+n)\,(2+3 n)\,\sqrt{a+b x^n}}$$

Problem 2495: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^n)^{3/2}}{x^2}\, dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$-\frac{(a+b x^n)^{5/2}\,\text{Hypergeometric2F1}\left[1,\ \frac{5}{2}-\frac{1}{n},\ -\frac{1-n}{n},\ -\frac{b x^n}{a}\right]}{a\,x}$$

Result (type 5, 100 leaves):

$$\frac{2\,(a+b x^n)\,\left(a\,(-2+4 n)+b\,(-2+n)\,x^n\right)-3 a^2 n^2 \sqrt{1+\frac{b x^n}{a}}\ \text{Hypergeometric2F1}\left[\frac{1}{2},\ -\frac{1}{n},\ \frac{-1+n}{n},\ -\frac{b x^n}{a}\right]}{(-2+n)\,(-2+3 n)\,x\,\sqrt{a+b x^n}}$$

Problem 2497: Result more than twice size of optimal antiderivative.

$$\int x\,(a+b x^n)^{5/2}\, dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{x^2\,(a+b x^n)^{7/2}\,\text{Hypergeometric2F1}\left[1,\ \frac{7}{2}+\frac{2}{n},\ \frac{2+n}{n},\ -\frac{b x^n}{a}\right]}{2 a}$$

Result (type 5, 144 leaves):

$$\left(x^2 \left(4 (a+b x^n) \left(a^2 \left(16+36 n+23 n^2\right)+a b \left(32+52 n+11 n^2\right) x^n+b^2 \left(16+16 n+3 n^2\right) x^{2 n}\right)+\right.\right.$$
$$\left.\left.15 a^3 n^3 \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{n},\frac{2+n}{n},-\frac{b x^n}{a}\right]\right)\right) \Big/ \left(2 (4+n) (4+3 n) (4+5 n) \sqrt{a+b x^n}\right)$$

- **Problem 2498: Result more than twice size of optimal antiderivative.**

$$\int (a+b x^n)^{5/2} \, dx$$

Optimal (type 5, 39 leaves, 2 steps):

$$\frac{x (a+b x^n)^{7/2} \text{Hypergeometric2F1}\left[1,\frac{7}{2}+\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]}{a}$$

Result (type 5, 135 leaves):

$$\left(x \left(2 (a+b x^n) \left(a^2 \left(4+18 n+23 n^2\right)+a b \left(8+26 n+11 n^2\right) x^n+b^2 \left(4+8 n+3 n^2\right) x^{2 n}\right)+\right.\right.$$
$$\left.\left.15 a^3 n^3 \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]\right)\right) \Big/ \left((2+n) (2+3 n) (2+5 n) \sqrt{a+b x^n}\right)$$

- **Problem 2500: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b x^n)^{5/2}}{x^2} \, dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$-\frac{(a+b x^n)^{7/2} \text{Hypergeometric2F1}\left[1,\frac{7}{2}-\frac{1}{n},-\frac{1-n}{n},-\frac{b x^n}{a}\right]}{a x}$$

Result (type 5, 141 leaves):

$$\left(2 (a+b x^n) \left(a^2 \left(4-18 n+23 n^2\right)+a b \left(8-26 n+11 n^2\right) x^n+b^2 \left(4-8 n+3 n^2\right) x^{2 n}\right)-\right.$$
$$\left.15 a^3 n^3 \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{1}{n},\frac{-1+n}{n},-\frac{b x^n}{a}\right]\right) \Big/ \left((-2+n) (-2+3 n) (-2+5 n) x \sqrt{a+b x^n}\right)$$

Problem 2501: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^n)^{5/2}}{x^3} \, dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$-\frac{(a+b x^n)^{7/2} \text{Hypergeometric2F1}\left[1, \frac{7}{2}-\frac{2}{n}, -\frac{2-n}{n}, -\frac{b x^n}{a}\right]}{2 a x^2}$$

Result (type 5, 144 leaves):

$$\left(4 (a+b x^n) \left(a^2 \left(16-36 n+23 n^2\right)+a b \left(32-52 n+11 n^2\right) x^n+b^2 \left(16-16 n+3 n^2\right) x^{2 n}\right)-15 a^3 n^3 \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2}{n}, \frac{-2+n}{n}, -\frac{b x^n}{a}\right]\right) \Big/ \left(2 (-4+n) (-4+3 n) (-4+5 n) x^2 \sqrt{a+b x^n}\right)$$

Problem 2512: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a+b x^n)^{5/2}} \, dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{x^2 \text{Hypergeometric2F1}\left[1, -\frac{3}{2}+\frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right]}{2 a (a+b x^n)^{3/2}}$$

Result (type 5, 100 leaves):

$$\frac{1}{6 a^2 n^2 (a+b x^n)^{3/2}} x^2 \left(4 a n+4 (-4+3 n) (a+b x^n)+\left(16-16 n+3 n^2\right) (a+b x^n) \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{n}, \frac{2+n}{n}, -\frac{b x^n}{a}\right]\right)$$

Problem 2513: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b x^n)^{5/2}} \, dx$$

Optimal (type 5, 39 leaves, 2 steps):

$$\frac{x \text{Hypergeometric2F1}\left[1, -\frac{3}{2}+\frac{1}{n}, 1+\frac{1}{n}, -\frac{b x^n}{a}\right]}{a (a+b x^n)^{3/2}}$$

Result (type 5, 94 leaves):

$$\frac{1}{3 a^2 n^2 (a+b x^n)^{3/2}} x \left(2 a n + 2 (-2+3 n) (a+b x^n) + \left(4-8 n+3 n^2\right) (a+b x^n) \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{n}, 1+\frac{1}{n}, -\frac{b x^n}{a}\right]\right)$$

- **Problem 2515: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^2 (a+b x^n)^{5/2}} \, dx$$

Optimal (type 5, 49 leaves, 2 steps) :

$$-\frac{\text{Hypergeometric2F1}\left[1, -\frac{3}{2}-\frac{1}{n}, -\frac{1-n}{n}, -\frac{b x^n}{a}\right]}{a x (a+b x^n)^{3/2}}$$

Result (type 5, 101 leaves) :

$$\frac{2 a n + 2 (2+3 n) (a+b x^n) - \left(4+8 n+3 n^2\right) (a+b x^n) \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1}{n}, \frac{-1+n}{n}, -\frac{b x^n}{a}\right]}{3 a^2 n^2 x (a+b x^n)^{3/2}}$$

- **Problem 2517: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x^n)^{1/3}}{x} \, dx$$

Optimal (type 3, 106 leaves, 6 steps) :

$$\frac{3 (a+b x^n)^{1/3}}{n} - \frac{\sqrt{3} a^{1/3} \text{ArcTan}\left[\frac{a^{1/3}+2 (a+b x^n)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{n} - \frac{1}{2} a^{1/3} \text{Log}[x] + \frac{3 a^{1/3} \text{Log}\left[a^{1/3} - (a+b x^n)^{1/3}\right]}{2 n}$$

Result (type 5, 68 leaves) :

$$\frac{6 (a+b x^n) - 3 a \left(1+\frac{a x^{-n}}{b}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a x^{-n}}{b}\right]}{2 n (a+b x^n)^{2/3}}$$

- **Problem 2561: Result more than twice size of optimal antiderivative.**

$$\int x^{-1-6 n} (a+b x^n)^5 \, dx$$

Optimal (type 3, 24 leaves, 1 step) :

$$-\frac{x^{-6 n} (a+b x^n)^6}{6 a n}$$

Result (type 3, 72 leaves) :

$$-\frac{x^{-6 n} \left(a^5 + 6 a^4 b x^n + 15 a^3 b^2 x^{2 n} + 20 a^2 b^3 x^{3 n} + 15 a b^4 x^{4 n} + 6 b^5 x^{5 n}\right)}{6 n}$$

Problem 2573: Result more than twice size of optimal antiderivative.

$$\int x^{-1+2n} (a+b x^n)^8 \, dx$$

Optimal (type 3, 40 leaves, 3 steps):

$$-\frac{a (a+b x^n)^9}{9 b^2 n}+\frac{(a+b x^n)^{10}}{10 b^2 n}$$

Result (type 3, 113 leaves):

$$\frac{1}{90 n} x^{2n} \left(45 a^8+240 a^7 b x^n+630 a^6 b^2 x^{2n}+1008 a^5 b^3 x^{3n}+1050 a^4 b^4 x^{4n}+720 a^3 b^5 x^{5n}+315 a^2 b^6 x^{6n}+80 a b^7 x^{7n}+9 b^8 x^{8n}\right)$$

Problem 2584: Result more than twice size of optimal antiderivative.

$$\int x^{-1-9n} (a+b x^n)^8 \, dx$$

Optimal (type 3, 24 leaves, 1 step):

$$-\frac{x^{-9n} (a+b x^n)^9}{9 a n}$$

Result (type 3, 111 leaves):

$$-\frac{1}{9 n} x^{-9n} \left(a^8+9 a^7 b x^n+36 a^6 b^2 x^{2n}+84 a^5 b^3 x^{3n}+126 a^4 b^4 x^{4n}+126 a^3 b^5 x^{5n}+84 a^2 b^6 x^{6n}+36 a b^7 x^{7n}+9 b^8 x^{8n}\right)$$

Problem 2585: Result more than twice size of optimal antiderivative.

$$\int x^{-1-10n} (a+b x^n)^8 \, dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$-\frac{x^{-10n} (a+b x^n)^9}{10 a n}+\frac{b x^{-9n} (a+b x^n)^9}{90 a^2 n}$$

Result (type 3, 113 leaves):

$$-\frac{1}{90 n} x^{-10n} \left(9 a^8+80 a^7 b x^n+315 a^6 b^2 x^{2n}+720 a^5 b^3 x^{3n}+1050 a^4 b^4 x^{4n}+1008 a^3 b^5 x^{5n}+630 a^2 b^6 x^{6n}+240 a b^7 x^{7n}+45 b^8 x^{8n}\right)$$

Problem 2592: Result more than twice size of optimal antiderivative.

$$\int x^{12} \left(a+b x^{13}\right)^{12} \, dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{\left(a+b x^{13}\right)^{13}}{169 b}$$

Result (type 1, 160 leaves):

$$\frac{a^{12} x^{13}}{13}+\frac{6}{13} a^{11} b x^{26}+\frac{22}{13} a^{10} b^2 x^{39}+\frac{55}{13} a^9 b^3 x^{52}+\frac{99}{13} a^8 b^4 x^{65}+\frac{132}{13} a^7 b^5 x^{78}+$$
$$\frac{132}{13} a^6 b^6 x^{91}+\frac{99}{13} a^5 b^7 x^{104}+\frac{55}{13} a^4 b^8 x^{117}+\frac{22}{13} a^3 b^9 x^{130}+\frac{6}{13} a^2 b^{10} x^{143}+\frac{1}{13} a b^{11} x^{156}+\frac{b^{12} x^{169}}{169}$$

- **Problem 2593: Result more than twice size of optimal antiderivative.**

$$\int x^{24} \left(a+b x^{25}\right)^{12} dx$$

Optimal (type 1, 16 leaves, 1 step) :

$$\frac{\left(a+b x^{25}\right)^{13}}{325 b}$$

Result (type 1, 160 leaves) :

$$\frac{a^{12} x^{25}}{25}+\frac{6}{25} a^{11} b x^{50}+\frac{22}{25} a^{10} b^2 x^{75}+\frac{11}{5} a^9 b^3 x^{100}+\frac{99}{25} a^8 b^4 x^{125}+\frac{132}{25} a^7 b^5 x^{150}+$$
$$\frac{132}{25} a^6 b^6 x^{175}+\frac{99}{25} a^5 b^7 x^{200}+\frac{11}{5} a^4 b^8 x^{225}+\frac{22}{25} a^3 b^9 x^{250}+\frac{6}{25} a^2 b^{10} x^{275}+\frac{1}{25} a b^{11} x^{300}+\frac{b^{12} x^{325}}{325}$$

- **Problem 2594: Result more than twice size of optimal antiderivative.**

$$\int x^{36} \left(a+b x^{37}\right)^{12} dx$$

Optimal (type 1, 16 leaves, 1 step) :

$$\frac{\left(a+b x^{37}\right)^{13}}{481 b}$$

Result (type 1, 160 leaves) :

$$\frac{a^{12} x^{37}}{37}+\frac{6}{37} a^{11} b x^{74}+\frac{22}{37} a^{10} b^2 x^{111}+\frac{55}{37} a^9 b^3 x^{148}+\frac{99}{37} a^8 b^4 x^{185}+\frac{132}{37} a^7 b^5 x^{222}+$$
$$\frac{132}{37} a^6 b^6 x^{259}+\frac{99}{37} a^5 b^7 x^{296}+\frac{55}{37} a^4 b^8 x^{333}+\frac{22}{37} a^3 b^9 x^{370}+\frac{6}{37} a^2 b^{10} x^{407}+\frac{1}{37} a b^{11} x^{444}+\frac{b^{12} x^{481}}{481}$$

- **Problem 2640: Result is not expressed in closed-form.**

$$\int \frac{x^{-1-\frac{2 n}{3}}}{a+b x^n} dx$$

Optimal (type 3, 160 leaves, 8 steps) :

$$-\frac{3 x^{-2 n/3}}{2 a n}+\frac{\sqrt{3}\, b^{2/3} \operatorname{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} x^{n/3}}{\sqrt{3}\, a^{1/3}}\right]}{a^{5/3} n}-\frac{b^{2/3} \operatorname{Log}\left[a^{1/3}+b^{1/3} x^{n/3}\right]}{a^{5/3} n}+\frac{b^{2/3} \operatorname{Log}\left[a^{2/3}-a^{1/3} b^{1/3} x^{n/3}+b^{2/3} x^{2 n/3}\right]}{2 a^{5/3} n}$$

Result (type 7, 60 leaves) :

$$\frac{-9\,a\,x^{-2\,n/3}+2\,b\,\text{RootSum}\left[b+a\,\#1^3\,\&,\ \frac{n\,\text{Log}[x]+3\,\text{Log}\left[x^{-n/3}-\#1\right]}{\#1}\,\&\right]}{6\,a^2\,n}$$

Problem 2641: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 236 leaves, 11 steps):

$$-\frac{4\,x^{-3\,n/4}}{3\,a\,n}+\frac{\sqrt{2}\,b^{3/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,b^{1/4}\,x^{n/4}}{a^{1/4}}\right]}{a^{7/4}\,n}-\frac{\sqrt{2}\,b^{3/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,b^{1/4}\,x^{n/4}}{a^{1/4}}\right]}{a^{7/4}\,n}+$$
$$\frac{b^{3/4}\,\text{Log}\left[\sqrt{a}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{n/4}+\sqrt{b}\,x^{n/2}\right]}{\sqrt{2}\,a^{7/4}\,n}-\frac{b^{3/4}\,\text{Log}\left[\sqrt{a}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{n/4}+\sqrt{b}\,x^{n/2}\right]}{\sqrt{2}\,a^{7/4}\,n}$$

Result (type 7, 60 leaves):

$$\frac{-16\,a\,x^{-3\,n/4}+3\,b\,\text{RootSum}\left[b+a\,\#1^4\,\&,\ \frac{n\,\text{Log}[x]+4\,\text{Log}\left[x^{-n/4}-\#1\right]}{\#1}\,\&\right]}{12\,a^2\,n}$$

Problem 2644: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{n}{3}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 158 leaves, 9 steps):

$$-\frac{3\,x^{-n/3}}{a\,n}-\frac{\sqrt{3}\,b^{1/3}\,\text{ArcTan}\left[\frac{b^{1/3}-2\,a^{1/3}\,x^{-n/3}}{\sqrt{3}\,b^{1/3}}\right]}{a^{4/3}\,n}+\frac{b^{1/3}\,\text{Log}\left[b^{1/3}+a^{1/3}\,x^{-n/3}\right]}{a^{4/3}\,n}-\frac{b^{1/3}\,\text{Log}\left[b^{2/3}+a^{2/3}\,x^{-2\,n/3}-a^{1/3}\,b^{1/3}\,x^{-n/3}\right]}{2\,a^{4/3}\,n}$$

Result (type 7, 59 leaves):

$$\frac{-9\,a\,x^{-n/3}+b\,\text{RootSum}\left[b+a\,\#1^3\,\&,\ \frac{n\,\text{Log}[x]+3\,\text{Log}\left[x^{-n/3}-\#1\right]}{\#1^2}\,\&\right]}{3\,a^2\,n}$$

Problem 2645: Result is not expressed in closed-form.

$$\int \frac{x^{-1-\frac{n}{4}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 234 leaves, 12 steps):

$$-\frac{4\,x^{-n/4}}{a\,n}-\frac{\sqrt{2}\;b^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\;a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{5/4}\,n}+\frac{\sqrt{2}\;b^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\;a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{5/4}\,n}-\frac{b^{1/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\;x^{-n/2}-\sqrt{2}\;a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\;a^{5/4}\,n}+\frac{b^{1/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\;x^{-n/2}+\sqrt{2}\;a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\;a^{5/4}\,n}$$

Result (type 7, 59 leaves):

$$\frac{-16\,a\,x^{-n/4}+b\,\text{RootSum}\left[b+a\,\#1^4\,\&,\,\frac{n\,\text{Log}[x]+4\,\text{Log}\left[x^{-n/4}-\#1\right]}{\#1^3}\,\&\right]}{4\,a^2\,n}$$

Problem 2647: Result is not expressed in closed-form.

$$\int\frac{x^{-1-\frac{4n}{3}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$-\frac{3\,x^{-4n/3}}{4\,a\,n}+\frac{3\,b\,x^{-n/3}}{a^2\,n}+\frac{\sqrt{3}\;b^{4/3}\,\text{ArcTan}\left[\frac{b^{1/3}-2\,a^{1/3}\,x^{-n/3}}{\sqrt{3}\;b^{1/3}}\right]}{a^{7/3}\,n}-\frac{b^{4/3}\,\text{Log}\left[b^{1/3}+a^{1/3}\,x^{-n/3}\right]}{a^{7/3}\,n}+\frac{b^{4/3}\,\text{Log}\left[b^{2/3}+a^{2/3}\,x^{-2n/3}-a^{1/3}\,b^{1/3}\,x^{-n/3}\right]}{2\,a^{7/3}\,n}$$

Result (type 7, 70 leaves):

$$-\frac{9\,a\,x^{-4n/3}\,(a-4\,b\,x^n)+4\,b^2\,\text{RootSum}\left[b+a\,\#1^3\,\&,\,\frac{n\,\text{Log}[x]+3\,\text{Log}\left[x^{-n/3}-\#1\right]}{\#1^2}\,\&\right]}{12\,a^3\,n}$$

Problem 2648: Result is not expressed in closed-form.

$$\int\frac{x^{-1-\frac{5n}{4}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 252 leaves, 13 steps):

$$-\frac{4\,x^{-5n/4}}{5\,a\,n}+\frac{4\,b\,x^{-n/4}}{a^2\,n}+\frac{\sqrt{2}\;b^{5/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\;a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{9/4}\,n}-\frac{\sqrt{2}\;b^{5/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\;a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{9/4}\,n}+\frac{b^{5/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\;x^{-n/2}-\sqrt{2}\;a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\;a^{9/4}\,n}-\frac{b^{5/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\;x^{-n/2}+\sqrt{2}\;a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\;a^{9/4}\,n}$$

Result (type 7, 70 leaves):

$$-\frac{16\,a\,x^{-5n/4}\,(a-5\,b\,x^n)+5\,b^2\,\text{RootSum}\left[b+a\,\#1^4\,\&,\,\frac{n\,\text{Log}[x]+4\,\text{Log}\left[x^{-n/4}-\#1\right]}{\#1^3}\,\&\right]}{20\,a^3\,n}$$

Problem 2672: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{(a+b x^n)^3} \, dx$$

Optimal (type 5, 40 leaves, 1 step):

$$\frac{x^{1+m} \text{Hypergeometric2F1}\left[3, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]}{a^3 (1+m)}$$

Result (type 5, 100 leaves):

$$\frac{x^{1+m} \left(\frac{a^2 n}{(a+b x^n)^2} - \frac{a (1+m-2 n)}{a+b x^n} + \frac{\left(1+m^2+m (2-3 n)-3 n+2 n^2\right) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]}{1+m}\right)}{2 a^3 n^2}$$

Problem 2673: Result more than twice size of optimal antiderivative.

$$\int x^m (a+b x^n)^{3/2} \, dx$$

Optimal (type 5, 55 leaves, 2 steps):

$$\frac{x^{1+m} (a+b x^n)^{5/2} \text{Hypergeometric2F1}\left[1, \frac{5}{2}+\frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]}{a (1+m)}$$

Result (type 5, 124 leaves):

$$\left(x^{1+m} \left(2 (1+m) (a+b x^n) (2 a (1+m+2 n) + b (2+2 m+n) x^n) + 3 a^2 n^2 \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]\right)\right) \Big/$$
$$\left((1+m) (2+2 m+n) (2+2 m+3 n) \sqrt{a+b x^n}\right)$$

Problem 2677: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{(a+b x^n)^{5/2}} \, dx$$

Optimal (type 5, 55 leaves, 2 steps):

$$\frac{x^{1+m} \text{Hypergeometric2F1}\left[1, -\frac{3}{2}+\frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]}{a (1+m) (a+b x^n)^{3/2}}$$

Result (type 5, 129 leaves):

$$\left(x^{1+m}\left(2\,(1+m)\,(a\,n-(2+2\,m-3\,n)\,(a+b\,x^n))+\left(4+4\,m^2-8\,m\,(-1+n)-8\,n+3\,n^2\right)\,(a+b\,x^n)\,\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]\right)\right)\Big/\left(3\,a^2\,(1+m)\,n^2\,(a+b\,x^n)^{3/2}\right)$$

- **Problem 2700: Result unnecessarily involves higher level functions.**

$$\int\frac{x^m}{\left(a+b\,x^{3\,(1+m)}\right)^{1/3}}\,dx$$

Optimal (type 3, 97 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x^{1+m}}{\left(a+b\,x^{3\,(1+m)}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\;b^{1/3}\,(1+m)}-\frac{\text{Log}\left[b^{1/3}\,x^{1+m}-\left(a+b\,x^{3\,(1+m)}\right)^{1/3}\right]}{2\,b^{1/3}\,(1+m)}$$

Result (type 5, 68 leaves):

$$\frac{x^{1+m}\left(\frac{a+b\,x^{3+3\,m}}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{b\,x^{3+3\,m}}{a}\right]}{(1+m)\,\left(a+b\,x^{3+3\,m}\right)^{1/3}}$$

- **Problem 2701: Result unnecessarily involves higher level functions.**

$$\int x^m\left(a+b\,x^{-\frac{3}{2}\,(1+m)}\right)^{2/3}dx$$

Optimal (type 3, 139 leaves, 3 steps):

$$\frac{x^{1+m}\left(a+b\,x^{-\frac{3}{2}\,(1+m)}\right)^{2/3}}{1+m}-\frac{2\,b^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x^{\frac{1}{2}\,(-1-m)}}{\left(a+b\,x^{-\frac{3}{2}\,(1+m)}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\;(1+m)}+\frac{b^{2/3}\,\text{Log}\left[b^{1/3}\,x^{\frac{1}{2}\,(-1-m)}-\left(a+b\,x^{-\frac{3}{2}\,(1+m)}\right)^{1/3}\right]}{1+m}$$

Result (type 5, 73 leaves):

$$\frac{x^{1+m}\left(a+b\,x^{-\frac{3}{2}\,(1+m)}\right)^{2/3}\text{Hypergeometric2F1}\left[-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\frac{b\,x^{-\frac{3}{2}\,(1+m)}}{a}\right]}{(1+m)\left(1+\frac{b\,x^{-\frac{3}{2}\,(1+m)}}{a}\right)^{2/3}}$$

Problem 2702: Result unnecessarily involves higher level functions.

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+b x^n)^{1/3}}\,dx$$

Optimal (type 3, 89 leaves, 2 steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x^{n/3}}{(a+b x^n)^{1/3}}}{\sqrt{3}}\right]}{b^{1/3} n}-\frac{3\,\text{Log}\left[b^{1/3} x^{n/3}-(a+b x^n)^{1/3}\right]}{2 b^{1/3} n}$$

Result (type 5, 57 leaves):

$$\frac{3 x^{n/3}\left(\frac{a+b x^n}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{b x^n}{a}\right]}{n\,(a+b x^n)^{1/3}}$$

Problem 2703: Result unnecessarily involves higher level functions.

$$\int x^{-1-\frac{2n}{3}}\,(a+b x^n)^{2/3}\,dx$$

Optimal (type 3, 114 leaves, 3 steps):

$$-\frac{3 x^{-2n/3}(a+b x^n)^{2/3}}{2 n}+\frac{\sqrt{3}\,b^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x^{n/3}}{(a+b x^n)^{1/3}}}{\sqrt{3}}\right]}{n}-\frac{3 b^{2/3}\,\text{Log}\left[b^{1/3} x^{n/3}-(a+b x^n)^{1/3}\right]}{2 n}$$

Result (type 5, 71 leaves):

$$-\frac{3 x^{-2n/3}\left(a+b x^n-2 b x^n\left(1+\frac{b x^n}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{b x^n}{a}\right]\right)}{2 n\,(a+b x^n)^{1/3}}$$

Problem 2705: Result unnecessarily involves higher level functions.

$$\int (a+b x^n)^{-4-\frac{1}{n}}\,dx$$

Optimal (type 3, 146 leaves, 4 steps):

$$\frac{x\,(a+b x^n)^{-3-\frac{1}{n}}}{a\,(1+3n)}+\frac{3 n x\,(a+b x^n)^{-2-\frac{1}{n}}}{a^2\left(1+5n+6n^2\right)}+\frac{6 n^2 x\,(a+b x^n)^{-1-\frac{1}{n}}}{a^3\,(1+n)\,(1+2n)\,(1+3n)}+\frac{6 n^3 x\,(a+b x^n)^{-1/n}}{a^4\,(1+n)\,(1+2n)\,(1+3n)}$$

Result (type 5, 55 leaves):

$$\frac{x\,(a+b x^n)^{-1/n}\left(1+\frac{b x^n}{a}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[4+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]}{a^4}$$

- **Problem 2706: Result unnecessarily involves higher level functions.**

$$\int (a+b x^n)^{-3-\frac{1}{n}} \, dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$\frac{x (a+b x^n)^{-2-\frac{1}{n}}}{a (1+2 n)}+\frac{2 n x (a+b x^n)^{-1-\frac{1}{n}}}{a^2 (1+n) (1+2 n)}+\frac{2 n^2 x (a+b x^n)^{-1/n}}{a^3 (1+n) (1+2 n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a+b x^n)^{-1/n} \left(1+\frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[3+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]}{a^3}$$

- **Problem 2707: Result unnecessarily involves higher level functions.**

$$\int (a+b x^n)^{-2-\frac{1}{n}} \, dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{x (a+b x^n)^{-1-\frac{1}{n}}}{a (1+n)}+\frac{n x (a+b x^n)^{-1/n}}{a^2 (1+n)}$$

Result (type 5, 55 leaves):

$$\frac{x (a+b x^n)^{-1/n} \left(1+\frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{b x^n}{a}\right]}{a^2}$$

- **Problem 2725: Result more than twice size of optimal antiderivative.**

$$\int x^{-1-9 n} (a+b x^n)^8 \, dx$$

Optimal (type 3, 24 leaves, 1 step):

$$-\frac{x^{-9 n} (a+b x^n)^9}{9 a n}$$

Result (type 3, 111 leaves):

$$-\frac{1}{9 n} x^{-9 n} \left(a^8+9 a^7 b x^n+36 a^6 b^2 x^{2 n}+84 a^5 b^3 x^{3 n}+126 a^4 b^4 x^{4 n}+126 a^3 b^5 x^{5 n}+84 a^2 b^6 x^{6 n}+36 a b^7 x^{7 n}+9 b^8 x^{8 n}\right)$$

- **Problem 2727: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^3\right)^8}{x^{28}} \, dx$$

Optimal (type 1, 19 leaves, 1 step):

$$-\frac{\left(a+b x^3\right)^9}{27 a x^{27}}$$

Result (type 1, 108 leaves) :

$$-\frac{a^8}{27 x^{27}}-\frac{a^7 b}{3 x^{24}}-\frac{4 a^6 b^2}{3 x^{21}}-\frac{28 a^5 b^3}{9 x^{18}}-\frac{14 a^4 b^4}{3 x^{15}}-\frac{14 a^3 b^5}{3 x^{12}}-\frac{28 a^2 b^6}{9 x^9}-\frac{4 a b^7}{3 x^6}-\frac{b^8}{3 x^3}$$

- **Problem 2730: Result unnecessarily involves higher level functions.**

$$\int (a+b x^n)^{-\frac{1+4 n}{n}} \, dx$$

Optimal (type 3, 147 leaves, 4 steps) :

$$\frac{x (a+b x^n)^{-3-\frac{1}{n}}}{a (1+3 n)}+\frac{3 n x (a+b x^n)^{-2-\frac{1}{n}}}{a^2 \left(1+5 n+6 n^2\right)}+\frac{6 n^3 x (a+b x^n)^{-1/n}}{a^4 (1+n) (1+2 n) (1+3 n)}+\frac{6 n^2 x (a+b x^n)^{-\frac{1+n}{n}}}{a^3 (1+n) (1+2 n) (1+3 n)}$$

Result (type 5, 55 leaves) :

$$\frac{x (a+b x^n)^{-1/n} \left(1+\frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[4+\frac{1}{n}, \frac{1}{n}, 1+\frac{1}{n}, -\frac{b x^n}{a}\right]}{a^4}$$

- **Problem 2731: Result unnecessarily involves higher level functions.**

$$\int (a+b x^n)^{-\frac{1+3 n}{n}} \, dx$$

Optimal (type 3, 97 leaves, 3 steps) :

$$\frac{x (a+b x^n)^{-2-\frac{1}{n}}}{a (1+2 n)}+\frac{2 n^2 x (a+b x^n)^{-1/n}}{a^3 (1+n) (1+2 n)}+\frac{2 n x (a+b x^n)^{-\frac{1+n}{n}}}{a^2 (1+n) (1+2 n)}$$

Result (type 5, 55 leaves) :

$$\frac{x (a+b x^n)^{-1/n} \left(1+\frac{b x^n}{a}\right)^{\frac{1}{n}} \text{Hypergeometric2F1}\left[3+\frac{1}{n}, \frac{1}{n}, 1+\frac{1}{n}, -\frac{b x^n}{a}\right]}{a^3}$$

- **Problem 2732: Result unnecessarily involves higher level functions.**

$$\int (a+b x^n)^{-\frac{1+2 n}{n}} \, dx$$

Optimal (type 3, 51 leaves, 2 steps) :

$$\frac{n x (a+b x^n)^{-1/n}}{a^2 (1+n)}+\frac{x (a+b x^n)^{-\frac{1+n}{n}}}{a (1+n)}$$

Result (type 5, 55 leaves) :

$$\frac{x\,(a+b\,x^n)^{-1/n}\,\left(1+\frac{b\,x^n}{a}\right)^{\frac{1}{n}}\,\text{Hypergeometric2F1}\left[2+\frac{1}{n},\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{b\,x^n}{a}\right]}{a^2}$$

- **Problem 2763: Result is not expressed in closed-form.**

$$\int\frac{(c\,x)^{-1-\frac{2n}{3}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 222 leaves, 9 steps):

$$-\frac{3\,(c\,x)^{-2n/3}}{2\,a\,c\,n}+\frac{\sqrt{3}\,b^{2/3}\,x^{2n/3}\,(c\,x)^{-2n/3}\,\text{ArcTan}\left[\frac{a^{1/3}-2\,b^{1/3}\,x^{n/3}}{\sqrt{3}\,a^{1/3}}\right]}{a^{5/3}\,c\,n}-\frac{b^{2/3}\,x^{2n/3}\,(c\,x)^{-2n/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,x^{n/3}\right]}{a^{5/3}\,c\,n}+\frac{b^{2/3}\,x^{2n/3}\,(c\,x)^{-2n/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x^{n/3}+b^{2/3}\,x^{2n/3}\right]}{2\,a^{5/3}\,c\,n}$$

Result (type 7, 72 leaves):

$$\frac{(c\,x)^{-2n/3}\,\left(-9\,a+2\,b\,x^{2n/3}\,\text{RootSum}\left[b+a\,\#1^3\,\&,\,\frac{n\,\text{Log}[x]+3\,\text{Log}\left[x^{-n/3}-\#1\right]}{\#1}\,\&\right]\right)}{6\,a^2\,c\,n}$$

- **Problem 2764: Result is not expressed in closed-form.**

$$\int\frac{(c\,x)^{-1-\frac{3n}{4}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 317 leaves, 12 steps):

$$-\frac{4\,(c\,x)^{-3n/4}}{3\,a\,c\,n}+\frac{\sqrt{2}\,b^{3/4}\,x^{3n/4}\,(c\,x)^{-3n/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,b^{1/4}\,x^{n/4}}{a^{1/4}}\right]}{a^{7/4}\,c\,n}-\frac{\sqrt{2}\,b^{3/4}\,x^{3n/4}\,(c\,x)^{-3n/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,b^{1/4}\,x^{n/4}}{a^{1/4}}\right]}{a^{7/4}\,c\,n}+\frac{b^{3/4}\,x^{3n/4}\,(c\,x)^{-3n/4}\,\text{Log}\left[\sqrt{a}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{n/4}+\sqrt{b}\,x^{n/2}\right]}{\sqrt{2}\,a^{7/4}\,c\,n}-\frac{b^{3/4}\,x^{3n/4}\,(c\,x)^{-3n/4}\,\text{Log}\left[\sqrt{a}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{n/4}+\sqrt{b}\,x^{n/2}\right]}{\sqrt{2}\,a^{7/4}\,c\,n}$$

Result (type 7, 72 leaves):

$$\frac{(c\,x)^{-3n/4}\,\left(-16\,a+3\,b\,x^{3n/4}\,\text{RootSum}\left[b+a\,\#1^4\,\&,\,\frac{n\,\text{Log}[x]+4\,\text{Log}\left[x^{-n/4}-\#1\right]}{\#1}\,\&\right]\right)}{12\,a^2\,c\,n}$$

- **Problem 2767: Result is not expressed in closed-form.**

$$\int\frac{(c\,x)^{-1-\frac{n}{3}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 220 leaves, 10 steps):

$$-\frac{3\,(c\,x)^{-n/3}}{a\,c\,n}-\frac{\sqrt{3}\,b^{1/3}\,x^{n/3}\,(c\,x)^{-n/3}\,\text{ArcTan}\left[\frac{b^{1/3}-2\,a^{1/3}\,x^{-n/3}}{\sqrt{3}\,b^{1/3}}\right]}{a^{4/3}\,c\,n}+\frac{b^{1/3}\,x^{n/3}\,(c\,x)^{-n/3}\,\text{Log}\left[b^{1/3}+a^{1/3}\,x^{-n/3}\right]}{a^{4/3}\,c\,n}-\frac{b^{1/3}\,x^{n/3}\,(c\,x)^{-n/3}\,\text{Log}\left[b^{2/3}+a^{2/3}\,x^{-2\,n/3}-a^{1/3}\,b^{1/3}\,x^{-n/3}\right]}{2\,a^{4/3}\,c\,n}$$

Result (type 7, 71 leaves) :

$$\frac{(c\,x)^{-n/3}\left(-9\,a+b\,x^{n/3}\,\text{RootSum}\left[b+a\,\sharp1^3\,\&,\,\frac{n\,\text{Log}[x]+3\,\text{Log}\left[x^{-n/3}-\sharp1\right]}{\sharp1^2}\,\&\right]\right)}{3\,a^2\,c\,n}$$

Problem 2768: Result is not expressed in closed-form.

$$\int\frac{(c\,x)^{-1-\frac{n}{4}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 315 leaves, 13 steps) :

$$-\frac{4\,(c\,x)^{-n/4}}{a\,c\,n}-\frac{\sqrt{2}\,b^{1/4}\,x^{n/4}\,(c\,x)^{-n/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{5/4}\,c\,n}+\frac{\sqrt{2}\,b^{1/4}\,x^{n/4}\,(c\,x)^{-n/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{5/4}\,c\,n}-\frac{b^{1/4}\,x^{n/4}\,(c\,x)^{-n/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\,x^{-n/2}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\,a^{5/4}\,c\,n}+\frac{b^{1/4}\,x^{n/4}\,(c\,x)^{-n/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\,x^{-n/2}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\,a^{5/4}\,c\,n}$$

Result (type 7, 71 leaves) :

$$\frac{(c\,x)^{-n/4}\left(-16\,a+b\,x^{n/4}\,\text{RootSum}\left[b+a\,\sharp1^4\,\&,\,\frac{n\,\text{Log}[x]+4\,\text{Log}\left[x^{-n/4}-\sharp1\right]}{\sharp1^3}\,\&\right]\right)}{4\,a^2\,c\,n}$$

Problem 2770: Result is not expressed in closed-form.

$$\int\frac{(c\,x)^{-1-\frac{4n}{3}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 246 leaves, 11 steps) :

$$-\frac{3\,(c\,x)^{-4\,n/3}}{4\,a\,c\,n}+\frac{3\,b\,x^n\,(c\,x)^{-4\,n/3}}{a^2\,c\,n}+\frac{\sqrt{3}\,b^{4/3}\,x^{4\,n/3}\,(c\,x)^{-4\,n/3}\,\text{ArcTan}\left[\frac{b^{1/3}-2\,a^{1/3}\,x^{-n/3}}{\sqrt{3}\,b^{1/3}}\right]}{a^{7/3}\,c\,n}-\frac{b^{4/3}\,x^{4\,n/3}\,(c\,x)^{-4\,n/3}\,\text{Log}\left[b^{1/3}+a^{1/3}\,x^{-n/3}\right]}{a^{7/3}\,c\,n}+\frac{b^{4/3}\,x^{4\,n/3}\,(c\,x)^{-4\,n/3}\,\text{Log}\left[b^{2/3}+a^{2/3}\,x^{-2\,n/3}-a^{1/3}\,b^{1/3}\,x^{-n/3}\right]}{2\,a^{7/3}\,c\,n}$$

Result (type 7, 82 leaves) :

$$\frac{(c\,x)^{-4\,n/3}\left(-9\,a\,(a-4\,b\,x^n)-4\,b^2\,x^{4\,n/3}\,\text{RootSum}\left[b+a\,\sharp 1^3\,\&,\,\frac{n\,\text{Log}[x]+3\,\text{Log}\left[x^{-n/3}-\sharp 1\right]}{\sharp 1^2}\,\&\right]\right)}{12\,a^3\,c\,n}$$

- **Problem 2771: Result is not expressed in closed-form.**

$$\int \frac{(c\,x)^{-1-\frac{5\,n}{4}}}{a+b\,x^n}\,dx$$

Optimal (type 3, 341 leaves, 14 steps):

$$-\frac{4\,(c\,x)^{-5\,n/4}}{5\,a\,c\,n}+\frac{4\,b\,x^n\,(c\,x)^{-5\,n/4}}{a^2\,c\,n}+\frac{\sqrt{2}\,b^{5/4}\,x^{5\,n/4}\,(c\,x)^{-5\,n/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{9/4}\,c\,n}-\frac{\sqrt{2}\,b^{5/4}\,x^{5\,n/4}\,(c\,x)^{-5\,n/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,a^{1/4}\,x^{-n/4}}{b^{1/4}}\right]}{a^{9/4}\,c\,n}+\frac{b^{5/4}\,x^{5\,n/4}\,(c\,x)^{-5\,n/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\,x^{-n/2}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\,a^{9/4}\,c\,n}-\frac{b^{5/4}\,x^{5\,n/4}\,(c\,x)^{-5\,n/4}\,\text{Log}\left[\sqrt{b}+\sqrt{a}\,x^{-n/2}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x^{-n/4}\right]}{\sqrt{2}\,a^{9/4}\,c\,n}$$

Result (type 7, 82 leaves):

$$\frac{(c\,x)^{-5\,n/4}\left(-16\,a\,(a-5\,b\,x^n)-5\,b^2\,x^{5\,n/4}\,\text{RootSum}\left[b+a\,\sharp 1^4\,\&,\,\frac{n\,\text{Log}[x]+4\,\text{Log}\left[x^{-n/4}-\sharp 1\right]}{\sharp 1^3}\,\&\right]\right)}{20\,a^3\,c\,n}$$

- **Problem 2799: Result unnecessarily involves higher level functions.**

$$\int (c\,x)^{-1-2\,n-n\,p}\,(a+b\,x^n)^p\,dx$$

Optimal (type 3, 79 leaves, 2 steps):

$$-\frac{(c\,x)^{-n\,(2+p)}\,(a+b\,x^n)^{1+p}}{a\,c\,n\,(1+p)}+\frac{(c\,x)^{-n\,(2+p)}\,(a+b\,x^n)^{2+p}}{a^2\,c\,n\,(1+p)\,(2+p)}$$

Result (type 5, 69 leaves):

$$-\frac{x\,(c\,x)^{-1-n\,(2+p)}\,(a+b\,x^n)^p\,\left(1+\frac{b\,x^n}{a}\right)^{-p}\,\text{Hypergeometric2F1}\left[-2-p,\,-p,\,-1-p,\,-\frac{b\,x^n}{a}\right]}{n\,(2+p)}$$

- **Problem 2800: Result unnecessarily involves higher level functions.**

$$\int (c\,x)^{-1-3\,n-n\,p}\,(a+b\,x^n)^p\,dx$$

Optimal (type 3, 127 leaves, 3 steps):

$$-\frac{(c\,x)^{-n\,(3+p)}\,(a+b\,x^n)^{1+p}}{a\,c\,n\,(1+p)}+\frac{2\,(c\,x)^{-n\,(3+p)}\,(a+b\,x^n)^{2+p}}{a^2\,c\,n\,(1+p)\,(2+p)}-\frac{2\,(c\,x)^{-n\,(3+p)}\,(a+b\,x^n)^{3+p}}{a^3\,c\,n\,(1+p)\,(2+p)\,(3+p)}$$

Result (type 5, 69 leaves):

$$-\frac{x\,(c\,x)^{-1-n\,(3+p)}\,(a+b\,x^n)^p\,\left(1+\frac{b\,x^n}{a}\right)^{-p}\,\text{Hypergeometric2F1}\left[-3-p,\,-p,\,-2-p,\,-\frac{b\,x^n}{a}\right]}{n\,(3+p)}$$

- **Problem 2801: Result unnecessarily involves higher level functions.**

$$\int (c\,x)^{-1-4\,n-n\,p}\,(a+b\,x^n)^p\,dx$$

Optimal (type 3, 179 leaves, 4 steps):

$$-\frac{(c\,x)^{-n\,(4+p)}\,(a+b\,x^n)^{1+p}}{a\,c\,n\,(1+p)}+\frac{3\,(c\,x)^{-n\,(4+p)}\,(a+b\,x^n)^{2+p}}{a^2\,c\,n\,(1+p)\,(2+p)}-\frac{6\,(c\,x)^{-n\,(4+p)}\,(a+b\,x^n)^{3+p}}{a^3\,c\,n\,(1+p)\,(2+p)\,(3+p)}+\frac{6\,(c\,x)^{-n\,(4+p)}\,(a+b\,x^n)^{4+p}}{a^4\,c\,n\,(1+p)\,(2+p)\,(3+p)\,(4+p)}$$

Result (type 5, 69 leaves):

$$-\frac{x\,(c\,x)^{-1-n\,(4+p)}\,(a+b\,x^n)^p\,\left(1+\frac{b\,x^n}{a}\right)^{-p}\,\text{Hypergeometric2F1}\left[-4-p,\,-p,\,-3-p,\,-\frac{b\,x^n}{a}\right]}{n\,(4+p)}$$

- **Problem 2841: Result more than twice size of optimal antiderivative.**

$$\int (c+d\,x)^3\,\left(a+b\,(c+d\,x)^2\right)\,dx$$

Optimal (type 1, 31 leaves, 3 steps):

$$\frac{a\,(c+d\,x)^4}{4\,d}+\frac{b\,(c+d\,x)^6}{6\,d}$$

Result (type 1, 77 leaves):

$$\frac{1}{12}\,x\,(2\,c+d\,x)\,\left(3\,a\,\left(2\,c^2+2\,c\,d\,x+d^2\,x^2\right)+2\,b\,\left(3\,c^4+6\,c^3\,d\,x+7\,c^2\,d^2\,x^2+4\,c\,d^3\,x^3+d^4\,x^4\right)\right)$$

- **Problem 2842: Result more than twice size of optimal antiderivative.**

$$\int (c+d\,x)^3\,\left(a+b\,(c+d\,x)^2\right)^2\,dx$$

Optimal (type 1, 51 leaves, 4 steps):

$$\frac{a^2\,(c+d\,x)^4}{4\,d}+\frac{a\,b\,(c+d\,x)^6}{3\,d}+\frac{b^2\,(c+d\,x)^8}{8\,d}$$

Result (type 1, 172 leaves):

$$c^3\,\left(a+b\,c^2\right)^2\,x+\frac{1}{2}\,c^2\,\left(3\,a^2+10\,a\,b\,c^2+7\,b^2\,c^4\right)\,d\,x^2+\frac{1}{3}\,c\,\left(3\,a^2+20\,a\,b\,c^2+21\,b^2\,c^4\right)\,d^2\,x^3+$$
$$\frac{1}{4}\,\left(a^2+20\,a\,b\,c^2+35\,b^2\,c^4\right)\,d^3\,x^4+b\,c\,\left(2\,a+7\,b\,c^2\right)\,d^4\,x^5+\frac{1}{6}\,b\,\left(2\,a+21\,b\,c^2\right)\,d^5\,x^6+b^2\,c\,d^6\,x^7+\frac{1}{8}\,b^2\,d^7\,x^8$$

Problem 2843: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 \left(a+b (c+d x)^2\right)^3 \, dx$$

Optimal (type 1, 48 leaves, 4 steps):

$$-\frac{a \left(a+b (c+d x)^2\right)^4}{8 b^2 d}+\frac{\left(a+b (c+d x)^2\right)^5}{10 b^2 d}$$

Result (type 1, 249 leaves):

$$c^3 \left(a+b c^2\right)^3 x+\frac{3}{2} c^2 \left(a+b c^2\right)^2 \left(a+3 b c^2\right) d x^2+c \left(a^3+10 a^2 b c^2+21 a b^2 c^4+12 b^3 c^6\right) d^2 x^3+\frac{1}{4} \left(a^3+30 a^2 b c^2+105 a b^2 c^4+84 b^3 c^6\right) d^3 x^4+\frac{3}{5} b c \left(5 a^2+35 a b c^2+42 b^2 c^4\right) d^4 x^5+\frac{1}{2} b \left(a^2+21 a b c^2+42 b^2 c^4\right) d^5 x^6+3 b^2 c \left(a+4 b c^2\right) d^6 x^7+\frac{3}{8} b^2 \left(a+12 b c^2\right) d^7 x^8+b^3 c d^8 x^9+\frac{1}{10} b^3 d^9 x^{10}$$

Problem 2853: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 \left(a+b (c+d x)^3\right) \, dx$$

Optimal (type 1, 31 leaves, 3 steps):

$$\frac{a (c+d x)^4}{4 d}+\frac{b (c+d x)^7}{7 d}$$

Result (type 1, 98 leaves):

$$c^3 \left(a+b c^3\right) x+\frac{3}{2} c^2 \left(a+2 b c^3\right) d x^2+c \left(a+5 b c^3\right) d^2 x^3+\frac{1}{4} \left(a+20 b c^3\right) d^3 x^4+3 b c^2 d^4 x^5+b c d^5 x^6+\frac{1}{7} b d^6 x^7$$

Problem 2854: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 \left(a+b (c+d x)^3\right)^2 \, dx$$

Optimal (type 1, 51 leaves, 3 steps):

$$\frac{a^2 (c+d x)^4}{4 d}+\frac{2 a b (c+d x)^7}{7 d}+\frac{b^2 (c+d x)^{10}}{10 d}$$

Result (type 1, 203 leaves):

$$c^3 \left(a+b c^3\right)^2 x+\frac{3}{2} c^2 \left(a^2+4 a b c^3+3 b^2 c^6\right) d x^2+c \left(a^2+10 a b c^3+12 b^2 c^6\right) d^2 x^3+\frac{1}{4} \left(a^2+40 a b c^3+84 b^2 c^6\right) d^3 x^4+\frac{6}{5} b c^2 \left(5 a+21 b c^3\right) d^4 x^5+b c \left(2 a+21 b c^3\right) d^5 x^6+\frac{2}{7} b \left(a+42 b c^3\right) d^6 x^7+\frac{9}{2} b^2 c^2 d^7 x^8+b^2 c d^8 x^9+\frac{1}{10} b^2 d^9 x^{10}$$

Problem 2855: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 \left(a+b (c+d x)^3\right)^3 \, dx$$

Optimal (type 1, 71 leaves, 3 steps):

$$\frac{a^3 (c+d x)^4}{4 d}+\frac{3 a^2 b (c+d x)^7}{7 d}+\frac{3 a b^2 (c+d x)^{10}}{10 d}+\frac{b^3 (c+d x)^{13}}{13 d}$$

Result (type 1, 323 leaves):

$$c^3 \left(a+b c^3\right)^3 x+\frac{3}{2} c^2 \left(a+b c^3\right)^2 \left(a+4 b c^3\right) d x^2+c \left(a^3+15 a^2 b c^3+36 a b^2 c^6+22 b^3 c^9\right) d^2 x^3+\frac{1}{4} \left(a^3+60 a^2 b c^3+252 a b^2 c^6+220 b^3 c^9\right) d^3 x^4+\frac{9}{5} b c^2 \left(5 a^2+42 a b c^3+55 b^2 c^6\right) d^4 x^5+3 b c \left(a^2+21 a b c^3+44 b^2 c^6\right) d^5 x^6+\frac{3}{7} b \left(a^2+84 a b c^3+308 b^2 c^6\right) d^6 x^7+\frac{9}{2} b^2 c^2 \left(3 a+22 b c^3\right) d^7 x^8+b^2 c \left(3 a+55 b c^3\right) d^8 x^9+\frac{1}{10} b^2 \left(3 a+220 b c^3\right) d^9 x^{10}+6 b^3 c^2 d^{10} x^{11}+b^3 c d^{11} x^{12}+\frac{1}{13} b^3 d^{12} x^{13}$$

Problem 2856: Result more than twice size of optimal antiderivative.

$$\int (c e+d e x)^3 \left(a+b (c+d x)^3\right) dx$$

Optimal (type 1, 37 leaves, 3 steps):

$$\frac{a e^3 (c+d x)^4}{4 d}+\frac{b e^3 (c+d x)^7}{7 d}$$

Result (type 1, 102 leaves):

$$e^3 \left(c^3 \left(a+b c^3\right) x+\frac{3}{2} c^2 \left(a+2 b c^3\right) d x^2+c \left(a+5 b c^3\right) d^2 x^3+\frac{1}{4} \left(a+20 b c^3\right) d^3 x^4+3 b c^2 d^4 x^5+b c d^5 x^6+\frac{1}{7} b d^6 x^7\right)$$

Problem 2857: Result more than twice size of optimal antiderivative.

$$\int (c e+d e x)^3 \left(a+b (c+d x)^3\right)^2 dx$$

Optimal (type 1, 60 leaves, 3 steps):

$$\frac{a^2 e^3 (c+d x)^4}{4 d}+\frac{2 a b e^3 (c+d x)^7}{7 d}+\frac{b^2 e^3 (c+d x)^{10}}{10 d}$$

Result (type 1, 207 leaves):

$$e^3 \left(c^3 \left(a+b c^3\right)^2 x+\frac{3}{2} c^2 \left(a^2+4 a b c^3+3 b^2 c^6\right) d x^2+c \left(a^2+10 a b c^3+12 b^2 c^6\right) d^2 x^3+\frac{1}{4} \left(a^2+40 a b c^3+84 b^2 c^6\right) d^3 x^4+\frac{6}{5} b c^2 \left(5 a+21 b c^3\right) d^4 x^5+b c \left(2 a+21 b c^3\right) d^5 x^6+\frac{2}{7} b \left(a+42 b c^3\right) d^6 x^7+\frac{9}{2} b^2 c^2 d^7 x^8+b^2 c d^8 x^9+\frac{1}{10} b^2 d^9 x^{10}\right)$$

Problem 2858: Result more than twice size of optimal antiderivative.

$$\int (c e+d e x)^3 \left(a+b (c+d x)^3\right)^3 dx$$

Optimal (type 1, 83 leaves, 3 steps):

$$\frac{a^3 e^3 (c+d x)^4}{4 d}+\frac{3 a^2 b e^3 (c+d x)^7}{7 d}+\frac{3 a b^2 e^3 (c+d x)^{10}}{10 d}+\frac{b^3 e^3 (c+d x)^{13}}{13 d}$$

Result (type 1, 327 leaves):

$$e^3 \left(c^3 \left(a+b c^3\right)^3 x+\frac{3}{2} c^2 \left(a+b c^3\right)^2 \left(a+4 b c^3\right) d x^2+c \left(a^3+15 a^2 b c^3+36 a b^2 c^6+22 b^3 c^9\right) d^2 x^3+\frac{1}{4} \left(a^3+60 a^2 b c^3+252 a b^2 c^6+220 b^3 c^9\right) d^3 x^4+\frac{9}{5} b c^2 \left(5 a^2+42 a b c^3+55 b^2 c^6\right) d^4 x^5+3 b c \left(a^2+21 a b c^3+44 b^2 c^6\right) d^5 x^6+\frac{3}{7} b \left(a^2+84 a b c^3+308 b^2 c^6\right) d^6 x^7+\frac{9}{2} b^2 c^2 \left(3 a+22 b c^3\right) d^7 x^8+b^2 c \left(3 a+55 b c^3\right) d^8 x^9+\frac{1}{10} b^2 \left(3 a+220 b c^3\right) d^9 x^{10}+6 b^3 c^2 d^{10} x^{11}+b^3 c d^{11} x^{12}+\frac{1}{13} b^3 d^{12} x^{13}\right)$$

- **Problem 2911: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^3 \left(a+b (c+d x)^4\right) dx$$

Optimal (type 1, 23 leaves, 3 steps):

$$\frac{\left(a+b (c+d x)^4\right)^2}{8 b d}$$

Result (type 1, 80 leaves):

$$\frac{1}{8} x \left(4 c^3+6 c^2 d x+4 c d^2 x^2+d^3 x^3\right) \left(2 a+b \left(2 c^4+4 c^3 d x+6 c^2 d^2 x^2+4 c d^3 x^3+d^4 x^4\right)\right)$$

- **Problem 2912: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^3 \left(a+b (c+d x)^4\right)^2 dx$$

Optimal (type 1, 23 leaves, 2 steps):

$$\frac{\left(a+b (c+d x)^4\right)^3}{12 b d}$$

Result (type 1, 172 leaves):

$$\frac{1}{12} x \left(4 c^3+6 c^2 d x+4 c d^2 x^2+d^3 x^3\right) \left(3 a^2+3 a b \left(2 c^4+4 c^3 d x+6 c^2 d^2 x^2+4 c d^3 x^3+d^4 x^4\right)+b^2 \left(3 c^8+12 c^7 d x+34 c^6 d^2 x^2+60 c^5 d^3 x^3+71 c^4 d^4 x^4+56 c^3 d^5 x^5+28 c^2 d^6 x^6+8 c d^7 x^7+d^8 x^8\right)\right)$$

- **Problem 2913: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^3 \left(a+b (c+d x)^4\right)^3 dx$$

Optimal (type 1, 23 leaves, 2 steps):

$$\frac{\left(a+b (c+d x)^4\right)^4}{16 b d}$$

Result (type 1, 308 leaves) :

$$\frac{1}{16} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3\right) \left(4 a^3 + 6 a^2 b \left(2 c^4 + 4 c^3 d x + 6 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4\right) + 4 a b^2 \left(3 c^8 + 12 c^7 d x + 34 c^6 d^2 x^2 + 60 c^5 d^3 x^3 + 71 c^4 d^4 x^4 + 56 c^3 d^5 x^5 + 28 c^2 d^6 x^6 + 8 c d^7 x^7 + d^8 x^8\right) + b^3 \left(4 c^{12} + 24 c^{11} d x + 100 c^{10} d^2 x^2 + 280 c^9 d^3 x^3 + 566 c^8 d^4 x^4 + 848 c^7 d^5 x^5 + 952 c^6 d^6 x^6 + 800 c^5 d^7 x^7 + 496 c^4 d^8 x^8 + 220 c^3 d^9 x^9 + 66 c^2 d^{10} x^{10} + 12 c d^{11} x^{11} + d^{12} x^{12}\right)\right)$$

- **Problem 2917: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + b (c + d x)^4}} \, dx$$

Optimal (type 4, 111 leaves, 2 steps) :

$$\frac{\left(\sqrt{a} + \sqrt{b} (c + d x)^2\right) \sqrt{\frac{a+b (c+d x)^4}{\left(\sqrt{a}+\sqrt{b} (c+d x)^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} (c+d x)}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} b^{1/4} d \sqrt{a + b (c + d x)^4}}$$

Result (type 4, 90 leaves) :

$$-\frac{i \sqrt{\frac{a+b (c+d x)^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} (c + d x)\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{a + b (c + d x)^4}}$$

- **Problem 2918: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{x}{\sqrt{a + b (c + d x)^4}} \, dx$$

Optimal (type 4, 154 leaves, 7 steps) :

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b} (c+d x)^2}{\sqrt{a+b (c+d x)^4}}\right]}{2 \sqrt{b} d^2} - \frac{c \left(\sqrt{a} + \sqrt{b} (c + d x)^2\right) \sqrt{\frac{a+b (c+d x)^4}{\left(\sqrt{a}+\sqrt{b} (c+d x)^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} (c+d x)}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} b^{1/4} d^2 \sqrt{a + b (c + d x)^4}}$$

Result (type 4, 330 leaves) :

$$\left((-1)^{1/4}\sqrt{2}\sqrt{-\frac{i\left((-1)^{1/4}a^{1/4}+b^{1/4}(c+dx)\right)}{(-1)^{1/4}a^{1/4}-b^{1/4}(c+dx)}}\left(i\sqrt{a}+\sqrt{b}(c+dx)^2\right)\right.$$
$$\left(\left((-1)^{1/4}a^{1/4}-b^{1/4}c\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{i\left((-1)^{1/4}a^{1/4}+b^{1/4}(c+dx)\right)}{(-1)^{1/4}a^{1/4}-b^{1/4}(c+dx)}}\right],-1\right]-\right.$$
$$\left.\left.2(-1)^{1/4}a^{1/4}\,\text{EllipticPi}\left[-i,\text{ArcSin}\left[\sqrt{-\frac{i\left((-1)^{1/4}a^{1/4}+b^{1/4}(c+dx)\right)}{(-1)^{1/4}a^{1/4}-b^{1/4}(c+dx)}}\right],-1\right]\right)\right)\Big/$$
$$\left(a^{1/4}\sqrt{b}\,d^2\sqrt{\frac{i\sqrt{a}+\sqrt{b}(c+dx)^2}{\left((-1)^{1/4}a^{1/4}-b^{1/4}(c+dx)\right)^2}}\sqrt{a+b(c+dx)^4}\right)$$

- **Problem 2929: Unable to integrate problem.**

$$\int\frac{1}{1+\left(x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 83 leaves, 7 steps) :

$$-\frac{x\,\text{ArcTan}\left[\frac{1-2\sqrt{x^2}}{\sqrt{3}}\right]}{\sqrt{3}\sqrt{x^2}}-\frac{x\,\text{Log}\left[1+x^2-\sqrt{x^2}\right]}{6\sqrt{x^2}}+\frac{x\,\text{Log}\left[1+\sqrt{x^2}\right]}{3\sqrt{x^2}}$$

Result (type 8, 13 leaves) :

$$\int\frac{1}{1+\left(x^2\right)^{3/2}}\,dx$$

- **Problem 2933: Unable to integrate problem.**

$$\int\frac{\sqrt{a+b\sqrt{c\,x^2}}}{x}\,dx$$

Optimal (type 3, 51 leaves, 4 steps) :

$$2\sqrt{a+b\sqrt{c\,x^2}}-2\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^2}}}{\sqrt{a}}\right]$$

Result (type 8, 23 leaves) :

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x}\,dx$$

Problem 2934: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^3}\,dx$$

Optimal (type 3, 97 leaves, 5 steps):

$$-\frac{\sqrt{a+b\sqrt{c\,x^2}}}{2\,x^2}-\frac{b\,c\,\sqrt{a+b\sqrt{c\,x^2}}}{4\,a\,\sqrt{c\,x^2}}+\frac{b^2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^2}}}{\sqrt{a}}\right]}{4\,a^{3/2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^3}\,dx$$

Problem 2935: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^5}\,dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$-\frac{\sqrt{a+b\sqrt{c\,x^2}}}{4\,x^4}+\frac{5\,b^2\,c\,\sqrt{a+b\sqrt{c\,x^2}}}{96\,a^2\,x^2}-\frac{b\,c^2\,\sqrt{a+b\sqrt{c\,x^2}}}{24\,a\,\left(c\,x^2\right)^{3/2}}-\frac{5\,b^3\,c^2\,\sqrt{a+b\sqrt{c\,x^2}}}{64\,a^3\,\sqrt{c\,x^2}}+\frac{5\,b^4\,c^2\,\text{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^2}}}{\sqrt{a}}\right]}{64\,a^{7/2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^5}\,dx$$

Problem 2936: Unable to integrate problem.

$$\int x^4 \sqrt{a+b\sqrt{c\,x^2}}\,dx$$

Optimal (type 2, 191 leaves, 3 steps):

$$\frac{2\,a^4\,x^5\,\left(a+b\sqrt{c\,x^2}\right)^{3/2}}{3\,b^5\,\left(c\,x^2\right)^{5/2}}-\frac{8\,a^3\,x^5\,\left(a+b\sqrt{c\,x^2}\right)^{5/2}}{5\,b^5\,\left(c\,x^2\right)^{5/2}}+\frac{12\,a^2\,x^5\,\left(a+b\sqrt{c\,x^2}\right)^{7/2}}{7\,b^5\,\left(c\,x^2\right)^{5/2}}-\frac{8\,a\,x^5\,\left(a+b\sqrt{c\,x^2}\right)^{9/2}}{9\,b^5\,\left(c\,x^2\right)^{5/2}}+\frac{2\,x^5\,\left(a+b\sqrt{c\,x^2}\right)^{11/2}}{11\,b^5\,\left(c\,x^2\right)^{5/2}}$$

Result (type 8, 23 leaves):

$$\int x^4 \sqrt{a+b\sqrt{c\,x^2}}\,dx$$

Problem 2937: Unable to integrate problem.

$$\int x^2 \sqrt{a+b\sqrt{c\,x^2}}\,dx$$

Optimal (type 2, 113 leaves, 3 steps):

$$\frac{2\,a^2\,x^3\,\left(a+b\sqrt{c\,x^2}\right)^{3/2}}{3\,b^3\,\left(c\,x^2\right)^{3/2}}-\frac{4\,a\,x^3\,\left(a+b\sqrt{c\,x^2}\right)^{5/2}}{5\,b^3\,\left(c\,x^2\right)^{3/2}}+\frac{2\,x^3\,\left(a+b\sqrt{c\,x^2}\right)^{7/2}}{7\,b^3\,\left(c\,x^2\right)^{3/2}}$$

Result (type 8, 23 leaves):

$$\int x^2 \sqrt{a+b\sqrt{c\,x^2}}\,dx$$

Problem 2939: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^2}\,dx$$

Optimal (type 3, 67 leaves, 4 steps):

$$-\frac{\sqrt{a+b\sqrt{c\,x^2}}}{x}-\frac{b\,\sqrt{c\,x^2}\,\operatorname{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^2}}}{\sqrt{a}}\right]}{\sqrt{a}\,x}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^2}\,dx$$

Problem 2940: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^4}\,dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$-\frac{\sqrt{a+b\sqrt{c\,x^2}}}{3\,x^3}+\frac{b^2\,c\,\sqrt{a+b\sqrt{c\,x^2}}}{8\,a^2\,x}-\frac{b\,\left(c\,x^2\right)^{3/2}\,\sqrt{a+b\sqrt{c\,x^2}}}{12\,a\,c\,x^5}-\frac{b^3\,\left(c\,x^2\right)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^2}}}{\sqrt{a}}\right]}{8\,a^{5/2}\,x^3}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^4}\,dx$$

Problem 2941: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^6}\,dx$$

Optimal (type 3, 219 leaves, 8 steps):

$$-\frac{\sqrt{a+b\sqrt{c\,x^2}}}{5\,x^5}+\frac{7\,b^2\,c\,\sqrt{a+b\sqrt{c\,x^2}}}{240\,a^2\,x^3}+\frac{7\,b^4\,c^2\,\sqrt{a+b\sqrt{c\,x^2}}}{128\,a^4\,x}-$$

$$\frac{b\,\left(c\,x^2\right)^{5/2}\,\sqrt{a+b\sqrt{c\,x^2}}}{40\,a\,c^2\,x^9}-\frac{7\,b^3\,\left(c\,x^2\right)^{5/2}\,\sqrt{a+b\sqrt{c\,x^2}}}{192\,a^3\,c\,x^7}-\frac{7\,b^5\,\left(c\,x^2\right)^{5/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^2}}}{\sqrt{a}}\right]}{128\,a^{9/2}\,x^5}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^6}\,dx$$

Problem 2942: Unable to integrate problem.

$$\int x^8 \sqrt{a+b\left(c x^2\right)^{3/2}}\, dx$$

Optimal (type 2, 113 leaves, 4 steps):

$$\frac{2 a^2 x^9 \left(a+b\left(c x^2\right)^{3/2}\right)^{3/2}}{9 b^3 \left(c x^2\right)^{9/2}}-\frac{4 a x^9 \left(a+b\left(c x^2\right)^{3/2}\right)^{5/2}}{15 b^3 \left(c x^2\right)^{9/2}}+\frac{2 x^9 \left(a+b\left(c x^2\right)^{3/2}\right)^{7/2}}{21 b^3 \left(c x^2\right)^{9/2}}$$

Result (type 8, 23 leaves):

$$\int x^8 \sqrt{a+b\left(c x^2\right)^{3/2}}\, dx$$

Problem 2943: Unable to integrate problem.

$$\int x^5 \sqrt{a+b\left(c x^2\right)^{3/2}}\, dx$$

Optimal (type 2, 56 leaves, 4 steps):

$$-\frac{2 a \left(a+b\left(c x^2\right)^{3/2}\right)^{3/2}}{9 b^2 c^3}+\frac{2 \left(a+b\left(c x^2\right)^{3/2}\right)^{5/2}}{15 b^2 c^3}$$

Result (type 8, 23 leaves):

$$\int x^5 \sqrt{a+b\left(c x^2\right)^{3/2}}\, dx$$

Problem 2945: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x}\, dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{2}{3} \sqrt{a+b\left(c x^2\right)^{3/2}}-\frac{2}{3} \sqrt{a}\, \text{ArcTanh}\left[\frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{\sqrt{a}}\right]$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x}\,dx$$

Problem 2946: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^4}\,dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$-\frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{3\,x^3}-\frac{b\left(c\,x^2\right)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{\sqrt{a}}\right]}{3\,\sqrt{a}\,x^3}$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^4}\,dx$$

Problem 2947: Result unnecessarily involves higher level functions.

$$\int x^3\,\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 340 leaves, 4 steps):

$$\frac{2}{11}\,x^4\,\sqrt{a+b\left(c\,x^2\right)^{3/2}}+\frac{6\,a\,\sqrt{c\,x^2}\,\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{55\,b\,c^2}-$$
$$\left(4\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a^2\,\left(a^{1/3}+b^{1/3}\,\sqrt{c\,x^2}\right)\,\sqrt{\frac{a^{2/3}+b^{2/3}\,c\,x^2-a^{1/3}\,b^{1/3}\,\sqrt{c\,x^2}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,\sqrt{c\,x^2}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,\sqrt{c\,x^2}}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,\sqrt{c\,x^2}}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$
$$\left(55\,b^{4/3}\,c^2\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,\sqrt{c\,x^2}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,\sqrt{c\,x^2}\right)^2}}\,\sqrt{a+b\left(c\,x^2\right)^{3/2}}\right)$$

Result (type 5, 132 leaves):

$$\frac{1}{55\,b\,c^2\sqrt{a+b\left(c\,x^2\right)^{3/2}}}\left(16\,a\,b\,c^2\,x^4+6\,a^2\sqrt{c\,x^2}+10\,b^2\,c^3\,x^6\sqrt{c\,x^2}-6\,a^2\sqrt{c\,x^2}\sqrt{\frac{a+b\left(c\,x^2\right)^{3/2}}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{b\left(c\,x^2\right)^{3/2}}{a}\right]\right)$$

Problem 2948: Unable to integrate problem.

$$\int\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 306 leaves, 3 steps):

$$\frac{2}{5}\,x\sqrt{a+b\left(c\,x^2\right)^{3/2}}+$$
$$\left(2\times3^{3/4}\sqrt{2+\sqrt{3}}\;a\,x\left(a^{1/3}+b^{1/3}\sqrt{c\,x^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\,c\,x^2-a^{1/3}\,b^{1/3}\sqrt{c\,x^2}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c\,x^2}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c\,x^2}}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c\,x^2}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$
$$\left(5\,b^{1/3}\sqrt{c\,x^2}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\sqrt{c\,x^2}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c\,x^2}\right)^2}}\sqrt{a+b\left(c\,x^2\right)^{3/2}}\right)$$

Result (type 8, 19 leaves):

$$\int\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

Problem 2949: Unable to integrate problem.

$$\int\frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^3}\,dx$$

Optimal (type 4, 298 leaves, 3 steps):

$$-\frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{2 x^2}+\left(3^{3/4}\sqrt{2+\sqrt{3}}\, b^{2/3} c\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3}\sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\sqrt{a+b\left(c x^2\right)^{3/2}}\right)$$

Result (type 8, 23 leaves) :

$$\int\frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x^3}\,dx$$

Problem 2950: Unable to integrate problem.

$$\int\frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x^6}\,dx$$

Optimal (type 4, 352 leaves, 4 steps) :

$$-\frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{5 x^5}-\frac{3 b\left(c x^2\right)^{5/2}\sqrt{a+b\left(c x^2\right)^{3/2}}}{20 a c x^7}-\left(3^{3/4}\sqrt{2+\sqrt{3}}\, b^{5/3}\left(c x^2\right)^{5/2}\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3}\sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(20 a x^5\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\sqrt{a+b\left(c x^2\right)^{3/2}}\right)$$

Result (type 8, 23 leaves) :

$$\int \frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x^6}\,dx$$

■ Problem 2951: Unable to integrate problem.

$$\int x^4 \sqrt{a+b\left(c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 709 leaves, 6 steps) :

$$\frac{2}{13} x^5 \sqrt{a+b\left(c x^2\right)^{3/2}}+\frac{6 a c x^7 \sqrt{a+b\left(c x^2\right)^{3/2}}}{91 b \left(c x^2\right)^{5/2}}-\frac{24 a^2 x^5 \sqrt{a+b\left(c x^2\right)^{3/2}}}{91 b^{5/3} \left(c x^2\right)^{5/2} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}+$$

$$\left(12\times 3^{1/4} \sqrt{2-\sqrt{3}}\; a^{7/3} x^5 \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right) \sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3} \sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}\right],\,-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(91 b^{5/3} \left(c x^2\right)^{5/2} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}} \sqrt{a+b\left(c x^2\right)^{3/2}}\right)-$$

$$\left(8 \sqrt{2}\; 3^{3/4} a^{7/3} x^5 \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right) \sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3} \sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}\right],\,-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(91 b^{5/3} \left(c x^2\right)^{5/2} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}} \sqrt{a+b\left(c x^2\right)^{3/2}}\right)$$

Result (type 8, 23 leaves) :

$$\int x^4 \sqrt{a+b\left(c x^2\right)^{3/2}}\,dx$$

Problem 2952: Result unnecessarily involves higher level functions.

$$\int x \sqrt{a+b\left(c x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 642 leaves, 5 steps):

$$\frac{2}{7} x^2 \sqrt{a+b\left(c x^2\right)^{3/2}}+\frac{6 a \sqrt{a+b\left(c x^2\right)^{3/2}}}{7 b^{2/3} c\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}-$$

$$\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\ a^{4/3}\left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right) \sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3} \sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 b^{2/3} c \sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}} \sqrt{a+b\left(c x^2\right)^{3/2}}\right)+$$

$$\left(2 \sqrt{2}\ 3^{3/4} a^{4/3}\left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right) \sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3} \sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 b^{2/3} c \sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}} \sqrt{a+b\left(c x^2\right)^{3/2}}\right)$$

Result (type 5, 89 leaves):

$$\frac{x^2\left(4\left(a+b\left(c x^2\right)^{3/2}\right)+3 a \sqrt{\frac{a+b\left(c x^2\right)^{3/2}}{a}}\ \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{b\left(c x^2\right)^{3/2}}{a}\right]\right)}{14 \sqrt{a+b\left(c x^2\right)^{3/2}}}$$

Problem 2953: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x^2}\,dx$$

Optimal (type 4, 661 leaves, 5 steps):

$$-\frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x}+\frac{3\,b^{1/3}\sqrt{c x^2}\,\sqrt{a+b\left(c x^2\right)^{3/2}}}{x\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}\right)}-\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\,b^{1/3}\sqrt{c x^2}\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\,c\,x^2-a^{1/3}\,b^{1/3}\sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2\,x\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\;\sqrt{a+b\left(c x^2\right)^{3/2}}\right)+$$

$$\left(\sqrt{2}\;3^{3/4}\,a^{1/3}\,b^{1/3}\sqrt{c x^2}\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\,c\,x^2-a^{1/3}\,b^{1/3}\sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(x\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^2}\right)^2}}\;\sqrt{a+b\left(c x^2\right)^{3/2}}\right)$$

Result (type 8, 23 leaves):

$$\int \frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x^2}\,dx$$

Problem 2954: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x^5}\,dx$$

Optimal (type 4, 681 leaves, 6 steps):

$$-\frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{4 x^4}-\frac{3 b c^2 \sqrt{a+b\left(c x^2\right)^{3/2}}}{8 a \sqrt{c x^2}}+\frac{3 b^{4/3} c^2 \sqrt{a+b\left(c x^2\right)^{3/2}}}{8 a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}-$$

$$\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\; b^{4/3} c^2 \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right) \sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3} \sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(16 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}} \sqrt{a+b\left(c x^2\right)^{3/2}}\right)+$$

$$\left(3^{3/4} b^{4/3} c^2 \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right) \sqrt{\frac{a^{2/3}+b^{2/3} c x^2-a^{1/3} b^{1/3} \sqrt{c x^2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(4 \sqrt{2}\; a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} \sqrt{c x^2}\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} \sqrt{c x^2}\right)^2}} \sqrt{a+b\left(c x^2\right)^{3/2}}\right)$$

Result (type 8, 23 leaves) :

$$\int \frac{\sqrt{a+b\left(c x^2\right)^{3/2}}}{x^5}\, dx$$

- **Problem 2955: Unable to integrate problem.**

$$\int (d x)^m \sqrt{a+b\left(c x^2\right)^{3/2}}\, dx$$

Optimal (type 5, 86 leaves, 3 steps) :

$$\frac{(d x)^{1+m} \sqrt{a+b\left(c x^2\right)^{3/2}}\; \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{b\left(c x^2\right)^{3/2}}{a}\right]}{d (1+m) \sqrt{1+\frac{b\left(c x^2\right)^{3/2}}{a}}}$$

Result (type 8, 25 leaves) :

$$\int (d\,x)^m \sqrt{a + b\,(c\,x^2)^{3/2}}\,dx$$

- **Problem 2958: Unable to integrate problem.**

$$\int (d\,x)^m \sqrt{a + \frac{b}{(c\,x^2)^{3/2}}}\,dx$$

Optimal (type 5, 90 leaves, 4 steps) :

$$\frac{(d\,x)^{1+m} \sqrt{a + \frac{b}{(c\,x^2)^{3/2}}}\ \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{3}(-1-m), \frac{2-m}{3}, -\frac{b}{a\,(c\,x^2)^{3/2}}\right]}{d\,(1+m)\sqrt{1 + \frac{b}{a\,(c\,x^2)^{3/2}}}}$$

Result (type 8, 25 leaves) :

$$\int (d\,x)^m \sqrt{a + \frac{b}{(c\,x^2)^{3/2}}}\,dx$$

- **Problem 2959: Unable to integrate problem.**

$$\int \frac{1}{1 + (x^3)^{2/3}}\,dx$$

Optimal (type 3, 17 leaves, 2 steps) :

$$\frac{x\,\text{ArcTan}\left[(x^3)^{1/3}\right]}{(x^3)^{1/3}}$$

Result (type 8, 13 leaves) :

$$\int \frac{1}{1 + (x^3)^{2/3}}\,dx$$

- **Problem 2962: Unable to integrate problem.**

$$\int \frac{\sqrt{a + b\sqrt{c\,x^3}}}{x}\,dx$$

Optimal (type 3, 55 leaves, 5 steps) :

$$\frac{4}{3}\sqrt{a+b\sqrt{c\,x^3}}-\frac{4}{3}\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^3}}}{\sqrt{a}}\right]$$

Result (type 8, 23 leaves) :

$$\int\frac{\sqrt{a+b\sqrt{c\,x^3}}}{x}\,dx$$

Problem 2963: Unable to integrate problem.

$$\int\frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^4}\,dx$$

Optimal (type 3, 97 leaves, 6 steps) :

$$-\frac{\sqrt{a+b\sqrt{c\,x^3}}}{3\,x^3}-\frac{b\,c\,\sqrt{a+b\sqrt{c\,x^3}}}{6\,a\sqrt{c\,x^3}}+\frac{b^2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\sqrt{c\,x^3}}}{\sqrt{a}}\right]}{6\,a^{3/2}}$$

Result (type 8, 23 leaves) :

$$\int\frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^4}\,dx$$

Problem 2964: Unable to integrate problem.

$$\int x\sqrt{a+b\sqrt{c\,x^3}}\,dx$$

Optimal (type 4, 400 leaves, 5 steps) :

$$\frac{4}{11}x^2\sqrt{a+b\sqrt{c\,x^3}}+\frac{12\,a\,x^2\sqrt{a+b\sqrt{c\,x^3}}}{55\,b\sqrt{c\,x^3}}-$$

$$\left(8\times 3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\,c^{1/3}\,x-\frac{a^{1/3}\,b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(55\,b^{4/3}\,c^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\sqrt{a+b\sqrt{c\,x^3}}\right)$$

Result (type 8, 21 leaves):

$$\int x\sqrt{a+b\sqrt{c\,x^3}}\,dx$$

- **Problem 2965: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^2}\,dx$$

Optimal (type 4, 355 leaves, 4 steps):

$$-\frac{\sqrt{a+b\sqrt{c\,x^3}}}{x}+$$

$$\left(3^{3/4}\sqrt{2+\sqrt{3}}\;b^{2/3}\,c^{1/3}\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\,c^{1/3}\,x-\frac{a^{1/3}\,b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\sqrt{a+b\sqrt{c\,x^3}}\right)$$

Result (type 8, 23 leaves) :

$$\int\frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^2}\,dx$$

■ Problem 2966: Unable to integrate problem.

$$\int\frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^5}\,dx$$

Optimal (type 4, 434 leaves, 6 steps) :

$$-\frac{\sqrt{a+b\sqrt{c\,x^3}}}{4\,x^4}+\frac{21\,b^2\,c\,\sqrt{a+b\sqrt{c\,x^3}}}{160\,a^2\,x}-\frac{3\,b\,c^3\,x^5\,\sqrt{a+b\sqrt{c\,x^3}}}{40\,a\,\left(c\,x^3\right)^{5/2}}+$$

$$\left(7\times 3^{3/4}\sqrt{2+\sqrt{3}}\;b^{8/3}\,c^{4/3}\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\,c^{1/3}\,x-\frac{a^{1/3}\,b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(160\,a^2\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\sqrt{a+b\sqrt{c\,x^3}}\right)$$

Result (type 8, 23 leaves) :

$$\int\frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^5}\,dx$$

- **Problem 2967: Unable to integrate problem.**

$$\int x^3\sqrt{a+b\sqrt{c\,x^3}}\;dx$$

Optimal (type 4, 843 leaves, 8 steps) :

$$-\frac{120\,a^2\,x\,\sqrt{a+b\,\sqrt{c\,x^3}}}{1729\,b^2\,c}+\frac{4}{19}\,x^4\,\sqrt{a+b\,\sqrt{c\,x^3}}+\frac{12\,a\,x\,\sqrt{c\,x^3}\,\sqrt{a+b\,\sqrt{c\,x^3}}}{247\,b\,c}+\frac{480\,a^3\,\sqrt{a+b\,\sqrt{c\,x^3}}}{1729\,b^{8/3}\,c^{4/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}-$$

$$\left(240\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{10/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)\,\sqrt{\frac{a^{2/3}+b^{2/3}\,c^{1/3}\,x-\frac{a^{1/3}\,b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(1729\,b^{8/3}\,c^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\,\sqrt{a+b\,\sqrt{c\,x^3}}\right)+$$

$$\left(160\,\sqrt{2}\,3^{3/4}\,a^{10/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)\,\sqrt{\frac{a^{2/3}+b^{2/3}\,c^{1/3}\,x-\frac{a^{1/3}\,b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(1729\,b^{8/3}\,c^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\,\sqrt{a+b\,\sqrt{c\,x^3}}\right)$$

Result (type 8, 23 leaves) :

$$\int x^3\,\sqrt{a+b\,\sqrt{c\,x^3}}\,dx$$

Problem 2968: Unable to integrate problem.

$$\int \sqrt{a+b\,\sqrt{c\,x^3}}\,dx$$

Optimal (type 4, 770 leaves, 6 steps) :

$$\frac{4}{7}x\sqrt{a+b\sqrt{c x^3}}+\frac{12 a\sqrt{a+b\sqrt{c x^3}}}{7 b^{2/3} c^{1/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)}-$$

$$\left(6\times 3^{1/4}\sqrt{2-\sqrt{3}}\; a^{4/3}\left(a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3} c^{1/3} x-\frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(7 b^{2/3} c^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)^2}}\sqrt{a+b\sqrt{c x^3}}\right)+$$

$$\left(4\sqrt{2}\;3^{3/4} a^{4/3}\left(a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3} c^{1/3} x-\frac{a^{1/3} b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}{\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(7 b^{2/3} c^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+\frac{b^{1/3} c^{2/3} x^2}{\sqrt{c x^3}}\right)^2}}\sqrt{a+b\sqrt{c x^3}}\right)$$

Result (type 8, 19 leaves) :

$$\int\sqrt{a+b\sqrt{c x^3}}\,dx$$

- **Problem 2969: Unable to integrate problem.**

$$\int\frac{\sqrt{a+b\sqrt{c x^3}}}{x^3}\,dx$$

Optimal (type 4, 810 leaves, 7 steps) :

$$-\frac{\sqrt{a+b\sqrt{c\,x^3}}}{2\,x^2}-\frac{3\,b\,c\,x\,\sqrt{a+b\sqrt{c\,x^3}}}{4\,a\,\sqrt{c\,x^3}}+\frac{3\,b^{4/3}\,c^{2/3}\,\sqrt{a+b\sqrt{c\,x^3}}}{4\,a\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\;b^{4/3}\,c^{2/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)\,\sqrt{\frac{a^{2/3}+b^{2/3}\,c^{1/3}\,x-\frac{a^{1/3}\,b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(8\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\sqrt{a+b\,\sqrt{c\,x^3}}\right)+$$

$$\left(3^{3/4}\,b^{4/3}\,c^{2/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)\,\sqrt{\frac{a^{2/3}+b^{2/3}\,c^{1/3}\,x-\frac{a^{1/3}\,b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}{\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(2\,\sqrt{2}\;a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+\frac{b^{1/3}\,c^{2/3}\,x^2}{\sqrt{c\,x^3}}\right)^2}}\;\sqrt{a+b\,\sqrt{c\,x^3}}\right)$$

Result (type 8, 23 leaves) :

$$\int\frac{\sqrt{a+b\,\sqrt{c\,x^3}}}{x^3}\,dx$$

- **Problem 2970: Unable to integrate problem.**

$$\int x^{17}\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}\;dx$$

Optimal (type 2, 116 leaves, 4 steps) :

$$-\frac{4 a^3 \left(a+b \left(c x^3\right)^{3/2}\right)^{3/2}}{27 b^4 c^6}+\frac{4 a^2 \left(a+b \left(c x^3\right)^{3/2}\right)^{5/2}}{15 b^4 c^6}-\frac{4 a \left(a+b \left(c x^3\right)^{3/2}\right)^{7/2}}{21 b^4 c^6}+\frac{4 \left(a+b \left(c x^3\right)^{3/2}\right)^{9/2}}{81 b^4 c^6}$$

Result (type 8, 23 leaves) :

$$\int x^{17} \sqrt{a+b \left(c x^3\right)^{3/2}}\, dx$$

Problem 2971: Unable to integrate problem.

$$\int x^8 \sqrt{a+b \left(c x^3\right)^{3/2}}\, dx$$

Optimal (type 2, 56 leaves, 4 steps) :

$$-\frac{4 a \left(a+b \left(c x^3\right)^{3/2}\right)^{3/2}}{27 b^2 c^3}+\frac{4 \left(a+b \left(c x^3\right)^{3/2}\right)^{5/2}}{45 b^2 c^3}$$

Result (type 8, 23 leaves) :

$$\int x^8 \sqrt{a+b \left(c x^3\right)^{3/2}}\, dx$$

Problem 2972: Unable to integrate problem.

$$\int \frac{\sqrt{a+b \left(c x^3\right)^{3/2}}}{x}\, dx$$

Optimal (type 3, 55 leaves, 5 steps) :

$$\frac{4}{9} \sqrt{a+b \left(c x^3\right)^{3/2}}-\frac{4}{9} \sqrt{a}\, \text{ArcTanh}\left[\frac{\sqrt{a+b \left(c x^3\right)^{3/2}}}{\sqrt{a}}\right]$$

Result (type 8, 23 leaves) :

$$\int \frac{\sqrt{a+b \left(c x^3\right)^{3/2}}}{x}\, dx$$

Problem 2973: Unable to integrate problem.

$$\int \frac{\sqrt{a+b \left(c x^3\right)^{3/2}}}{x^{10}}\, dx$$

Optimal (type 3, 101 leaves, 6 steps) :

$$-\frac{\sqrt{a+b\left(c\,x^3\right)^{3/2}}}{9\,x^9}-\frac{b\,c^3\,\sqrt{a+b\left(c\,x^3\right)^{3/2}}}{18\,a\left(c\,x^3\right)^{3/2}}+\frac{b^2\,c^3\,\text{ArcTanh}\left[\frac{\sqrt{a+b\left(c\,x^3\right)^{3/2}}}{\sqrt{a}}\right]}{18\,a^{3/2}}$$

Result (type 8, 23 leaves) :

$$\int\frac{\sqrt{a+b\left(c\,x^3\right)^{3/2}}}{x^{10}}\,dx$$

Problem 2974: Result unnecessarily involves higher level functions.

$$\int x^2\,\sqrt{a+b\left(c\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 642 leaves, 7 steps) :

$$\frac{4}{21}x^3\sqrt{a+b\left(c x^3\right)^{3/2}}+\frac{4 a\sqrt{a+b\left(c x^3\right)^{3/2}}}{7 b^{2/3} c\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}\right)}-$$

$$\left(2\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^3}\right)\sqrt{\frac{a^{2/3}+b^{2/3} c x^3-a^{1/3} b^{1/3}\sqrt{c x^3}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7 b^{2/3} c\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^3}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}\right)^2}}\sqrt{a+b\left(c x^3\right)^{3/2}}\right)+$$

$$\left(4\sqrt{2}\,a^{4/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^3}\right)\sqrt{\frac{a^{2/3}+b^{2/3} c x^3-a^{1/3} b^{1/3}\sqrt{c x^3}}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{1/4} b^{2/3} c\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\sqrt{c x^3}\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\sqrt{c x^3}\right)^2}}\sqrt{a+b\left(c x^3\right)^{3/2}}\right)$$

Result (type 5, 89 leaves) :

$$\frac{x^3\left(4\left(a+b\left(c x^3\right)^{3/2}\right)+3 a\sqrt{\frac{a+b\left(c x^3\right)^{3/2}}{a}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{b\left(c x^3\right)^{3/2}}{a}\right]\right)}{21\sqrt{a+b\left(c x^3\right)^{3/2}}}$$

- **Problem 2975: Unable to integrate problem.**

$$\int x^9\sqrt{a+b\left(c x^3\right)^{3/2}}\,dx$$

Optimal (type 5, 170 leaves, 7 steps) :

$$-\frac{792\,a^2\,x\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}{19\,747\,b^2\,c^3}+\frac{4}{49}\,x^{10}\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}+$$

$$\frac{36\,a\,x\,\left(c\,x^3\right)^{3/2}\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}{1519\,b\,c^3}+\frac{792\,a^3\,x\,\sqrt{1+\frac{b\,\left(c\,x^3\right)^{3/2}}{a}}\,\text{Hypergeometric2F1}\left[\frac{2}{9},\,\frac{1}{2},\,\frac{11}{9},\,-\frac{b\,\left(c\,x^3\right)^{3/2}}{a}\right]}{19\,747\,b^2\,c^3\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}$$

Result (type 8, 23 leaves) :

$$\int x^9\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}\,dx$$

■ Problem 2976: Unable to integrate problem.

$$\int \sqrt{a+b\,\left(c\,x^3\right)^{3/2}}\,dx$$

Optimal (type 5, 91 leaves, 5 steps) :

$$\frac{4}{13}\,x\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}+\frac{9\,a\,x\,\sqrt{1+\frac{b\,\left(c\,x^3\right)^{3/2}}{a}}\,\text{Hypergeometric2F1}\left[\frac{2}{9},\,\frac{1}{2},\,\frac{11}{9},\,-\frac{b\,\left(c\,x^3\right)^{3/2}}{a}\right]}{13\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}$$

Result (type 8, 19 leaves) :

$$\int \sqrt{a+b\,\left(c\,x^3\right)^{3/2}}\,dx$$

■ Problem 2977: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}{x^9}\,dx$$

Optimal (type 5, 139 leaves, 6 steps) :

$$-\frac{\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}{8\,x^8}-\frac{9\,b\,c^3\,x\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}{112\,a\,\left(c\,x^3\right)^{3/2}}-\frac{45\,b^2\,c^3\,x\,\sqrt{1+\frac{b\,\left(c\,x^3\right)^{3/2}}{a}}\,\text{Hypergeometric2F1}\left[\frac{2}{9},\,\frac{1}{2},\,\frac{11}{9},\,-\frac{b\,\left(c\,x^3\right)^{3/2}}{a}\right]}{448\,a\,\sqrt{a+b\,\left(c\,x^3\right)^{3/2}}}$$

Result (type 8, 23 leaves) :

$$\int \frac{\sqrt{a+b\left(c\,x^3\right)^{3/2}}}{x^9}\,dx$$

Problem 2978: Unable to integrate problem.

$$\int (d\,x)^m \sqrt{a+b\left(c\,x^3\right)^{3/2}}\,dx$$

Optimal (type 5, 84 leaves, 5 steps) :

$$\frac{x\,(d\,x)^m \sqrt{a+b\left(c\,x^3\right)^{3/2}}\ \text{Hypergeometric2F1}\left[-\frac{1}{2},\frac{2\,(1+m)}{9},1+\frac{2\,(1+m)}{9},-\frac{b\left(c\,x^3\right)^{3/2}}{a}\right]}{(1+m)\sqrt{1+\frac{b\left(c\,x^3\right)^{3/2}}{a}}}$$

Result (type 8, 25 leaves) :

$$\int (d\,x)^m \sqrt{a+b\left(c\,x^3\right)^{3/2}}\,dx$$

Problem 2981: Unable to integrate problem.

$$\int (d\,x)^m \sqrt{a+\frac{b}{\left(c\,x^3\right)^{3/2}}}\,dx$$

Optimal (type 5, 102 leaves, 6 steps) :

$$\frac{x\,(d\,x)^m \sqrt{a+\frac{b\,c^3\,x^9}{\left(c\,x^3\right)^{9/2}}}\ \text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{2}{9}\,(1+m),\frac{1}{9}\,(7-2\,m),-\frac{b\,c^3\,x^9}{a\left(c\,x^3\right)^{9/2}}\right]}{(1+m)\sqrt{1+\frac{b\,c^3\,x^9}{a\left(c\,x^3\right)^{9/2}}}}$$

Result (type 8, 25 leaves) :

$$\int (d\,x)^m \sqrt{a+\frac{b}{\left(c\,x^3\right)^{3/2}}}\,dx$$

Problem 2995: Unable to integrate problem.

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}\,(d\,x)^m\,dx$$

Optimal (type 5, 102 leaves, 6 steps):

$$\frac{\sqrt{a+\frac{b\,c^3}{\left(\frac{c}{x}\right)^{3/2}x^3}}\;x\,(d\,x)^m\,\text{Hypergeometric2F1}\left[-\frac{1}{2},\,-\frac{2}{3}\,(1+m),\,\frac{1}{3}\,(1-2\,m),\,-\frac{b\,c^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right]}{(1+m)\sqrt{1+\frac{b\,c^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}}}$$

Result (type 8, 25 leaves):

$$\int\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}\,(d\,x)^m\,dx$$

Problem 2998: Unable to integrate problem.

$$\int\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\,(d\,x)^m\,dx$$

Optimal (type 5, 102 leaves, 5 steps):

$$\frac{x\,(d\,x)^m\sqrt{a+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{c^3}}\;\text{Hypergeometric2F1}\left[-\frac{1}{2},\,\frac{2\,(1+m)}{3},\,\frac{1}{3}\,(5+2\,m),\,-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{a\,c^3}\right]}{(1+m)\sqrt{1+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{a\,c^3}}}$$

Result (type 8, 25 leaves):

$$\int\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\,(d\,x)^m\,dx$$

Problem 2999: Unable to integrate problem.

$$\int\frac{(d\,x)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx$$

Optimal (type 5, 102 leaves, 6 steps):

$$\frac{\sqrt{1+\frac{b\,c^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}}\;x\,(d\,x)^m\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,-\frac{2}{3}\,(1+m),\,\frac{1}{3}\,(1-2\,m),\,-\frac{b\,c^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right]}{(1+m)\sqrt{a+\frac{b\,c^3}{\left(\frac{c}{x}\right)^{3/2}x^3}}}$$

Result (type 8, 25 leaves):

$$\int \frac{(d\,x)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx$$

- **Problem 3002: Unable to integrate problem.**

$$\int \frac{(d\,x)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx$$

Optimal (type 5, 102 leaves, 5 steps):

$$\frac{x\,(d\,x)^m\,\sqrt{1+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{a\,c^3}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{2\,(1+m)}{3},\,\frac{1}{3}\,(5+2\,m),\,-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{a\,c^3}\right]}{(1+m)\,\sqrt{a+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{c^3}}}$$

Result (type 8, 25 leaves):

$$\int \frac{(d\,x)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx$$

- **Problem 3006: Unable to integrate problem.**

$$\int \frac{x^3}{a+b\,(c\,x^n)^{\frac{1}{n}}}\,dx$$

Optimal (type 3, 101 leaves, 3 steps):

$$\frac{a^2\,x^4\,(c\,x^n)^{-3/n}}{b^3}-\frac{a\,x^4\,(c\,x^n)^{-2/n}}{2\,b^2}+\frac{x^4\,(c\,x^n)^{-1/n}}{3\,b}-\frac{a^3\,x^4\,(c\,x^n)^{-4/n}\,\text{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{b^4}$$

Result (type 8, 21 leaves):

$$\int \frac{x^3}{a+b\,(c\,x^n)^{\frac{1}{n}}}\,dx$$

- **Problem 3007: Unable to integrate problem.**

$$\int \frac{x^2}{a+b\,(c\,x^n)^{\frac{1}{n}}}\,dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{a\,x^3\,(c\,x^n)^{-2/n}}{b^2}+\frac{x^3\,(c\,x^n)^{-1/n}}{2\,b}+\frac{a^2\,x^3\,(c\,x^n)^{-3/n}\,\text{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{b^3}$$

Result (type 8, 21 leaves):

$$\int \frac{x^2}{a+b\,(c\,x^n)^{\frac{1}{n}}}\,dx$$

Problem 3008: Unable to integrate problem.

$$\int \frac{x}{a+b\,(c\,x^n)^{\frac{1}{n}}}\,dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{x^2\,(c\,x^n)^{-1/n}}{b}-\frac{a\,x^2\,(c\,x^n)^{-2/n}\,\text{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{b^2}$$

Result (type 8, 19 leaves):

$$\int \frac{x}{a+b\,(c\,x^n)^{\frac{1}{n}}}\,dx$$

Problem 3011: Unable to integrate problem.

$$\int \frac{1}{x^2\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}\,dx$$

Optimal (type 3, 60 leaves, 3 steps):

$$-\frac{1}{a\,x}-\frac{b\,(c\,x^n)^{\frac{1}{n}}\,\text{Log}[x]}{a^2\,x}+\frac{b\,(c\,x^n)^{\frac{1}{n}}\,\text{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{a^2\,x}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{x^2\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}\,dx$$

Problem 3012: Unable to integrate problem.

$$\int \frac{1}{x^3\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}\,dx$$

Optimal (type 3, 87 leaves, 3 steps):

$$-\frac{1}{2\,a\,x^2}+\frac{b\,(c\,x^n)^{\frac{1}{n}}}{a^2\,x^2}+\frac{b^2\,(c\,x^n)^{2/n}\,\mathrm{Log}[x]}{a^3\,x^2}-\frac{b^2\,(c\,x^n)^{2/n}\,\mathrm{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{a^3\,x^2}$$

Result (type 8, 21 leaves) :

$$\int\frac{1}{x^3\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}\,dx$$

Problem 3013: Unable to integrate problem.

$$\int\frac{x^3}{\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Optimal (type 3, 114 leaves, 3 steps) :

$$-\frac{2\,a\,x^4\,(c\,x^n)^{-3/n}}{b^3}+\frac{x^4\,(c\,x^n)^{-2/n}}{2\,b^2}+\frac{a^3\,x^4\,(c\,x^n)^{-4/n}}{b^4\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}+\frac{3\,a^2\,x^4\,(c\,x^n)^{-4/n}\,\mathrm{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{b^4}$$

Result (type 8, 21 leaves) :

$$\int\frac{x^3}{\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Problem 3014: Unable to integrate problem.

$$\int\frac{x^2}{\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Optimal (type 3, 90 leaves, 3 steps) :

$$\frac{x^3\,(c\,x^n)^{-2/n}}{b^2}-\frac{a^2\,x^3\,(c\,x^n)^{-3/n}}{b^3\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}-\frac{2\,a\,x^3\,(c\,x^n)^{-3/n}\,\mathrm{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{b^3}$$

Result (type 8, 21 leaves) :

$$\int\frac{x^2}{\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Problem 3015: Unable to integrate problem.

$$\int \frac{x}{\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Optimal (type 3, 67 leaves, 3 steps):

$$\frac{a\,x^2\,(c\,x^n)^{-2/n}}{b^2\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}+\frac{x^2\,(c\,x^n)^{-2/n}\,\text{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{b^2}$$

Result (type 8, 19 leaves):

$$\int \frac{x}{\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Problem 3018: Unable to integrate problem.

$$\int \frac{1}{x^2\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Optimal (type 3, 94 leaves, 3 steps):

$$-\frac{1}{a^2\,x}-\frac{b\,(c\,x^n)^{\frac{1}{n}}}{a^2\,x\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}-\frac{2\,b\,(c\,x^n)^{\frac{1}{n}}\,\text{Log}[x]}{a^3\,x}+\frac{2\,b\,(c\,x^n)^{\frac{1}{n}}\,\text{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{a^3\,x}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{x^2\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Problem 3019: Unable to integrate problem.

$$\int \frac{1}{x^3\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)^2}\,dx$$

Optimal (type 3, 125 leaves, 3 steps):

$$-\frac{1}{2\,a^2\,x^2}+\frac{2\,b\,(c\,x^n)^{\frac{1}{n}}}{a^3\,x^2}+\frac{b^2\,(c\,x^n)^{2/n}}{a^3\,x^2\left(a+b\,(c\,x^n)^{\frac{1}{n}}\right)}+\frac{3\,b^2\,(c\,x^n)^{2/n}\,\text{Log}[x]}{a^4\,x^2}-\frac{3\,b^2\,(c\,x^n)^{2/n}\,\text{Log}\left[a+b\,(c\,x^n)^{\frac{1}{n}}\right]}{a^4\,x^2}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{x^3 \left(a+b \left(c x^n\right)^{\frac{1}{n}}\right)^2} \, dx$$

- **Problem 3021: Unable to integrate problem.**

$$\int \frac{x}{\left(1+\left(x^n\right)^{\frac{1}{n}}\right)^2} \, dx$$

Optimal (type 3, 48 leaves, 3 steps):

$$\frac{x^2 \left(x^n\right)^{-2/n}}{1+\left(x^n\right)^{\frac{1}{n}}}+x^2 \left(x^n\right)^{-2/n} \text{Log}\left[1+\left(x^n\right)^{\frac{1}{n}}\right]$$

Result (type 8, 15 leaves):

$$\int \frac{x}{\left(1+\left(x^n\right)^{\frac{1}{n}}\right)^2} \, dx$$

- **Problem 3031: Unable to integrate problem.**

$$\int \frac{1}{a+b \left(c x^n\right)^{2/n}} \, dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{x \left(c x^n\right)^{-1/n} \text{ArcTan}\left[\frac{\sqrt{b} \left(c x^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{a+b \left(c x^n\right)^{2/n}} \, dx$$

- **Problem 3032: Unable to integrate problem.**

$$\int \frac{1}{\left(a+b \left(c x^n\right)^{2/n}\right)^2} \, dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{x}{2 a \left(a+b \left(c x^n\right)^{2/n}\right)}+\frac{x \left(c x^n\right)^{-1/n} \text{ArcTan}\left[\frac{\sqrt{b} \left(c x^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right]}{2 a^{3/2} \sqrt{b}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{\left(a+b\left(c\,x^n\right)^{2/n}\right)^2}\,dx$$

Problem 3033: Unable to integrate problem.

$$\int \frac{1}{\left(a+b\left(c\,x^n\right)^{2/n}\right)^3}\,dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{x}{4\,a\left(a+b\left(c\,x^n\right)^{2/n}\right)^2}+\frac{3\,x}{8\,a^2\left(a+b\left(c\,x^n\right)^{2/n}\right)}+\frac{3\,x\left(c\,x^n\right)^{-1/n}\,\text{ArcTan}\left[\frac{\sqrt{b}\left(c\,x^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right]}{8\,a^{5/2}\,\sqrt{b}}$$

Result (type 8, 19 leaves):

$$\int \frac{1}{\left(a+b\left(c\,x^n\right)^{2/n}\right)^3}\,dx$$

Problem 3034: Unable to integrate problem.

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x\,\text{ArcTan}\left[2\left(x^4\right)^{1/4}\right]}{2\left(x^4\right)^{1/4}}$$

Result (type 8, 15 leaves):

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx$$

Problem 3035: Unable to integrate problem.

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x\,\text{ArcTanh}\left[2\left(x^4\right)^{1/4}\right]}{2\left(x^4\right)^{1/4}}$$

Result (type 8, 15 leaves):

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx$$

Problem 3036: Unable to integrate problem.

$$\int \frac{1}{1+4\left(x^6\right)^{1/3}}\,dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x\,\text{ArcTan}\left[2\left(x^6\right)^{1/6}\right]}{2\left(x^6\right)^{1/6}}$$

Result (type 9, 142 leaves):

$$\frac{1}{24\left(-x^6\right)^{5/6}}\left(-2\,x\left(-x^{12}\right)^{1/3}\text{Beta}\left[-64\,x^6,\frac{1}{2},0\right]+2\,x\left(x^6\right)^{2/3}\text{Beta}\left[-64\,x^6,\frac{5}{6},0\right]+\left(-x^6\right)^{5/6}\left(-2\,\text{ArcTan}\left[\sqrt{3}-4\,x\right]+4\,\text{ArcTan}[2\,x]+2\,\text{ArcTan}\left[\sqrt{3}+4\,x\right]-\sqrt{3}\,\text{Log}\left[1-2\sqrt{3}\,x+4\,x^2\right]+\sqrt{3}\,\text{Log}\left[1+2\sqrt{3}\,x+4\,x^2\right]\right)\right)$$

Problem 3037: Unable to integrate problem.

$$\int \frac{1}{1-4\left(x^6\right)^{1/3}}\,dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$\frac{x\,\text{ArcTanh}\left[2\left(x^6\right)^{1/6}\right]}{2\left(x^6\right)^{1/6}}$$

Result (type 9, 123 leaves):

$$\frac{1}{24}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+4\,x}{\sqrt{3}}\right]+2\sqrt{3}\,\text{ArcTan}\left[\frac{1+4\,x}{\sqrt{3}}\right]+\frac{2\,x\,\text{Beta}\left[64\,x^6,\frac{1}{2},0\right]}{\left(x^6\right)^{1/6}}+\frac{2\,x\,\text{Beta}\left[64\,x^6,\frac{5}{6},0\right]}{\left(x^6\right)^{1/6}}-2\,\text{Log}[1-2\,x]+2\,\text{Log}[1+2\,x]-\text{Log}\left[1-2\,x+4\,x^2\right]+\text{Log}\left[1+2\,x+4\,x^2\right]\right)$$

Problem 3038: Unable to integrate problem.

$$\int \frac{1}{1+4\left(x^{2\,n}\right)^{\frac{1}{n}}}\,dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{1}{2} x \left(x^{2 n}\right)^{-\frac{1}{2}/n} \text{ArcTan}\left[2 \left(x^{2 n}\right)^{\frac{1}{2}/n}\right]$$

Result (type 8, 17 leaves) :

$$\int \frac{1}{1+4 \left(x^{2 n}\right)^{\frac{1}{n}}} \, dx$$

- **Problem 3039: Unable to integrate problem.**

$$\int \frac{1}{1-4 \left(x^{2 n}\right)^{\frac{1}{n}}} \, dx$$

Optimal (type 3, 34 leaves, 2 steps) :

$$\frac{1}{2} x \left(x^{2 n}\right)^{-\frac{1}{2}/n} \text{ArcTanh}\left[2 \left(x^{2 n}\right)^{\frac{1}{2}/n}\right]$$

Result (type 8, 17 leaves) :

$$\int \frac{1}{1-4 \left(x^{2 n}\right)^{\frac{1}{n}}} \, dx$$

- **Problem 3043: Unable to integrate problem.**

$$\int \frac{1}{a+b \left(c x^n\right)^{3/n}} \, dx$$

Optimal (type 3, 183 leaves, 7 steps) :

$$-\frac{x \left(c x^n\right)^{-1/n} \text{ArcTan}\left[\frac{a^{1/3}-2 b^{1/3} \left(c x^n\right)^{\frac{1}{n}}}{\sqrt{3} \, a^{1/3}}\right]}{\sqrt{3} \, a^{2/3} b^{1/3}}+\frac{x \left(c x^n\right)^{-1/n} \text{Log}\left[a^{1/3}+b^{1/3} \left(c x^n\right)^{\frac{1}{n}}\right]}{3 a^{2/3} b^{1/3}}-\frac{x \left(c x^n\right)^{-1/n} \text{Log}\left[a^{2/3}-a^{1/3} b^{1/3} \left(c x^n\right)^{\frac{1}{n}}+b^{2/3} \left(c x^n\right)^{2/n}\right]}{6 a^{2/3} b^{1/3}}$$

Result (type 8, 19 leaves) :

$$\int \frac{1}{a+b \left(c x^n\right)^{3/n}} \, dx$$

- **Problem 3044: Unable to integrate problem.**

$$\int \frac{1}{\left(a+b \left(c x^n\right)^{3/n}\right)^2} \, dx$$

Optimal (type 3, 210 leaves, 8 steps) :

$$\frac{x}{3\,a\left(a+b\,(c\,x^n)^{3/n}\right)}-\frac{2\,x\,(c\,x^n)^{-1/n}\,\mathrm{ArcTan}\left[\frac{a^{1/3}-2\,b^{1/3}\,(c\,x^n)^{\frac{1}{n}}}{\sqrt{3}\,a^{1/3}}\right]}{3\,\sqrt{3}\,a^{5/3}\,b^{1/3}}+$$

$$\frac{2\,x\,(c\,x^n)^{-1/n}\,\mathrm{Log}\left[a^{1/3}+b^{1/3}\,(c\,x^n)^{\frac{1}{n}}\right]}{9\,a^{5/3}\,b^{1/3}}-\frac{x\,(c\,x^n)^{-1/n}\,\mathrm{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,(c\,x^n)^{\frac{1}{n}}+b^{2/3}\,(c\,x^n)^{2/n}\right]}{9\,a^{5/3}\,b^{1/3}}$$

Result (type 8, 19 leaves) :

$$\int\frac{1}{\left(a+b\,(c\,x^n)^{3/n}\right)^2}\,dx$$

■ Problem 3045: Unable to integrate problem.

$$\int\frac{1}{\left(a+b\,(c\,x^n)^{3/n}\right)^3}\,dx$$

Optimal (type 3, 235 leaves, 9 steps) :

$$\frac{x}{6\,a\left(a+b\,(c\,x^n)^{3/n}\right)^2}+\frac{5\,x}{18\,a^2\left(a+b\,(c\,x^n)^{3/n}\right)}-\frac{5\,x\,(c\,x^n)^{-1/n}\,\mathrm{ArcTan}\left[\frac{a^{1/3}-2\,b^{1/3}\,(c\,x^n)^{\frac{1}{n}}}{\sqrt{3}\,a^{1/3}}\right]}{9\,\sqrt{3}\,a^{8/3}\,b^{1/3}}+$$

$$\frac{5\,x\,(c\,x^n)^{-1/n}\,\mathrm{Log}\left[a^{1/3}+b^{1/3}\,(c\,x^n)^{\frac{1}{n}}\right]}{27\,a^{8/3}\,b^{1/3}}-\frac{5\,x\,(c\,x^n)^{-1/n}\,\mathrm{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,(c\,x^n)^{\frac{1}{n}}+b^{2/3}\,(c\,x^n)^{2/n}\right]}{54\,a^{8/3}\,b^{1/3}}$$

Result (type 8, 19 leaves) :

$$\int\frac{1}{\left(a+b\,(c\,x^n)^{3/n}\right)^3}\,dx$$

■ Problem 3052: Unable to integrate problem.

$$\int\sqrt{a+b\,\sqrt{\frac{d}{x}+\frac{c}{x}}}\;x^m\,dx$$

Optimal (type 6, 230 leaves, 4 steps) :

$$\left(\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x^{1+m}\ \text{AppellF1}\left[-2\,(1+m),-\frac{1}{2},-\frac{1}{2},-1-2\,m,-\frac{2\,c\,\sqrt{\frac{d}{x}}}{\sqrt{d}\,\left(b\,\sqrt{d}-\sqrt{-4\,a\,c+b^2\,d}\right)},-\frac{2\,c\,\sqrt{\frac{d}{x}}}{\sqrt{d}\,\left(b\,\sqrt{d}+\sqrt{-4\,a\,c+b^2\,d}\right)}\right]\right)\Big/$$

$$\left((1+m)\,\sqrt{1+\frac{2\,c\,\sqrt{\frac{d}{x}}}{\sqrt{d}\,\left(b\,\sqrt{d}-\sqrt{-4\,a\,c+b^2\,d}\right)}}\,\sqrt{1+\frac{2\,c\,\sqrt{\frac{d}{x}}}{\sqrt{d}\,\left(b\,\sqrt{d}+\sqrt{-4\,a\,c+b^2\,d}\right)}}\right)$$

Result (type 8, 28 leaves) :

$$\int\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x^m\,dx$$

- **Problem 3053: Unable to integrate problem.**

$$\int\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x^2\,dx$$

Optimal (type 3, 333 leaves, 9 steps) :

$$-\frac{3\,b\,d^3\,\left(a+b\,\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10\,a^2\,\left(\frac{d}{x}\right)^{5/2}}+\frac{7\,b\,d^2\,\left(28\,a\,c-15\,b^2\,d\right)\,\left(a+b\,\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{480\,a^4\,\left(\frac{d}{x}\right)^{3/2}}+\frac{\left(16\,a^2\,c^2-56\,a\,b^2\,c\,d+21\,b^4\,d^2\right)\,\left(2\,a+b\,\sqrt{\frac{d}{x}}\right)\,\sqrt{a+b\,\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x}{256\,a^5}-$$

$$\frac{\left(20\,a\,c-21\,b^2\,d\right)\,\left(a+b\,\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}\,x^2}{80\,a^3}+\frac{\left(a+b\,\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}\,x^3}{3\,a}+\frac{\left(4\,a\,c-b^2\,d\right)\,\left(16\,a^2\,c^2-56\,a\,b^2\,c\,d+21\,b^4\,d^2\right)\,\text{ArcTanh}\left[\frac{2\,a+b\,\sqrt{\frac{d}{x}}}{2\,\sqrt{a}\,\sqrt{a+b\,\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{512\,a^{11/2}}$$

Result (type 8, 28 leaves) :

$$\int\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x^2\,dx$$

Problem 3054: Unable to integrate problem.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}\,x}\,dx$$

Optimal (type 3, 209 leaves, 7 steps):

$$-\frac{5\,b\,d^2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{12\,a^2\left(\frac{d}{x}\right)^{3/2}}-\frac{\left(4\,a\,c-5\,b^2\,d\right)\left(2\,a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{32\,a^3}+$$

$$\frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^2}{2\,a}-\frac{\left(4\,a\,c-5\,b^2\,d\right)\left(4\,a\,c-b^2\,d\right)\text{ArcTanh}\left[\frac{2\,a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{64\,a^{7/2}}$$

Result (type 8, 26 leaves):

$$\int \sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}\,x}\,dx$$

Problem 3055: Unable to integrate problem.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}\,dx$$

Optimal (type 3, 113 leaves, 5 steps):

$$\frac{\left(2\,a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{2\,a}+\frac{\left(4\,a\,c-b^2\,d\right)\text{ArcTanh}\left[\frac{2\,a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{4\,a^{3/2}}$$

Result (type 8, 24 leaves):

$$\int \sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}\,dx$$

Problem 3056: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$

Optimal (type 3, 145 leaves, 8 steps):

$$-2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}+2\sqrt{a}\,\text{ArcTanh}\left[\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]-\frac{b\sqrt{d}\,\text{ArcTanh}\left[\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{\sqrt{c}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$

Problem 3057: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

Optimal (type 3, 155 leaves, 6 steps):

$$\frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{4c^2}-\frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c}+\frac{b\sqrt{d}\,\left(4ac-b^2d\right)\text{ArcTanh}\left[\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{8c^{5/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

Problem 3058: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

Optimal (type 3, 233 leaves, 7 steps):

$$-\frac{b\left(12 a c-7 b^2 d\right)\left(b d+2 c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{64 c^4}+\frac{\left(32 a c-35 b^2 d+42 b c\sqrt{\frac{d}{x}}\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{120 c^3}-$$

$$\frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{5 c x}-\frac{b\sqrt{d}\left(12 a c-7 b^2 d\right)\left(4 a c-b^2 d\right)\text{ArcTanh}\left[\frac{b d+2 c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{128 c^{9/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

Problem 3059: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$$

Optimal (type 3, 371 leaves, 9 steps):

$$\frac{b\left(80 a^2 c^2-120 a b^2 c d+33 b^4 d^2\right)\left(b d+2 c \sqrt{\frac{d}{x}}\right) \sqrt{a+b \sqrt{\frac{d}{x}}+\frac{c}{x}}}{512 c^6}-$$

$$\frac{\left(1024 a^2 c^2-3276 a b^2 c d+1155 b^4 d^2+18 b c\left(148 a c-77 b^2 d\right) \sqrt{\frac{d}{x}}\right)\left(a+b \sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{6720 c^5}+\frac{11 b\left(a+b \sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}\left(\frac{d}{x}\right)^{3/2}}{42 c^2 d}-$$

$$\frac{2\left(a+b \sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{7 c x^2}+\frac{\left(32 a c-33 b^2 d\right)\left(a+b \sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{140 c^3 x}+\frac{b \sqrt{d}\left(4 a c-b^2 d\right)\left(80 a^2 c^2-120 a b^2 c d+33 b^4 d^2\right) \operatorname{ArcTanh}\left[\frac{b d+2 c \sqrt{\frac{d}{x}}}{2 \sqrt{c} \sqrt{d} \sqrt{a+b \sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{1024 c^{13/2}}$$

Result (type 8, 28 leaves) :

$$\int \frac{\sqrt{a+b \sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4} \, dx$$

Problem 3060: Unable to integrate problem.

$$\int \frac{x^m}{\sqrt{a+b \sqrt{\frac{d}{x}}+\frac{c}{x}}} \, dx$$

Optimal (type 6, 230 leaves, 4 steps) :

$$\frac{1}{(1+m) \sqrt{a+b \sqrt{\frac{d}{x}}+\frac{c}{x}}} \sqrt{1+\frac{2 c \sqrt{\frac{d}{x}}}{\sqrt{d}\left(b \sqrt{d}-\sqrt{-4 a c+b^2 d}\right)}} \sqrt{1+\frac{2 c \sqrt{\frac{d}{x}}}{\sqrt{d}\left(b \sqrt{d}+\sqrt{-4 a c+b^2 d}\right)}}$$

$$x^{1+m} \operatorname{AppellF1}\left[-2(1+m), \frac{1}{2}, \frac{1}{2},-1-2 m,-\frac{2 c \sqrt{\frac{d}{x}}}{\sqrt{d}\left(b \sqrt{d}-\sqrt{-4 a c+b^2 d}\right)},-\frac{2 c \sqrt{\frac{d}{x}}}{\sqrt{d}\left(b \sqrt{d}+\sqrt{-4 a c+b^2 d}\right)}\right]$$

Result (type 8, 28 leaves) :

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

- **Problem 3061: Unable to integrate problem.**

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Optimal (type 3, 386 leaves, 10 steps):

$$-\frac{11\,b\,d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30\,a^2\left(\frac{d}{x}\right)^{5/2}}+\frac{b\,d^2\left(156\,a\,c-77\,b^2\,d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160\,a^4\left(\frac{d}{x}\right)^{3/2}}-\frac{7\,b\,d\left(528\,a^2\,c^2-680\,a\,b^2\,c\,d+165\,b^4\,d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{1280\,a^6\sqrt{\frac{d}{x}}}+$$

$$\frac{\left(400\,a^2\,c^2-1176\,a\,b^2\,c\,d+385\,b^4\,d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x}{640\,a^5}-\frac{\left(100\,a\,c-99\,b^2\,d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x^2}{240\,a^3}+$$

$$\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x^3}{3\,a}-\frac{\left(320\,a^3\,c^3-1680\,a^2\,b^2\,c^2\,d+1260\,a\,b^4\,c\,d^2-231\,b^6\,d^3\right)\text{ArcTanh}\left[\frac{2\,a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{512\,a^{13/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

- **Problem 3062: Unable to integrate problem.**

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Optimal (type 3, 248 leaves, 8 steps):

$$-\frac{7\,b\,d^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12\,a^2\left(\frac{d}{x}\right)^{3/2}}+\frac{5\,b\,d\left(44\,a\,c-21\,b^2\,d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{96\,a^4\sqrt{\frac{d}{x}}}-\frac{\left(36\,a\,c-35\,b^2\,d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x}{48\,a^3}+\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x^2}{2\,a}+\frac{\left(48\,a^2\,c^2-120\,a\,b^2\,c\,d+35\,b^4\,d^2\right)\text{ArcTanh}\left[\frac{2\,a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{64\,a^{9/2}}$$

Result (type 8, 26 leaves):

$$\int\frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Problem 3063: Unable to integrate problem.

$$\int\frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$-\frac{3\,b\,d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2\,a^2\sqrt{\frac{d}{x}}}+\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x}{a}-\frac{\left(4\,a\,c-3\,b^2\,d\right)\text{ArcTanh}\left[\frac{2\,a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{4\,a^{5/2}}$$

Result (type 8, 24 leaves):

$$\int\frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Problem 3064: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x}\,dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{\sqrt{a}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x}\,dx$$

Problem 3065: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x^2}\,dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{c}+\frac{b\sqrt{d}\,\text{ArcTanh}\left[\frac{b\,d+2\,c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{c^{3/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\;x^2}\,dx$$

Problem 3066: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x^3}\,dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\frac{\left(16\,a\,c-15\,b^2\,d+10\,b\,c\,\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12\,c^3}-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{3\,c\,x}-\frac{b\,\sqrt{d}\,\left(12\,a\,c-5\,b^2\,d\right)\,\mathrm{ArcTanh}\left[\frac{b\,d+2\,c\,\sqrt{\frac{d}{x}}}{2\,\sqrt{c}\,\sqrt{d}\,\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{8\,c^{7/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x^3}\,dx$$

Problem 3067: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\ x^4}\,dx$$

Optimal (type 3, 289 leaves, 8 steps):

$$-\frac{\left(1024\,a^2\,c^2-2940\,a\,b^2\,c\,d+945\,b^4\,d^2+14\,b\,c\,\left(92\,a\,c-45\,b^2\,d\right)\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{960\,c^5}+\frac{9\,b\,\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,\left(\frac{d}{x}\right)^{3/2}}{20\,c^2\,d}-$$

$$\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{5\,c\,x^2}+\frac{\left(64\,a\,c-63\,b^2\,d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{120\,c^3\,x}+\frac{b\,\sqrt{d}\,\left(240\,a^2\,c^2-280\,a\,b^2\,c\,d+63\,b^4\,d^2\right)\,\mathrm{ArcTanh}\left[\frac{b\,d+2\,c\,\sqrt{\frac{d}{x}}}{2\,\sqrt{c}\,\sqrt{d}\,\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right]}{128\,c^{11/2}}$$

Result (type 8, 28 leaves):

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}\,x^4}}}\,dx$$

Problem 3068: Unable to integrate problem.

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$$

Optimal (type 2, 26 leaves, 2 steps):

$$\frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}$$

Result (type 8, 17 leaves):

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$$

Problem 3069: Unable to integrate problem.

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$$

Optimal (type 3, 75 leaves, 5 steps):

$$\frac{1}{4}\left(4+\sqrt{\frac{1}{x}}\right)\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\;x+\frac{7\,\text{ArcTanh}\left[\frac{4+\sqrt{\frac{1}{x}}}{2\sqrt{2}\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}}\right]}{8\sqrt{2}}$$

Result (type 8, 18 leaves):

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$$

- **Problem 3074: Unable to integrate problem.**

$$\int \frac{(c\, x^n)^{\frac{1}{n}}}{a + b\, (c\, x^n)^{\frac{1}{n}}}\, dx$$

Optimal (type 3, 38 leaves, 4 steps):

$$\frac{x}{b} - \frac{a\, x\, (c\, x^n)^{-1/n}\, \text{Log}\left[a + b\, (c\, x^n)^{\frac{1}{n}}\right]}{b^2}$$

Result (type 8, 27 leaves):

$$\int \frac{(c\, x^n)^{\frac{1}{n}}}{a + b\, (c\, x^n)^{\frac{1}{n}}}\, dx$$

- **Problem 3075: Unable to integrate problem.**

$$\int \frac{(c\, x^n)^{\frac{1}{n}}}{\left(a + b\, (c\, x^n)^{\frac{1}{n}}\right)^2}\, dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$\frac{a\, x\, (c\, x^n)^{-1/n}}{b^2 \left(a + b\, (c\, x^n)^{\frac{1}{n}}\right)} + \frac{x\, (c\, x^n)^{-1/n}\, \text{Log}\left[a + b\, (c\, x^n)^{\frac{1}{n}}\right]}{b^2}$$

Result (type 8, 27 leaves):

$$\int \frac{(c\, x^n)^{\frac{1}{n}}}{\left(a + b\, (c\, x^n)^{\frac{1}{n}}\right)^2}\, dx$$

Test results for the 385 problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

- **Problem 34: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a + b\, x^3\right)^{7/3}}{a - b\, x^3}\, dx$$

Optimal (type 5, 483 leaves, 22 steps):

$$-\frac{7}{5} a x \left(a+b x^3\right)^{1/3}-\frac{1}{5} x \left(a+b x^3\right)^{4/3}-\frac{4\times 2^{1/3} a^{5/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-\frac{2\times 2^{1/3} a^{5/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-$$
$$\frac{7 a^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{5 \left(a+b x^3\right)^{2/3}}-\frac{2\times 2^{1/3} a^{5/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+$$
$$\frac{2\times 2^{1/3} a^{5/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}-\frac{4\times 2^{1/3} a^{5/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+\frac{2^{1/3} a^{5/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}$$

Result (type 6, 330 leaves) :

$$\frac{1}{20 \left(a+b x^3\right)^{2/3}} \left(-4 \left(8 a^2 x+9 a b x^4+b^2 x^7\right)+\left(208 a^4 x \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\right.\right.$$
$$\left.\left(4 a \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3 \left(3 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-2 \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)+$$
$$\left(189 a^3 b x^4 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\right.$$
$$\left.\left.\left(7 a \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3 \left(3 \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-2 \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 35: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{4/3}}{a-b x^3}\, dx$$

Optimal (type 5, 464 leaves, 21 steps) :

$$-\frac{1}{2} x \left(a+b x^3\right)^{1/3}-\frac{2\times 2^{1/3} a^{2/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-\frac{2^{1/3} a^{2/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-$$
$$\frac{a x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 \left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} a^{2/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+$$
$$\frac{2^{1/3} a^{2/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}-\frac{2\times 2^{1/3} a^{2/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+\frac{a^{2/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{1/3}}$$

Result (type 6, 316 leaves) :

$$\frac{1}{8\left(a+b x^3\right)^{2/3}}x\left(-4\left(a+b x^3\right)+\left(48 a^3 \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\left(4 a \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-2 \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)+\left(35 a^2 b x^3 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\left(7 a \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3 \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-2 \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{a-b x^3}\, dx$$

Optimal (type 3, 398 leaves, 14 steps):

$$-\frac{2^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{1/3} b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}\, a^{1/3} b^{1/3}}-\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{1/3} b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{1/3} b^{1/3}}-\frac{2^{1/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{1/3} b^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{1/3} b^{1/3}}$$

Result (type 6, 151 leaves):

$$\left(4 a x\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\left(4 a \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3 \text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 37: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^3\right)\left(a+b x^3\right)^{2/3}}\, dx$$

Optimal (type 5, 452 leaves, 17 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}\,a^{4/3}\,b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}\,a^{4/3}\,b^{1/3}}+\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2\,a\left(a+b x^3\right)^{2/3}}-$$

$$\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}x}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3}\,a^{4/3}\,b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3}\,a^{4/3}\,b^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\,a^{4/3}\,b^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3}\,a^{4/3}\,b^{1/3}}$$

Result (type 6, 153 leaves) :

$$\left(4\,a\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\left(a+b x^3\right)^{2/3}\right.$$
$$\left.\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)$$

- **Problem 38: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\left(a-b x^3\right)\left(a+b x^3\right)^{5/3}}\,dx$$

Optimal (type 5, 473 leaves, 21 steps) :

$$\frac{x}{4\,a^2\left(a+b x^3\right)^{2/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}\,a^{7/3}\,b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{2/3}\sqrt{3}\,a^{7/3}\,b^{1/3}}+\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2\,a^2\left(a+b x^3\right)^{2/3}}-$$

$$\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}x}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3}\,a^{7/3}\,b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3}\,a^{7/3}\,b^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3}\,a^{7/3}\,b^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{24\times 2^{2/3}\,a^{7/3}\,b^{1/3}}$$

Result (type 6, 308 leaves) :

$$\frac{1}{16\,(a+b\,x^3)^{2/3}}\,x\left(\left(48\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left((a-b\,x^3)\right.\right.$$

$$\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\Bigg)+$$

$$\left.\frac{4-\frac{7\,a\,b\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]}{(a-b\,x^3)\left(7\,a\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)}}{a^2}\right)$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int\frac{1}{(a-b\,x^3)\,(a+b\,x^3)^{8/3}}\,dx$$

Optimal (type 5, 492 leaves, 22 steps):

$$\frac{x}{10\,a^2\,(a+b\,x^3)^{5/3}}+\frac{13\,x}{40\,a^3\,(a+b\,x^3)^{2/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{4\times2^{2/3}\,\sqrt{3}\,a^{10/3}\,b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{8\times2^{2/3}\,\sqrt{3}\,a^{10/3}\,b^{1/3}}+\frac{9\,x\left(1+\frac{b\,x^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{20\,a^3\,(a+b\,x^3)^{2/3}}-$$

$$\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{24\times2^{2/3}\,a^{10/3}\,b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(a^{1/3}+b^{1/3}\,x)^2}{(a+b\,x^3)^{2/3}}-\frac{2^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{(a+b\,x^3)^{1/3}}\right]}{24\times2^{2/3}\,a^{10/3}\,b^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{(a+b\,x^3)^{1/3}}\right]}{12\times2^{2/3}\,a^{10/3}\,b^{1/3}}+\frac{\text{Log}\left[2\times2^{1/3}+\frac{(a^{1/3}+b^{1/3}\,x)^2}{(a+b\,x^3)^{2/3}}+\frac{2^{2/3}\,(a^{1/3}+b^{1/3}\,x)}{(a+b\,x^3)^{1/3}}\right]}{48\times2^{2/3}\,a^{10/3}\,b^{1/3}}$$

Result (type 6, 334 leaves):

$$\frac{1}{160\,a^3\,(a+b\,x^3)^{5/3}}\,x\left(16\,a+52\,(a+b\,x^3)+\left(368\,a^2\,(a+b\,x^3)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left((a-b\,x^3)\right.\right.$$

$$\left.\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)-$$

$$\left(91\,a\,b\,x^3\,(a+b\,x^3)\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left((a-b\,x^3)\right.$$

$$\left.\left.\left(7\,a\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 86: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{8/3}}{c+d x^3}\,dx$$

Optimal (type 3, 331 leaves, 5 steps):

$$-\frac{b\,(6 b c-11 a d)\,x\,\left(a+b x^3\right)^{2/3}}{18 d^2}+\frac{b x\,\left(a+b x^3\right)^{5/3}}{6 d}+\frac{b^{2/3}\left(9 b^2 c^2-24 a b c d+20 a^2 d^2\right)\mathrm{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}\,d^3}-\frac{(b c-a d)^{8/3}\,\mathrm{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3} d^3}-\frac{(b c-a d)^{8/3}\,\mathrm{Log}\left[c+d x^3\right]}{6 c^{2/3} d^3}+\frac{(b c-a d)^{8/3}\,\mathrm{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} d^3}-\frac{b^{2/3}\left(9 b^2 c^2-24 a b c d+20 a^2 d^2\right)\mathrm{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 d^3}$$

Result (type 6, 669 leaves):

$$-\left(7 a b c\left(9 b^2 c^2-24 a b c d+20 a^2 d^2\right) x^4\,\mathrm{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(36 d^2\left(a+b x^3\right)^{1/3}\left(c+d x^3\right)\left(-7 a c\,\mathrm{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d\,\mathrm{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c\,\mathrm{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+\frac{1}{54 c^{2/3} d^2 (b c-a d)^{1/3}}\left(-18 b^2 c^{5/3}(b c-a d)^{1/3} x\left(a+b x^3\right)^{2/3}+42 a b c^{2/3} d\,(b c-a d)^{1/3} x\left(a+b x^3\right)^{2/3}+9 b^2 c^{2/3} d\,(b c-a d)^{1/3} x^4\left(a+b x^3\right)^{2/3}+2\sqrt{3}\,a\left(3 b^2 c^2-7 a b c d+9 a^2 d^2\right)\mathrm{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 a\left(3 b^2 c^2-7 a b c d+9 a^2 d^2\right)\mathrm{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+3 a b^2 c^2\,\mathrm{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3}(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]-7 a^2 b c d\,\mathrm{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3}(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+9 a^3 d^2\,\mathrm{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3}(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)$$

Problem 87: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{5/3}}{c+d x^3}\,dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\frac{b\,x\,\left(a+b\,x^3\right)^{2/3}}{3\,d}-\frac{b^{2/3}\,(3\,b\,c-5\,a\,d)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,d^2}+\frac{(b\,c-a\,d)^{5/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,d^2}+$$
$$\frac{(b\,c-a\,d)^{5/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,d^2}-\frac{(b\,c-a\,d)^{5/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{2/3}\,d^2}+\frac{b^{2/3}\,(3\,b\,c-5\,a\,d)\,\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{6\,d^2}$$

Result (type 6, 474 leaves) :

$$\frac{1}{36}$$
$$\left(-\left(21\,a\,b\,c\,(-3\,b\,c+5\,a\,d)\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(d\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.\right.$$
$$\left.\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\frac{1}{c^{2/3}\,d\,(b\,c-a\,d)^{1/3}}$$
$$\left(12\,b\,c^{2/3}\,(b\,c-a\,d)^{1/3}\,x\,\left(a+b\,x^3\right)^{2/3}+4\,\sqrt{3}\,a\,(-b\,c+3\,a\,d)\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(b+a\,x^3\right)^{1/3}}}{\sqrt{3}}\right]+4\,a\,(b\,c-3\,a\,d)\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]-\right.$$
$$\left.\left.2\,a\,b\,c\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]+6\,a^2\,d\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)\right)$$

- **Problem 88: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{2/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 233 leaves, 3 steps) :

$$\frac{b^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d}-\frac{(b\,c-a\,d)^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,d}-$$
$$\frac{(b\,c-a\,d)^{2/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,d}+\frac{(b\,c-a\,d)^{2/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{2/3}\,d}-\frac{b^{2/3}\,\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{2\,d}$$

Result (type 6, 161 leaves) :

$$\left(4\,a\,c\,x\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(-3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 93: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{c+d\,x^3}\,dx$$

Optimal (type 6, 60 leaves, 2 steps) :

$$\frac{a\,x\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Result (type 6, 435 leaves) :

$$\frac{1}{8\,d\,\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)}x\left(-\left(16\,a^2\,c\,\left(-b\,c+2\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+\left(b\left(-7\,a\,c\,\left(4\,a\,c+2\,b\,c\,x^3+7\,a\,d\,x^3+4\,b\,d\,x^6\right)\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+4\,x^3\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\Big/\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 94: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{c+d\,x^3}\,dx$$

Optimal (type 6, 59 leaves, 2 steps) :

$$\frac{x\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Result (type 6, 160 leaves) :

$$\left(4\,a\,c\,x\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\bigg/\left(\left(c+d\,x^3\right)\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(-3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 95: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b\,x^3\right)^{2/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\left(1+\frac{b\,x^3}{a}\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c\left(a+b\,x^3\right)^{2/3}}$$

Result (type 6, 161 leaves):

$$-\left(4\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\bigg/\left(\left(a+b\,x^3\right)^{2/3}\left(c+d\,x^3\right)\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 96: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b\,x^3\right)^{5/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x\left(1+\frac{b\,x^3}{a}\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},\frac{5}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a\,c\left(a+b\,x^3\right)^{2/3}}$$

Result (type 6, 342 leaves):

$$\frac{1}{8\,(-b\,c+a\,d)\,\left(a+b\,x^3\right)^{2/3}}$$

$$x\left(-\frac{4\,b}{a}+\left(16\,c\,(b\,c-2\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(7\,b\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 97: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(a+b\,x^3\right)^{8/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x\left(1+\frac{b\,x^3}{a}\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},\frac{8}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a^2\,c\,\left(a+b\,x^3\right)^{2/3}}$$

Result (type 6, 407 leaves):

$$\left(x\left(\frac{4\,b\left(-11\,a^2\,d+4\,b^2\,c\,x^3+a\,b\left(6\,c-9\,d\,x^3\right)\right)}{a+b\,x^3}+\left(16\,a\,c\left(4\,b^2\,c^2-9\,a\,b\,c\,d+10\,a^2\,d^2\right)\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(7\,a\,b\,c\,d\,(-4\,b\,c+9\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)\right)\Big/\left(40\,a^2\,(b\,c-a\,d)^2\left(a+b\,x^3\right)^{2/3}\right)$$

Problem 98: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,x^3\right)^{8/3}}{\left(c+d\,x^3\right)^2}\,dx$$

Optimal (type 3, 351 leaves, 5 steps):

$$\frac{b\,(2bc-ad)\,x\,(a+bx^3)^{2/3}}{3cd^2}-\frac{(bc-ad)\,x\,(a+bx^3)^{5/3}}{3cd\,(c+dx^3)}-\frac{2b^{5/3}\,(3bc-4ad)\,\mathrm{ArcTan}\Big[\frac{1+\frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\Big]}{3\sqrt{3}\,d^3}+\frac{2\,(bc-ad)^{5/3}\,(3bc+ad)\,\mathrm{ArcTan}\Big[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\Big]}{3\sqrt{3}\,c^{5/3}d^3}+$$
$$\frac{(bc-ad)^{5/3}\,(3bc+ad)\,\mathrm{Log}\big[c+dx^3\big]}{9c^{5/3}d^3}-\frac{(bc-ad)^{5/3}\,(3bc+ad)\,\mathrm{Log}\Big[\frac{(bc-ad)^{1/3}x}{c^{1/3}}-(a+bx^3)^{1/3}\Big]}{3c^{5/3}d^3}+\frac{b^{5/3}\,(3bc-4ad)\,\mathrm{Log}\big[-b^{1/3}x+(a+bx^3)^{1/3}\big]}{3d^3}$$

Result (type 6, 914 leaves) :

$$\frac{1}{18\,c^{5/3}d^2\,(c+dx^3)}$$
$$\Bigg(-\Big(21\,a\,b^2c^{8/3}\,(-3bc+4ad)\,x^4\,\mathrm{AppellF1}\Big[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\Big]\Big)\Big/\Big((a+bx^3)^{1/3}\Big(-7ac\,\mathrm{AppellF1}\Big[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\Big]+$$
$$x^3\Big(3ad\,\mathrm{AppellF1}\Big[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\Big]+bc\,\mathrm{AppellF1}\Big[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\Big]\Big)\Big)\Big)+$$
$$\frac{1}{(bc-ad)^{1/3}}\,2\,\Big(3b^2c^{8/3}(bc-ad)^{1/3}x\,(a+bx^3)^{2/3}-6abc^{5/3}d\,(bc-ad)^{1/3}x\,(a+bx^3)^{2/3}+3a^2c^{2/3}d^2\,(bc-ad)^{1/3}x\,(a+bx^3)^{2/3}+$$
$$3b^2c^{5/3}(bc-ad)^{1/3}x\,(a+bx^3)^{2/3}\,(c+dx^3)-2\sqrt{3}\,ab^2c^2\,(c+dx^3)\,\mathrm{ArcTan}\Big[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(b+ax^3)^{1/3}}}{\sqrt{3}}\Big]+$$
$$2\sqrt{3}\,a^2bcd\,(c+dx^3)\,\mathrm{ArcTan}\Big[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(b+ax^3)^{1/3}}}{\sqrt{3}}\Big]+2\sqrt{3}\,a^3d^2\,(c+dx^3)\,\mathrm{ArcTan}\Big[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(b+ax^3)^{1/3}}}{\sqrt{3}}\Big]+$$
$$2ab^2c^2\,(c+dx^3)\,\mathrm{Log}\Big[c^{1/3}-\frac{(bc-ad)^{1/3}x}{(b+ax^3)^{1/3}}\Big]-2a^2bcd\,(c+dx^3)\,\mathrm{Log}\Big[c^{1/3}-\frac{(bc-ad)^{1/3}x}{(b+ax^3)^{1/3}}\Big]-$$
$$2a^3d^2\,(c+dx^3)\,\mathrm{Log}\Big[c^{1/3}-\frac{(bc-ad)^{1/3}x}{(b+ax^3)^{1/3}}\Big]-ab^2c^2\,(c+dx^3)\,\mathrm{Log}\Big[c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{c^{1/3}(bc-ad)^{1/3}x}{(b+ax^3)^{1/3}}\Big]+$$
$$a^2bcd\,(c+dx^3)\,\mathrm{Log}\Big[c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{c^{1/3}(bc-ad)^{1/3}x}{(b+ax^3)^{1/3}}\Big]+a^3d^2\,(c+dx^3)\,\mathrm{Log}\Big[c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{c^{1/3}(bc-ad)^{1/3}x}{(b+ax^3)^{1/3}}\Big]\Big)\Bigg)$$

Problem 99: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{5/3}}{\left(c+d x^3\right)^2}\,dx$$

Optimal (type 3, 301 leaves, 4 steps):

$$-\frac{(b c-a d)\, x \left(a+b x^3\right)^{2/3}}{3 c d \left(c+d x^3\right)}+\frac{b^{5/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2}-\frac{(b c-a d)^{2/3} (3 b c+2 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, c^{5/3} d^2}-$$
$$\frac{(b c-a d)^{2/3} (3 b c+2 a d) \operatorname{Log}\left[c+d x^3\right]}{18 c^{5/3} d^2}+\frac{(b c-a d)^{2/3} (3 b c+2 a d) \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{6 c^{5/3} d^2}-\frac{b^{5/3} \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 d^2}$$

Result (type 6, 554 leaves):

$$-\frac{(b c-a d)\, x \left(a+b x^3\right)^{2/3}}{3 c d \left(c+d x^3\right)}-$$
$$\left(7 a b^2 c x^4 \operatorname{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(4 d \left(a+b x^3\right)^{1/3} \left(c+d x^3\right) \left(-7 a c \operatorname{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3 \left(3 a d \operatorname{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \operatorname{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+\frac{1}{9 c^{5/3} (b c-a d)^{1/3}}$$
$$a^2 \left(2 \sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+\operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)+$$
$$\frac{1}{18 c^{2/3} d\, (b c-a d)^{1/3}} a b \left(2 \sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+\operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{\left(c+d x^3\right)^2}\,dx$$

Optimal (type 6, 60 leaves, 2 steps):

$$\frac{a x \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3},-\frac{4}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 440 leaves):

$$\Big(x\,\Big(-\Big(16\,a^2\,(b\,c+2\,a\,d)\,\text{AppellF1}\Big[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big/\Big(-4\,a\,c\,\text{AppellF1}\Big[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+$$

$$x^3\,\Big(3\,a\,d\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{2}{3},\,2,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+2\,b\,c\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{5}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big)+$$

$$\Big(7\,a\,c\,\big(4\,a^2\,d-2\,b^2\,c\,x^3+a\,b\,(-4\,c+5\,d\,x^3)\big)\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+4\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)$$

$$\Big(3\,a\,d\,\text{AppellF1}\Big[\frac{7}{3},\,\frac{2}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+2\,b\,c\,\text{AppellF1}\Big[\frac{7}{3},\,\frac{5}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big)\Big/$$

$$\Big(c\,\Big(7\,a\,c\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]-x^3\,\Big(3\,a\,d\,\text{AppellF1}\Big[\frac{7}{3},\,\frac{2}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+$$

$$2\,b\,c\,\text{AppellF1}\Big[\frac{7}{3},\,\frac{5}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big)\Big)\Big)\Big)\Big/\Big(12\,d\,(a+b\,x^3)^{2/3}\,(c+d\,x^3)\Big)$$

- **Problem 105: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b\,x^3)^{1/3}}{(c+d\,x^3)^2}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,(a+b\,x^3)^{1/3}\,\text{AppellF1}\Big[\frac{1}{3},\,-\frac{1}{3},\,2,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]}{c^2\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Result (type 6, 322 leaves):

$$\frac{1}{12\,(a+b\,x^3)^{2/3}\,(c+d\,x^3)}x\,\Big(\frac{4\,(a+b\,x^3)}{c}-\Big(32\,a^2\,\text{AppellF1}\Big[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big/\Big(-4\,a\,c\,\text{AppellF1}\Big[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+$$

$$x^3\,\Big(3\,a\,d\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{2}{3},\,2,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+2\,b\,c\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{5}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big)-$$

$$\Big(7\,a\,b\,x^3\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big/\Big(-7\,a\,c\,\text{AppellF1}\Big[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+$$

$$x^3\,\Big(3\,a\,d\,\text{AppellF1}\Big[\frac{7}{3},\,\frac{2}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]+2\,b\,c\,\text{AppellF1}\Big[\frac{7}{3},\,\frac{5}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\Big]\Big)\Big)\Big)$$

- **Problem 106: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b\,x^3)^{2/3}\,(c+d\,x^3)^2}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2\left(a+b x^3\right)^{2/3}}$$

Result (type 6, 341 leaves):

$$\left(x\left(-\frac{4 d\left(a+b x^3\right)}{c}+\left(16 a(-3 b c+2 a d)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-4 a c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+\left(7 a b d x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-7 a c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)\Big/\left(12(b c-a d)\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)\right)$$

- **Problem 107: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\left(a+b x^3\right)^{5/3}\left(c+d x^3\right)^2}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},\frac{5}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a c^2\left(a+b x^3\right)^{2/3}}$$

Result (type 6, 485 leaves):

$$\left(x\left(-\left(16\left(3 b^2 c^2-12 a b c d+4 a^2 d^2\right)\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-4 a c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+\left(7 a c\left(8 a^2 d^2+10 a b d^2 x^3+3 b^2 c\left(4 c+5 d x^3\right)\right)\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-4 x^3\left(2 a^2 d^2+2 a b d^2 x^3+3 b^2 c\left(c+d x^3\right)\right)\left(3 a d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\Big/\left(a c\left(7 a c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-x^3\left(3 a d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)\right)\Big/\left(24(b c-a d)^2\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)\right)$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{8/3} \left(c+d x^3\right)^2} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{8}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2 c^2 \left(a+b x^3\right)^{2/3}}$$

Result (type 6, 637 leaves):

$$\frac{1}{60 a^2 (b c-a d)^3 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} x \left(\left(16 a \left(-6 b^3 c^3+21 a b^2 c^2 d-45 a^2 b c d^2+10 a^3 d^3\right) \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(-4 a c \, \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(3 a d \, \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \, \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+ \left(-7 a c \left(20 a^4 d^3+45 a^3 b d^3 x^3-6 b^4 c^2 x^3 \left(4 c+5 d x^3\right)+a^2 b^2 d \left(96 c^2+117 c d x^3+25 d^2 x^6\right)+3 a b^3 c \left(-12 c^2+14 c d x^3+35 d^2 x^6\right)\right) \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ 4 x^3 \left(5 a^4 d^3+10 a^3 b d^3 x^3-6 b^4 c^2 x^3 \left(c+d x^3\right)+a^2 b^2 d \left(24 c^2+24 c d x^3+5 d^2 x^6\right)+3 a b^3 c \left(-3 c^2+4 c d x^3+7 d^2 x^6\right)\right) \left(3 a d \, \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \, \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right) \Big/ \left(c \left(a+b x^3\right) \left(7 a c \, \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-x^3 \left(3 a d \, \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \, \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 109: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{14/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 541 leaves, 7 steps):

$$-\frac{b\,(2 b c-a d)\left(18 b^2 c^2-18 a b c d-5 a^2 d^2\right) x \left(a+b x^3\right)^{2/3}}{18 c^2 d^4}+\frac{b\left(18 b^2 c^2-10 a b c d-5 a^2 d^2\right) x\left(a+b x^3\right)^{5/3}}{18 c^2 d^3}-$$

$$\frac{(b c-a d)\, x\left(a+b x^3\right)^{11/3}}{6 c d\left(c+d x^3\right)^2}-\frac{(b c-a d)\,(12 b c+5 a d)\, x\left(a+b x^3\right)^{8/3}}{18 c^2 d^2\left(c+d x^3\right)}+\frac{b^{8/3}\left(54 b^2 c^2-126 a b c d+77 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, d^5}-$$

$$\frac{(b c-a d)^{8/3}\left(54 b^2 c^2+18 a b c d+5 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, c^{8/3} d^5}-\frac{(b c-a d)^{8/3}\left(54 b^2 c^2+18 a b c d+5 a^2 d^2\right) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} d^5}+$$

$$\frac{(b c-a d)^{8/3}\left(54 b^2 c^2+18 a b c d+5 a^2 d^2\right) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} d^5}-\frac{b^{8/3}\left(54 b^2 c^2-126 a b c d+77 a^2 d^2\right) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 d^5}$$

Result (type 6, 1509 leaves) :

$$\left(a+b x^3\right)^{2/3}\left(-\frac{b^3 (9 b c-13 a d)\, x}{9 d^4}+\frac{b^4 x^4}{6 d^3}+\frac{(b c-a d)^4 x}{6 c d^4\left(c+d x^3\right)^2}-\frac{(b c-a d)^3 (21 b c+5 a d)\, x}{18 c^2 d^4\left(c+d x^3\right)}\right)-$$

$$\left(21 a b^5 c^3 x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(2 d^4\left(a+b x^3\right)^{1/3}\left(c+d x^3\right)\left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+$$

$$\left(49 a^2 b^4 c^2 x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(2 d^3\left(a+b x^3\right)^{1/3}\left(c+d x^3\right)\left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)-$$

$$\left(539 a^3 b^3 c x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(36 d^2\left(a+b x^3\right)^{1/3}\left(c+d x^3\right)\left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+\frac{1}{54 c^{8/3} (b c-a d)^{1/3}}$$

$$5 a^5\left(2 \sqrt{3}\, \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)+$$

$$\frac{1}{3 d^4 (b c-a d)^{1/3}}$$

$$a\,b^4\,c^{4/3}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(b+a\,x^3)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(b+a\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]\right)-$$

$$\frac{1}{3\,d^3\,(b\,c-a\,d)^{1/3}}$$

$$2\,a^2\,b^3\,c^{1/3}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(b+a\,x^3)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(b+a\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]\right)+$$

$$\frac{1}{18\,c^{2/3}\,d^2\,(b\,c-a\,d)^{1/3}}$$

$$5\,a^3\,b^2\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(b+a\,x^3)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(b+a\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]\right)+$$

$$\frac{1}{18\,c^{5/3}\,d\,(b\,c-a\,d)^{1/3}}$$

$$a^4\,b\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(b+a\,x^3)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(b+a\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]\right)$$

- **Problem 110: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{(a+b\,x^3)^{11/3}}{(c+d\,x^3)^3}\,dx$$

Optimal (type 3, 458 leaves, 6 steps) :

$$\frac{b\left(18 b^2 c^2-7 a b c d-5 a^2 d^2\right) x\left(a+b x^3\right)^{2/3}}{18 c^2 d^3}-\frac{(b c-a d) x\left(a+b x^3\right)^{8/3}}{6 c d\left(c+d x^3\right)^2}-$$

$$\frac{(b c-a d)(9 b c+5 a d) x\left(a+b x^3\right)^{5/3}}{18 c^2 d^2\left(c+d x^3\right)}-\frac{b^{8/3}(9 b c-11 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^4}+$$

$$\frac{(b c-a d)^{5/3}\left(27 b^2 c^2+12 a b c d+5 a^2 d^2\right) \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^4}+\frac{(b c-a d)^{5/3}\left(27 b^2 c^2+12 a b c d+5 a^2 d^2\right) \operatorname{Log}\left[c+d x^3\right]}{54 c^{8/3} d^4}-$$

$$\frac{(b c-a d)^{5/3}\left(27 b^2 c^2+12 a b c d+5 a^2 d^2\right) \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} d^4}+\frac{b^{8/3}(9 b c-11 a d) \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 d^4}$$

Result (type 6, 1114 leaves) :

$$\frac{1}{108}\left(\frac{6\,x\,\left(a+b\,x^3\right)^{2/3}\,\left(6\,b^3-\frac{3\,(b\,c-a\,d)^3}{c\,\left(c+d\,x^3\right)^2}+\frac{5\,(b\,c-a\,d)^2\,(3\,b\,c+a\,d)}{c^2\,\left(c+d\,x^3\right)}\right)}{d^3}+\right.$$

$$\left(567\,a\,b^4\,c^2\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\bigg/\left(d^3\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.$$

$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)-$$

$$\left(693\,a^2\,b^3\,c\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\bigg/\left(d^2\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.$$

$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\frac{1}{c^{8/3}\,(b\,c-a\,d)^{1/3}}$$

$$10\,a^4\left(2\,\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(b+a\,x^3\right)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)-$$

$$\frac{1}{d^3\,(b\,c-a\,d)^{1/3}}18\,a\,b^3\,c^{1/3}\left(2\,\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(b+a\,x^3\right)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)+$$

$$\frac{1}{c^{2/3}\,d^2\,(b\,c-a\,d)^{1/3}}16\,a^2\,b^2\left(2\,\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(b+a\,x^3\right)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)+$$

$$\left.\frac{1}{c^{5/3}\,d\,(b\,c-a\,d)^{1/3}}4\,a^3\,b\left(2\,\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(b+a\,x^3\right)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)\right)$$

- **Problem 111: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{8/3}}{\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 3, 391 leaves, 5 steps) :

$$-\frac{(b c-a d) x \left(a+b x^3\right)^{5/3}}{6 c d \left(c+d x^3\right)^2}-\frac{(b c-a d) (6 b c+5 a d) x \left(a+b x^3\right)^{2/3}}{18 c^2 d^2 \left(c+d x^3\right)}+\frac{b^{8/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3}-$$

$$\frac{(b c-a d)^{2/3} \left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^3}-\frac{(b c-a d)^{2/3} \left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} d^3}+$$

$$\frac{(b c-a d)^{2/3} \left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} d^3}-\frac{b^{8/3} \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 d^3}$$

Result (type 6, 753 leaves):

$$\frac{1}{108 c^{8/3}}\left(\frac{6 c^{2/3} (-b c+a d) x \left(a+b x^3\right)^{2/3} \left(3 b c \left(2 c+3 d x^3\right)+a d \left(8 c+5 d x^3\right)\right)}{d^2 \left(c+d x^3\right)^2}-\right.$$

$$\left(189 a b^3 c^{11/3} x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(d^2 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right) \left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$

$$\left.\left.x^3 \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+\frac{1}{(b c-a d)^{1/3}}$$

$$10 a^3 \left(2 \sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)+$$

$$\frac{1}{d^2 (b c-a d)^{1/3}} 6 a b^2 c^2 \left(2 \sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)+$$

$$\left.\frac{1}{d (b c-a d)^{1/3}} 2 a^2 b c \left(2 \sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)\right)$$

- **Problem 117: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^3\right)^{4/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 6, 60 leaves, 2 steps):

$$\frac{a\,x\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{4}{3},3,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c^3\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Result (type 6, 520 leaves):

$$\frac{1}{72\,c^2\,d\,\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)^2}$$
$$x\left(\left(16\,a^2\,c\,(b\,c+10\,a\,d)\,\left(c+d\,x^3\right)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.\right.$$
$$\left.x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+$$
$$\left(7\,a\,c\,\left(-2\,b^2\,c\,x^3\,\left(c-5\,d\,x^3\right)+4\,a^2\,d\,\left(8\,c+5\,d\,x^3\right)+a\,b\,\left(-4\,c^2+45\,c\,d\,x^3+25\,d^2\,x^6\right)\right)\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.$$
$$4\,x^3\,\left(-b^2\,c\,x^3\,\left(c-2\,d\,x^3\right)+a^2\,d\,\left(8\,c+5\,d\,x^3\right)+a\,b\,\left(-c^2+10\,c\,d\,x^3+5\,d^2\,x^6\right)\right)\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.$$
$$\left.\left.2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\Big/\left(7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.$$
$$\left.\left.x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 118: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},3,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c^3\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Result (type 6, 725 leaves):

$$\frac{1}{72\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)^2}$$
$$x\left(\frac{4\left(a+b x^3\right)\left(b c\left(7 c+4 d x^3\right)-a d\left(8 c+5 d x^3\right)\right)}{c^2(b c-a d)}+\left(160 a^3 d\left(c+d x^3\right) \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(c(b c-a d)\left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+\right.$$
$$\left(176 a^2 b\left(c+d x^3\right) \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left((-b c+a d)\left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+$$
$$\left(35 a^2 b d x^3\left(c+d x^3\right) \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(c(b c-a d)\left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+$$
$$\left.\left(28 a b^2 x^3\left(c+d x^3\right) \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left((-b c+a d)\left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 119: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)^3}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^3\left(a+b x^3\right)^{2/3}}$$

Result (type 6, 545 leaves):

$$\frac{1}{72\,c^2\,(b\,c-a\,d)^2\,\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)^2}$$
$$x\left(\left(16\,a\,c\,\left(18\,b^2\,c^2-23\,a\,b\,c\,d+10\,a^2\,d^2\right)\,\left(c+d\,x^3\right)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.\right.$$
$$\left.x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+$$
$$\left(d\left(7\,a\,c\,\left(4\,a^2\,d\,\left(8\,c+5\,d\,x^3\right)-2\,b^2\,c\,x^3\,\left(31\,c+25\,d\,x^3\right)+a\,b\,\left(-52\,c^2-3\,c\,d\,x^3+25\,d^2\,x^6\right)\right)\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.\right.$$
$$4\,x^3\,\left(a^2\,d\,\left(8\,c+5\,d\,x^3\right)-b^2\,c\,x^3\,\left(13\,c+10\,d\,x^3\right)+a\,b\,\left(-13\,c^2-2\,c\,d\,x^3+5\,d^2\,x^6\right)\right)\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.$$
$$\left.\left.\left.2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\Big/\left(7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.$$
$$\left.\left.x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 120: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b\,x^3\right)^{5/3}\,\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},\frac{5}{3},3,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a\,c^3\,\left(a+b\,x^3\right)^{2/3}}$$

Result (type 6, 627 leaves):

$$\frac{1}{72\,c^2\,(b\,c-a\,d)^3\,(a+b\,x^3)^{2/3}\,(c+d\,x^3)^2}$$

$$\times\left(\left(16\,c\,\left(-9\,b^3\,c^3+54\,a\,b^2\,c^2\,d-35\,a^2\,b\,c\,d^2+10\,a^3\,d^3\right)\,(c+d\,x^3)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+\right.$$

$$\left(-7\,a\,c\,\left(4\,a^3\,d^3\,(8\,c+5\,d\,x^3)-4\,a\,b^2\,c\,d^2\,x^3\,(23\,c+20\,d\,x^3)-9\,b^3\,c^2\,(4\,c^2+9\,c\,d\,x^3+5\,d^2\,x^6)+a^2\,b\,d^2\,(-76\,c^2-27\,c\,d\,x^3+25\,d^2\,x^6)\right)\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-4\,x^3\,\left(9\,b^3\,c^2\,(c+d\,x^3)^2-a^3\,d^3\,(8\,c+5\,d\,x^3)+a\,b^2\,c\,d^2\,x^3\,(19\,c+16\,d\,x^3)+a^2\,b\,d^2\,(19\,c^2+8\,c\,d\,x^3-5\,d^2\,x^6)\right)\right.$$

$$\left.\left.\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\Big/\left(a\left(7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 121: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{(a+b\,x^3)^{8/3}\,(c+d\,x^3)^3}\,dx$$

Optimal (type 6, 62 leaves, 2 steps) :

$$\frac{x\left(1+\frac{b\,x^3}{a}\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},\frac{8}{3},3,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a^2\,c^3\,(a+b\,x^3)^{2/3}}$$

Result (type 6, 615 leaves) :

$$\frac{1}{360\,a^2\,c^2\,(b\,c-a\,d)^4\,\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)^2}$$

$$x\left(\frac{1}{a+b\,x^3}4\,\left(36\,b^5\,c^3\,x^3\,\left(c+d\,x^3\right)^2+9\,a\,b^4\,c^2\,\left(6\,c-19\,d\,x^3\right)\,\left(c+d\,x^3\right)^2+5\,a^5\,d^4\,\left(8\,c+5\,d\,x^3\right)+5\,a^3\,b^2\,d^3\,x^3\,\left(-50\,c^2-36\,c\,d\,x^3+5\,d^2\,x^6\right)+\right.\right.$$
$$\left.5\,a^4\,b\,d^3\,\left(-25\,c^2-6\,c\,d\,x^3+10\,d^2\,x^6\right)-a^2\,b^3\,c\,d\,\left(189\,c^3+378\,c^2\,d\,x^3+314\,c\,d^2\,x^6+110\,d^3\,x^9\right)\right)+$$
$$\left(16\,a\,c\,\left(36\,b^4\,c^4-171\,a\,b^3\,c^3\,d+540\,a^2\,b^2\,c^2\,d^2-235\,a^3\,b\,c\,d^3+50\,a^4\,d^4\right)\,\left(c+d\,x^3\right)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/$$
$$\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.$$
$$\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)-$$
$$\left(7\,a\,b\,c\,d\,\left(36\,b^3\,c^3-171\,a\,b^2\,c^2\,d-110\,a^2\,b\,c\,d^2+25\,a^3\,d^3\right)\,x^3\,\left(c+d\,x^3\right)\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/$$
$$\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.$$
$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x^3\right)^{7/4}}{\left(c+d\,x^3\right)^{37/12}}\,dx$$

Optimal (type 5, 155 leaves, 3 steps):

$$\frac{4\,x\,\left(a+b\,x^3\right)^{7/4}}{25\,c\,\left(c+d\,x^3\right)^{25/12}}+\frac{84\,a\,x\,\left(a+b\,x^3\right)^{3/4}}{325\,c^2\,\left(c+d\,x^3\right)^{13/12}}+\frac{189\,a^2\,x\,\left(\frac{c\,(a+b\,x^3)}{a\,(c+d\,x^3)}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(b\,c-a\,d)\,x^3}{a\,(c+d\,x^3)}\right]}{325\,c^3\,\left(a+b\,x^3\right)^{1/4}\,\left(c+d\,x^3\right)^{1/12}}$$

Result (type 6, 479 leaves):

$$\frac{1}{325\,c^3\,\left(a+b\,x^3\right)^{1/4}\,\left(c+d\,x^3\right)^{1/12}}4\,x\left(\frac{1}{(b\,c-a\,d)\,\left(c+d\,x^3\right)^2}\left(13\,b^3\,c^3\,x^6+a\,b^2\,c^2\,x^3\,\left(47\,c+8\,d\,x^3\right)-a^3\,d\,\left(223\,c^2+399\,c\,d\,x^3+189\,d^2\,x^6\right)+a^2\,b\,\left(34\,c^3-215\,c^2\,d\,x^3-399\,c\,d^2\,x^6-189\,d^3\,x^9\right)\right)-\left(756\,a^3\,c\,(b\,c+3\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left((b\,c-a\,d)\left(-16\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{13}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(3969\,a^3\,b\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left((-b\,c+a\,d)\left(-28\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{4},\frac{13}{12},\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{4},\frac{1}{12},\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x^3\right)^{3/4}}{\left(c+d\,x^3\right)^{25/12}}\,dx$$

Optimal (type 5, 122 leaves, 2 steps):

$$\frac{4\,x\,\left(a+b\,x^3\right)^{3/4}}{13\,c\,\left(c+d\,x^3\right)^{13/12}}+\frac{9\,a\,x\,\left(\frac{c\,(a+b\,x^3)}{a\,(c+d\,x^3)}\right)^{1/4}\,\text{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(b\,c-a\,d)\,x^3}{a\,(c+d\,x^3)}\right]}{13\,c^2\,\left(a+b\,x^3\right)^{1/4}\,\left(c+d\,x^3\right)^{1/12}}$$

Result (type 6, 431 leaves):

$$\left(4\,x\left(\frac{b^2\,c^2\,x^3-a^2\,d\,\left(10\,c+9\,d\,x^3\right)+a\,b\,\left(c^2-10\,c\,d\,x^3-9\,d^2\,x^6\right)}{(b\,c-a\,d)\,\left(c+d\,x^3\right)}-\left(36\,a^2\,c\,(b\,c+3\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left((b\,c-a\,d)\left(-16\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{13}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(189\,a^2\,b\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left((-b\,c+a\,d)\left(-28\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{4},\frac{13}{12},\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{4},\frac{1}{12},\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)\Big/\left(13\,c^2\,\left(a+b\,x^3\right)^{1/4}\,\left(c+d\,x^3\right)^{1/12}\right)$$

- **Problem 126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b x^3\right)^{1/4} \left(c+d x^3\right)^{13/12}} \, dx$$

Optimal (type 5, 87 leaves, 1 step):

$$\frac{x \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{3}, \frac{4}{3}, -\frac{(b c-a d) x^3}{a \left(c+d x^3\right)}\right]}{c \left(a+b x^3\right)^{1/4} \left(c+d x^3\right)^{1/12}}$$

Result (type 6, 374 leaves):

$$\frac{1}{\left(a+b x^3\right)^{1/4} \left(c+d x^3\right)^{1/12}} 4 x \left(-\frac{d \left(a+b x^3\right)}{b c^2-a c d}-\left(4 a (b c+3 a d) \text{AppellF1}\left[\frac{1}{3}, \frac{1}{4}, \frac{1}{12}, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]\right) \Big/ \left((b c-a d) \left(-16 a c \text{AppellF1}\left[\frac{1}{3}, \frac{1}{4}, \frac{1}{12}, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]+ x^3 \left(a d \text{AppellF1}\left[\frac{4}{3}, \frac{1}{4}, \frac{13}{12}, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]+3 b c \text{AppellF1}\left[\frac{4}{3}, \frac{5}{4}, \frac{1}{12}, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]\right)\right)\right)+ \left(21 a b d x^3 \text{AppellF1}\left[\frac{4}{3}, \frac{1}{4}, \frac{1}{12}, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]\right) \Big/ \left((-b c+a d) \left(-28 a c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{4}, \frac{1}{12}, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]+ x^3 \left(a d \text{AppellF1}\left[\frac{7}{3}, \frac{1}{4}, \frac{13}{12}, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]+3 b c \text{AppellF1}\left[\frac{7}{3}, \frac{5}{4}, \frac{1}{12}, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b x^3\right)^{5/4} \left(c+d x^3\right)^{1/12}} \, dx$$

Optimal (type 5, 87 leaves, 1 step):

$$\frac{x \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{5/4} \left(c+d x^3\right)^{11/12} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{5}{4}, \frac{4}{3}, -\frac{(b c-a d) x^3}{a \left(c+d x^3\right)}\right]}{c \left(a+b x^3\right)^{5/4}}$$

Result (type 6, 356 leaves):

$$\left(4x\left(-\frac{b\left(c+dx^3\right)}{a}-\left(4c\,(bc+3ad)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\Big/\left(-16ac\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]+x^3\left(ad\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{13}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]+3bc\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{4},\frac{1}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\right)-\left(21bcdx^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\Big/\left(-28ac\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]+x^3\left(ad\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{4},\frac{13}{12},\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]+3bc\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{4},\frac{1}{12},\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\right)\right)\right)\Big/\left(3\,(-bc+ad)\left(a+bx^3\right)^{1/4}\left(c+dx^3\right)^{1/12}\right)$$

- **Problem 130: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}}\,dx$$

Optimal (type 5, 121 leaves, 2 steps):

$$\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}+\frac{11x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\left(c+dx^3\right)^{11/12}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)\,x^3}{a\left(c+dx^3\right)}\right]}{15a\left(a+bx^3\right)^{5/4}}$$

Result (type 6, 391 leaves):

$$\left(4x\left(\frac{\left(-14abc+3a^2d-11b^2cx^3\right)\left(c+dx^3\right)}{a+bx^3}+\left(44ac^2\,(bc+3ad)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\Big/\left(16ac\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]-x^3\left(ad\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{13}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]+3bc\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{4},\frac{1}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\right)-\left(231abc^2dx^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\Big/\left(-28ac\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]+x^3\left(ad\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{4},\frac{13}{12},\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]+3bc\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{4},\frac{1}{12},\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]\right)\right)\right)\right)\Big/\left(45a^2\,(-bc+ad)\left(a+bx^3\right)^{1/4}\left(c+dx^3\right)^{1/12}\right)$$

- **Problem 132: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+dx^3\right)^{23/12}}{\left(a+bx^3\right)^{13/4}}\,dx$$

Optimal (type 5, 153 leaves, 3 steps):

$$\frac{92\,c\,x\,\left(c+d\,x^3\right)^{11/12}}{405\,a^2\,\left(a+b\,x^3\right)^{5/4}}+\frac{4\,x\,\left(c+d\,x^3\right)^{23/12}}{27\,a\,\left(a+b\,x^3\right)^{9/4}}+\frac{253\,c\,x\,\left(\frac{c\,(a+b\,x^3)}{a\,(c+d\,x^3)}\right)^{5/4}\,\left(c+d\,x^3\right)^{11/12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(b\,c-a\,d)\,x^3}{a\,(c+d\,x^3)}\right]}{405\,a^2\,\left(a+b\,x^3\right)^{5/4}}$$

Result (type 6, 426 leaves):

$$\left(4\,x\left(\frac{\left(c+d\,x^3\right)\left(-575\,a\,b^2\,c^2\,x^3-253\,b^3\,c^2\,x^6+3\,a^3\,d\,\left(38\,c+15\,d\,x^3\right)+a^2\,b\,c\,\left(-367\,c+24\,d\,x^3\right)\right)}{\left(a+b\,x^3\right)^2}+\right.\right.$$
$$\left(1012\,a\,c^3\,(b\,c+3\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(16\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{4},\frac{1}{12},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.$$
$$\left.x^3\left(a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{13}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)-$$
$$\left(5313\,a\,b\,c^3\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-28\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{4},\frac{1}{12},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.$$
$$\left.\left.\left.x^3\left(a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{4},\frac{13}{12},\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{4},\frac{1}{12},\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)\Big/$$
$$\left(1215\,a^3\,(-b\,c+a\,d)\,\left(a+b\,x^3\right)^{1/4}\,\left(c+d\,x^3\right)^{1/12}\right)$$

Problem 133: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\,x^3\right)^m\,\left(c+d\,x^3\right)^p\,dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x\,\left(a+b\,x^3\right)^m\,\left(1+\frac{b\,x^3}{a}\right)^{-m}\,\left(c+d\,x^3\right)^p\,\left(1+\frac{d\,x^3}{c}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{3},-m,-p,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]$$

Result (type 6, 172 leaves):

$$\left(4\,a\,c\,x\,\left(a+b\,x^3\right)^m\,\left(c+d\,x^3\right)^p\,\text{AppellF1}\left[\frac{1}{3},-m,-p,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},-m,-p,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.$$
$$\left.3\,x^3\left(b\,c\,m\,\text{AppellF1}\left[\frac{4}{3},1-m,-p,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+a\,d\,p\,\text{AppellF1}\left[\frac{4}{3},-m,1-p,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(c+d\,x^3\right)^q}{a+b\,x^3}\,dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x\left(c+d x^3\right)^q\left(1+\frac{d x^3}{c}\right)^{-q}\text{AppellF1}\left[\frac{1}{3},1,-q,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a}$$

Result (type 6, 162 leaves):

$$\left(4 a c x\left(c+d x^3\right)^q\text{AppellF1}\left[\frac{1}{3},-q,1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right)\left(4 a c\,\text{AppellF1}\left[\frac{1}{3},-q,1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(a d q\,\text{AppellF1}\left[\frac{4}{3},1-q,1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-b c\,\text{AppellF1}\left[\frac{4}{3},-q,2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

- **Problem 137: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(c+d x^3\right)^q}{\left(a+b x^3\right)^2}\,dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x\left(c+d x^3\right)^q\left(1+\frac{d x^3}{c}\right)^{-q}\text{AppellF1}\left[\frac{1}{3},2,-q,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2}$$

Result (type 6, 162 leaves):

$$\left(4 a c x\left(c+d x^3\right)^q\text{AppellF1}\left[\frac{1}{3},2,-q,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\left(a+b x^3\right)^2\left(4 a c\,\text{AppellF1}\left[\frac{1}{3},2,-q,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+3 x^3\left(a d q\,\text{AppellF1}\left[\frac{4}{3},2,1-q,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-2 b c\,\text{AppellF1}\left[\frac{4}{3},3,-q,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

- **Problem 141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\left(a+b x^3\right)^m\,dx$$

Optimal (type 5, 44 leaves, 2 steps):

$$x\left(a+b x^3\right)^m\left(1+\frac{b x^3}{a}\right)^{-m}\text{Hypergeometric2F1}\left[\frac{1}{3},-m,\frac{4}{3},-\frac{b x^3}{a}\right]$$

Result (type 6, 196 leaves):

$$\frac{1}{b^{1/3}(1+m)}2^{-m}\left((-1)^{2/3}a^{1/3}+b^{1/3}x\right)\left(\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}\right)^{-m}\left(\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3 i+\sqrt{3}}\right)^{-m}\left(a+b x^3\right)^m\text{AppellF1}\left[1+m,-m,-m,2+m,-\frac{i\left((-1)^{2/3}a^{1/3}+b^{1/3}x\right)}{\sqrt{3}\,a^{1/3}},\frac{i+\sqrt{3}-\frac{2 i b^{1/3}x}{a^{1/3}}}{3 i+\sqrt{3}}\right]$$

Problem 142: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^3)^m}{c+d x^3} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^m \left(1+\frac{b x^3}{a}\right)^{-m} \text{AppellF1}\left[\frac{1}{3},-m,1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c}$$

Result (type 6, 162 leaves):

$$-\left(4 a c x \left(a+b x^3\right)^m \text{AppellF1}\left[\frac{1}{3},-m,1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(-4 a c \,\text{AppellF1}\left[\frac{1}{3},-m,1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ 3 x^3 \left(-b c m \,\text{AppellF1}\left[\frac{4}{3},1-m,1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+a d \,\text{AppellF1}\left[\frac{4}{3},-m,2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

Problem 143: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^3)^m}{(c+d x^3)^2} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^m \left(1+\frac{b x^3}{a}\right)^{-m} \text{AppellF1}\left[\frac{1}{3},-m,2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2}$$

Result (type 6, 162 leaves):

$$-\left(4 a c x \left(a+b x^3\right)^m \text{AppellF1}\left[\frac{1}{3},-m,2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right)^2 \left(-4 a c \,\text{AppellF1}\left[\frac{1}{3},-m,2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]- 3 x^3 \left(b c m \,\text{AppellF1}\left[\frac{4}{3},1-m,2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-2 a d \,\text{AppellF1}\left[\frac{4}{3},-m,3,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

Problem 144: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^3)^m}{(c+d x^3)^3} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^m \left(1+\frac{b x^3}{a}\right)^{-m} \text{AppellF1}\left[\frac{1}{3},-m,3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^3}$$

Result (type 6, 162 leaves):

$$-\left(4\,a\,c\,x\,\left(a+b\,x^3\right)^m\,\text{AppellF1}\left[\frac{1}{3},\,-m,\,3,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\bigg/\left(\left(c+d\,x^3\right)^3\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\,-m,\,3,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]-3\,x^3\left(b\,c\,m\,\text{AppellF1}\left[\frac{4}{3},\,1-m,\,3,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]-3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\,-m,\,4,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 145: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\left(a+b\,x^3\right)^{-1-\frac{b\,c}{3\,b\,c-3\,a\,d}}\left(c+d\,x^3\right)^{-1+\frac{a\,d}{3\,b\,c-3\,a\,d}}\,dx$$

Optimal (type 3, 53 leaves, 1 step) :

$$\frac{x\,\left(a+b\,x^3\right)^{-\frac{b\,c}{3\,b\,c-3\,a\,d}}\left(c+d\,x^3\right)^{\frac{a\,d}{3\,b\,c-3\,a\,d}}}{a\,c}$$

Result (type 6, 594 leaves) :

$$4\,a\,c\,x\,\left(a+b\,x^3\right)^{\frac{b\,c}{-3\,b\,c+3\,a\,d}}\left(c+d\,x^3\right)^{\frac{a\,d}{3\,b\,c-3\,a\,d}}\left(\left(d\,\text{AppellF1}\left[\frac{1}{3},\,\frac{b\,c}{3\,b\,c-3\,a\,d},\,1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\bigg/\left(\left(c+d\,x^3\right)\left(4\,a\,c\,\left(-b\,c+a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},\,\frac{b\,c}{3\,b\,c-3\,a\,d},\,1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+x^3\left(a\,d\,\left(3\,b\,c-4\,a\,d\right)\,\text{AppellF1}\left[\frac{4}{3},\,\frac{b\,c}{3\,b\,c-3\,a\,d},\,2+\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b^2\,c^2\,\text{AppellF1}\left[\frac{4}{3},\,1+\frac{b\,c}{3\,b\,c-3\,a\,d},\,1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(b\,\text{AppellF1}\left[\frac{1}{3},\,1+\frac{b\,c}{3\,b\,c-3\,a\,d},\,\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\bigg/\left(\left(a+b\,x^3\right)\left(4\,a\,c\,\left(b\,c-a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},\,1+\frac{b\,c}{3\,b\,c-3\,a\,d},\,\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+x^3\left(a^2\,d^2\,\text{AppellF1}\left[\frac{4}{3},\,1+\frac{b\,c}{3\,b\,c-3\,a\,d},\,1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\left(-4\,b\,c+3\,a\,d\right)\,\text{AppellF1}\left[\frac{4}{3},\,2+\frac{b\,c}{3\,b\,c-3\,a\,d},\,\frac{a\,d}{-3\,b\,c+3\,a\,d},\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 173: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a-b\,x^4\right)^{5/2}}{c-d\,x^4}\,dx$$

Optimal (type 4, 321 leaves, 10 steps) :

$$-\frac{b\,(7\,b\,c-13\,a\,d)\,x\,\sqrt{a-b\,x^4}}{21\,d^2}+\frac{b\,x\,\left(a-b\,x^4\right)^{3/2}}{7\,d}+\frac{a^{1/4}\,b^{3/4}\,\left(21\,b^2\,c^2-56\,a\,b\,c\,d+47\,a^2\,d^2\right)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{21\,d^3\,\sqrt{a-b\,x^4}}-$$

$$\frac{a^{1/4}\,(b\,c-a\,d)^3\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^3\,\sqrt{a-b\,x^4}}-\frac{a^{1/4}\,(b\,c-a\,d)^3\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^3\,\sqrt{a-b\,x^4}}$$

Result (type 6, 385 leaves):

$$\frac{1}{105\,d^2\,\sqrt{a-b\,x^4}}$$

$$x\,\Bigg(5\,b\,\left(-a+b\,x^4\right)\,\left(7\,b\,c-16\,a\,d+3\,b\,d\,x^4\right)+\Bigg(25\,a^2\,c\,\left(7\,b^2\,c^2-16\,a\,b\,c\,d+21\,a^2\,d^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\Bigg)\Big/\Bigg(\left(c-d\,x^4\right)$$

$$\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\Bigg)-$$

$$\Bigg(9\,a\,b\,c\,\left(21\,b^2\,c^2-56\,a\,b\,c\,d+47\,a^2\,d^2\right)\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\Bigg)\Big/\Bigg(\left(c-d\,x^4\right)$$

$$\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\Bigg)\Bigg)$$

- **Problem 174: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a-b\,x^4\right)^{3/2}}{c-d\,x^4}\,dx$$

Optimal (type 4, 277 leaves, 9 steps):

$$\frac{b\,x\,\sqrt{a-b\,x^4}}{3\,d}-\frac{a^{1/4}\,b^{3/4}\,(3\,b\,c-5\,a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{3\,d^2\,\sqrt{a-b\,x^4}}+$$

$$\frac{a^{1/4}\,(b\,c-a\,d)^2\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^2\,\sqrt{a-b\,x^4}}+\frac{a^{1/4}\,(b\,c-a\,d)^2\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^2\,\sqrt{a-b\,x^4}}$$

Result (type 6, 419 leaves):

$$\frac{1}{15\,d\,\sqrt{a-b\,x^4}\,\left(-c+d\,x^4\right)}x\left(-\left(25\,a^2\,c\,(-b\,c+3\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\right.$$
$$\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)+$$
$$\left(b\left(-9\,a\,c\left(-2\,b\,c\,x^4+5\,b\,d\,x^8+5\,a\left(c-2\,d\,x^4\right)\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]-\right.\right.$$
$$\left.\left.\left.10\,x^4\left(a-b\,x^4\right)\left(c-d\,x^4\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)\Big/$$
$$\left.\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)$$

- **Problem 175: Result unnecessarily involves higher level functions.**

$$\int\frac{\sqrt{a-b\,x^4}}{c-d\,x^4}\,dx$$

Optimal (type 4, 240 leaves, 8 steps):

$$\frac{a^{1/4}\,b^{3/4}\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{d\,\sqrt{a-b\,x^4}}-$$
$$\frac{a^{1/4}\,(b\,c-a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d\,\sqrt{a-b\,x^4}}-\frac{a^{1/4}\,(b\,c-a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d\,\sqrt{a-b\,x^4}}$$

Result (type 6, 155 leaves):

$$-\left(5\,a\,c\,x\,\sqrt{a-b\,x^4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c-d\,x^4\right)\right.$$
$$\left.\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(-2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)$$

- **Problem 176: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)}\,dx$$

Optimal (type 4, 162 leaves, 5 steps):

$$\frac{a^{1/4}\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\ -1\right]}{2 b^{1/4} c\sqrt{a-b x^4}}+\frac{a^{1/4}\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\ -1\right]}{2 b^{1/4} c\sqrt{a-b x^4}}$$

Result (type 6, 156 leaves) :

$$-\left(5 a c x \,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(\sqrt{a-b x^4}\,\left(-c+d x^4\right)\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)$$

Problem 177: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^4\right)^{3/2}\left(c-d x^4\right)}\,dx$$

Optimal (type 4, 281 leaves, 9 steps) :

$$\frac{b x}{2 a (b c-a d)\sqrt{a-b x^4}}+\frac{b^{3/4}\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 a^{3/4}(b c-a d)\sqrt{a-b x^4}}-$$

$$\frac{a^{1/4} d\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)\sqrt{a-b x^4}}-\frac{a^{1/4} d\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)\sqrt{a-b x^4}}$$

Result (type 6, 329 leaves) :

$$\frac{1}{10(-b c+a d)\sqrt{a-b x^4}}x\left(-\frac{5 b}{a}-\left(25 c (b c-2 a d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(\left(c-d x^4\right)\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)+\left(9 b c d x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(\left(c-d x^4\right)\left(9 a c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 178: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^4\right)^{5/2}\left(c-d x^4\right)}\,dx$$

Optimal (type 4, 334 leaves, 10 steps):

$$\frac{b\,x}{6\,a\,(b\,c-a\,d)\,\left(a-b\,x^4\right)^{3/2}}+\frac{b\,(5\,b\,c-11\,a\,d)\,x}{12\,a^2\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}}+\frac{b^{3/4}\,(5\,b\,c-11\,a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{12\,a^{7/4}\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}}+$$
$$\frac{a^{1/4}\,d^2\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}}+\frac{a^{1/4}\,d^2\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}}$$

Result (type 6, 396 leaves):

$$\left(x\left(\frac{5\,b\left(13\,a^2\,d+5\,b^2\,c\,x^4-a\,b\left(7\,c+11\,d\,x^4\right)\right)}{-a+b\,x^4}+\left(25\,a\,c\left(5\,b^2\,c^2-11\,a\,b\,c\,d+12\,a^2\,d^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c-d\,x^4\right)\right.\right.\right.$$
$$\left.\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)+$$
$$\left(9\,a\,b\,c\,d\,(-5\,b\,c+11\,a\,d)\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c-d\,x^4\right)\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+\right.\right.$$
$$\left.\left.\left.\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(60\,a^2\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}\right)$$

- **Problem 179: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^4\right)^{3/2}}{c+d\,x^4}\,dx$$

Optimal (type 4, 926 leaves, 10 steps):

$$\frac{b\,x\,\sqrt{a+b\,x^4}}{3\,d}-\frac{(b\,c-a\,d)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,d^{7/4}}-\frac{(-b\,c+a\,d)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,d^{7/4}}-$$

$$\frac{b^{3/4}\,(3\,b\,c-5\,a\,d)\,\left(\sqrt{a}+\sqrt{b}\;x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{6\,a^{1/4}\,d^2\,\sqrt{a+b\,x^4}}+$$

$$\frac{b^{1/4}\,\left(\sqrt{b}\;\sqrt{-c}-\sqrt{a}\;\sqrt{d}\right)\,(b\,c-a\,d)^2\,\left(\sqrt{a}+\sqrt{b}\;x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{4\,a^{1/4}\,\sqrt{-c}\;d^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}}+$$

$$\frac{b^{1/4}\,\left(\sqrt{b}\;\sqrt{-c}+\sqrt{a}\;\sqrt{d}\right)\,(b\,c-a\,d)^2\,\left(\sqrt{a}+\sqrt{b}\;x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{4\,a^{1/4}\,\sqrt{-c}\;d^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}}+$$

$$\left(\left(\sqrt{b}\;\sqrt{-c}+\sqrt{a}\;\sqrt{d}\right)^2\,(b\,c-a\,d)^2\,\left(\sqrt{a}+\sqrt{b}\;x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\;\sqrt{-c}-\sqrt{a}\;\sqrt{d}\right)^2}{4\,\sqrt{a}\;\sqrt{b}\;\sqrt{-c}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,b^{1/4}\,c\,d^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\;\sqrt{-c}-\sqrt{a}\;\sqrt{d}\right)^2\,(b\,c-a\,d)^2\,\left(\sqrt{a}+\sqrt{b}\;x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\;\sqrt{-c}+\sqrt{a}\;\sqrt{d}\right)^2}{4\,\sqrt{a}\;\sqrt{b}\;\sqrt{-c}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,b^{1/4}\,c\,d^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)$$

Result (type 6, 435 leaves) :

$$\frac{1}{15\,d\,\sqrt{a+b\,x^4}\,\left(c+d\,x^4\right)}x\left(\left(25\,a^2\,c\,(-b\,c+3\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]-\right.\right.$$
$$\left.\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+$$
$$\left(b\left(-9\,a\,c\left(5\,a\left(c+2\,d\,x^4\right)+b\,x^4\left(2\,c+5\,d\,x^4\right)\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+10\,x^4\left(a+b\,x^4\right)\left(c+d\,x^4\right)\right.\right.$$
$$\left.\left.\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\right.\right.$$
$$\left.\left.\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

- **Problem 180: Result unnecessarily involves higher level functions.**

$$\int\frac{\sqrt{a+b\,x^4}}{c+d\,x^4}\,dx$$

Optimal (type 4, 881 leaves, 9 steps) :

$$\frac{\sqrt{b c-a d}\ \text{ArcTan}\left[\frac{\sqrt{b c-a d}\ x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} d^{3/4}}-\frac{\sqrt{-b c+a d}\ \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\ x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} d^{3/4}}+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} d \sqrt{a+b x^4}}-$$

$$\frac{b^{1/4}\left(\sqrt{b}\ \sqrt{-c}-\sqrt{a}\ \sqrt{d}\right)(b c-a d)\left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} \sqrt{-c}\ d (b c+a d) \sqrt{a+b x^4}}-$$

$$\frac{b^{1/4}\left(\sqrt{b}+\frac{\sqrt{a}\ \sqrt{d}}{\sqrt{-c}}\right)(b c-a d)\left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} d (b c+a d) \sqrt{a+b x^4}}-$$

$$\left(\left(\sqrt{b}\ \sqrt{-c}+\sqrt{a}\ \sqrt{d}\right)^2 (b c-a d)\left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{b}\ \sqrt{-c}-\sqrt{a}\ \sqrt{d}\right)^2}{4 \sqrt{a}\ \sqrt{b}\ \sqrt{-c}\ \sqrt{d}}, 2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right)\Big/$$

$$\left(8 a^{1/4} b^{1/4} c d (b c+a d) \sqrt{a+b x^4}\right)-$$

$$\left(\left(\sqrt{b}\ \sqrt{-c}-\sqrt{a}\ \sqrt{d}\right)^2 (b c-a d)\left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticPi}\left[\frac{\left(\sqrt{b}\ \sqrt{-c}+\sqrt{a}\ \sqrt{d}\right)^2}{4 \sqrt{a}\ \sqrt{b}\ \sqrt{-c}\ \sqrt{d}}, 2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right)\Big/$$

$$\left(8 a^{1/4} b^{1/4} c d (b c+a d) \sqrt{a+b x^4}\right)$$

Result (type 6, 161 leaves) :

$$\left(5 a c x \sqrt{a+b x^4}\ \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\Big/\left(\left(c+d x^4\right)\left(5 a c\, \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+\right.\right.$$

$$\left.\left.2 x^4\left(-2 a d\, \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+b c\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)\right)$$

- **Problem 181: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+b x^4}\ \left(c+d x^4\right)}\, dx$$

Optimal (type 4, 742 leaves, 7 steps) :

$$-\frac{d^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,\sqrt{b\,c-a\,d}}-\frac{d^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,\sqrt{-b\,c+a\,d}}+$$

$$\frac{b^{1/4}\left(\sqrt{b}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}}+$$

$$\frac{b^{1/4}\left(\sqrt{b}\,c+\sqrt{a}\,\sqrt{-c}\,\sqrt{d}\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,c\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{4\,\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(8\,a^{1/4}\,b^{1/4}\,c\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)+$$
$$\left(\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\,\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(8\,a^{1/4}\,b^{1/4}\,c\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)$$

Result (type 6, 161 leaves) :

$$-\left(5\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\sqrt{a+b\,x^4}\,\left(c+d\,x^4\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+\right.\right.$$
$$\left.\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

Problem 182: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^4\right)^{3/2}\,\left(c+d\,x^4\right)}\,dx$$

Optimal (type 4, 913 leaves, 10 steps) :

$$\frac{b\,x}{2\,a\,(b\,c-a\,d)\,\sqrt{a+b\,x^4}}+\frac{d^{5/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,(b\,c-a\,d)^{3/2}}-$$

$$\frac{d^{5/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,(-b\,c+a\,d)^{3/2}}+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\;x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{5/4}\,(b\,c-a\,d)\,\sqrt{a+b\,x^4}}-$$

$$\frac{b^{1/4}\left(\sqrt{b}+\frac{\sqrt{a}\;\sqrt{d}}{\sqrt{-c}}\right)d\left(\sqrt{a}+\sqrt{b}\;x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}}-$$

$$\frac{b^{1/4}\left(\sqrt{b}\;c+\sqrt{a}\;\sqrt{-c}\;\sqrt{d}\right)d\left(\sqrt{a}+\sqrt{b}\;x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,c\,\left(b^2\,c^2-a^2\,d^2\right)\sqrt{a+b\,x^4}}-$$

$$\left(\left(\sqrt{b}\;\sqrt{-c}+\sqrt{a}\;\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}\;x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\;\sqrt{-c}-\sqrt{a}\;\sqrt{d}\right)^2}{4\,\sqrt{a}\;\sqrt{b}\;\sqrt{-c}\;\sqrt{d}},2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,b^{1/4}\,c\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\;\sqrt{-c}-\sqrt{a}\;\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}\;x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\;\sqrt{-c}+\sqrt{a}\;\sqrt{d}\right)^2}{4\,\sqrt{a}\;\sqrt{b}\;\sqrt{-c}\;\sqrt{d}},2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,b^{1/4}\,c\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)$$

Result (type 6, 342 leaves) :

$$\frac{1}{10\,(-b\,c+a\,d)\,\sqrt{a+b\,x^4}}$$

$$x\left(-\frac{5\,b}{a}+\left(25\,c\,(b\,c-2\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\,\frac{1}{2},\,1,\,\frac{5}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\,\frac{1}{2},\,1,\,\frac{5}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+\right.\right.\right.$$

$$\left.\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\,\frac{1}{2},\,2,\,\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\,\frac{3}{2},\,1,\,\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\right)\right)+$$

$$\left(9\,b\,c\,d\,x^4\,\text{AppellF1}\left[\frac{5}{4},\,\frac{1}{2},\,1,\,\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\,\frac{1}{2},\,1,\,\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+\right.\right.$$

$$\left.\left.\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\,\frac{1}{2},\,2,\,\frac{13}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\,\frac{3}{2},\,1,\,\frac{13}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

- **Problem 183: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(a+b\,x^4\right)^{5/2}\,\left(c+d\,x^4\right)}\,dx$$

Optimal (type 4, 976 leaves, 11 steps):

$$\frac{b\,x}{6\,a\,(b\,c-a\,d)\,\left(a+b\,x^4\right)^{3/2}}+\frac{b\,(5\,b\,c-11\,a\,d)\,x}{12\,a^2\,(b\,c-a\,d)^2\,\sqrt{a+b\,x^4}}-\frac{d^{9/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,(b\,c-a\,d)^{5/2}}-$$

$$\frac{d^{9/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,(-b\,c+a\,d)^{5/2}}+\frac{b^{3/4}\,(5\,b\,c-11\,a\,d)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{24\,a^{9/4}\,(b\,c-a\,d)^2\,\sqrt{a+b\,x^4}}+$$

$$\frac{b^{1/4}\,\left(\sqrt{b}\,c-\sqrt{a}\,\sqrt{-c}\,\sqrt{d}\right)\,d^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,c\,(b\,c-a\,d)^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}}+$$

$$\frac{b^{1/4}\,\left(\sqrt{b}\,c+\sqrt{a}\,\sqrt{-c}\,\sqrt{d}\right)\,d^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,c\,(b\,c-a\,d)^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2\,d^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{4\,\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\bigg/$$
$$\left(8\,a^{1/4}\,b^{1/4}\,c\,(b\,c-a\,d)^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2\,d^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\,\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\bigg/$$
$$\left(8\,a^{1/4}\,b^{1/4}\,c\,(b\,c-a\,d)^2\,(b\,c+a\,d)\,\sqrt{a+b\,x^4}\right)$$

Result (type 6, 406 leaves) :

$$\left(x\left(\frac{5\,b\left(-13\,a^2\,d+5\,b^2\,c\,x^4+a\,b\left(7\,c-11\,d\,x^4\right)\right)}{a+b\,x^4}+\right.\right.$$
$$\left(25\,a\,c\left(5\,b^2\,c^2-11\,a\,b\,c\,d+12\,a^2\,d^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]-\right.\right.$$
$$\left.\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+$$
$$\left(9\,a\,b\,c\,d\,(-5\,b\,c+11\,a\,d)\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+\right.\right.$$
$$\left.\left.\left.\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(60\,a^2\,(b\,c-a\,d)^2\sqrt{a+b\,x^4}\right)$$

- **Problem 184: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a-b\,x^4\right)^{7/2}}{\left(c-d\,x^4\right)^2}\,dx$$

Optimal (type 4, 426 leaves, 11 steps):

$$-\frac{b\left(77\,b^2\,c^2-122\,a\,b\,c\,d+21\,a^2\,d^2\right)x\sqrt{a-b\,x^4}}{84\,c\,d^3}+\frac{b\,(11\,b\,c-7\,a\,d)\,x\left(a-b\,x^4\right)^{3/2}}{28\,c\,d^2}-\frac{(b\,c-a\,d)\,x\left(a-b\,x^4\right)^{5/2}}{4\,c\,d\left(c-d\,x^4\right)}+$$
$$\frac{a^{1/4}\,b^{3/4}\left(231\,b^3\,c^3-553\,a\,b^2\,c^2\,d+349\,a^2\,b\,c\,d^2+21\,a^3\,d^3\right)\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{84\,c\,d^4\sqrt{a-b\,x^4}}-$$
$$\frac{a^{1/4}\,(b\,c-a\,d)^3\,(11\,b\,c+3\,a\,d)\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{8\,b^{1/4}\,c^2\,d^4\sqrt{a-b\,x^4}}-$$
$$\frac{a^{1/4}\,(b\,c-a\,d)^3\,(11\,b\,c+3\,a\,d)\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{8\,b^{1/4}\,c^2\,d^4\sqrt{a-b\,x^4}}$$

Result (type 6, 580 leaves):

$$\frac{1}{420\, d^3 \sqrt{a-b x^4}\, \left(c-d x^4\right)} x \left(\left(\left(25 a^2 \left(77 b^3 c^3-155 a b^2 c^2 d+63 a^2 b c d^2+63 a^3 d^3\right) \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\right.\right.$$
$$\left(5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4 \left(2 a d \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)+$$
$$\left(9 a c \left(105 a^4 d^3+a^2 b^2 c d \left(775 c-494 d x^4\right)-63 a^3 b d^2 \left(5 c+2 d x^4\right)+2 b^4 c x^4 \left(77 c^2-110 c d x^4-30 d^2 x^8\right)+\right.\right.$$
$$\left. a b^3 c \left(-385 c^2-2 c d x^4+520 d^2 x^8\right)\right) \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]-$$
$$10 x^4 \left(-a+b x^4\right) \left(-63 a^2 b c d^2+21 a^3 d^3+a b^2 c d \left(155 c-92 d x^4\right)+b^3 c \left(-77 c^2+44 c d x^4+12 d^2 x^8\right)\right)$$
$$\left.\left.\left(2 a d \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\Big/\left(c \left(9 a c\right.\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4 \left(2 a d \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

- **Problem 185: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a-b x^4\right)^{5/2}}{\left(c-d x^4\right)^2}\, dx$$

Optimal (type 4, 365 leaves, 10 steps):

$$\frac{b\,(7 b c-3 a d)\, x \sqrt{a-b x^4}}{12 c d^2}-\frac{(b c-a d)\, x \left(a-b x^4\right)^{3/2}}{4 c d \left(c-d x^4\right)}-\frac{a^{1/4} b^{3/4} \left(21 b^2 c^2-26 a b c d-3 a^2 d^2\right) \sqrt{1-\frac{b x^4}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{12 c d^3 \sqrt{a-b x^4}}+$$
$$\frac{a^{1/4} (b c-a d)^2 (7 b c+3 a d) \sqrt{1-\frac{b x^4}{a}}\, \text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d^3 \sqrt{a-b x^4}}+$$
$$\frac{a^{1/4} (b c-a d)^2 (7 b c+3 a d) \sqrt{1-\frac{b x^4}{a}}\, \text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d^3 \sqrt{a-b x^4}}$$

Result (type 6, 491 leaves):

$$\left(x\left(-\left(25 a^2\left(-7 b^2 c^2+6 a b c d+9 a^2 d^2\right) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right/\right.$$
$$\left(5 a c \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+2 x^4\left(2 a d \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right)+$$
$$\left(-9 a c\left(15 a^3 d^2+a b^2 c\left(35 c-16 d x^4\right)-6 a^2 b d\left(5 c+3 d x^4\right)+2 b^3 c x^4\left(-7 c+10 d x^4\right)\right) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]-\right.$$
$$10 x^4\left(a-b x^4\right)\left(-6 a b c d+3 a^2 d^2+b^2 c\left(7 c-4 d x^4\right)\right)\left(2 a d \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+\right.$$
$$\left.\left.b c \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right) \Big/ \left(c\left(9 a c \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+\right.\right.$$
$$\left.\left.\left.\left.2 x^4\left(2 a d \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right)\right)\right)\right) \Big/ \left(60 d^2 \sqrt{a-b x^4}\left(-c+d x^4\right)\right)$$

- **Problem 186: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a-b x^4\right)^{3/2}}{\left(c-d x^4\right)^2} \, dx$$

Optimal (type 4, 309 leaves, 9 steps):

$$-\frac{(b c-a d) x \sqrt{a-b x^4}}{4 c d\left(c-d x^4\right)}+\frac{a^{1/4} b^{3/4}(3 b c+a d) \sqrt{1-\frac{b x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{4 c d^2 \sqrt{a-b x^4}}-$$
$$\frac{3 a^{1/4}(b c-a d)(b c+a d) \sqrt{1-\frac{b x^4}{a}} \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d^2 \sqrt{a-b x^4}}-$$
$$\frac{3 a^{1/4}(b c-a d)(b c+a d) \sqrt{1-\frac{b x^4}{a}} \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d^2 \sqrt{a-b x^4}}$$

Result (type 6, 423 leaves):

$$\frac{1}{20\,d\,\sqrt{a-b\,x^4}\,\left(-c+d\,x^4\right)}x\left(-\left(25\,a^2\,(b\,c+3\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\right.$$

$$\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)+$$

$$\left(-9\,a\,c\,\left(5\,a^2\,d+2\,b^2\,c\,x^4-a\,b\,\left(5\,c+6\,d\,x^4\right)\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]-\right.$$

$$\left.10\,(-b\,c+a\,d)\,x^4\,\left(a-b\,x^4\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\Big/$$

$$\left.\left(c\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

- **Problem 187: Result unnecessarily involves higher level functions.**

$$\int\frac{\sqrt{a-b\,x^4}}{\left(c-d\,x^4\right)^2}\,dx$$

Optimal (type 4, 276 leaves, 9 steps):

$$\frac{x\,\sqrt{a-b\,x^4}}{4\,c\,\left(c-d\,x^4\right)}+\frac{a^{1/4}\,b^{3/4}\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{4\,c\,d\,\sqrt{a-b\,x^4}}-$$

$$\frac{a^{1/4}\,(b\,c-3\,a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{8\,b^{1/4}\,c^2\,d\,\sqrt{a-b\,x^4}}-\frac{a^{1/4}\,(b\,c-3\,a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{8\,b^{1/4}\,c^2\,d\,\sqrt{a-b\,x^4}}$$

Result (type 6, 310 leaves):

$$\frac{1}{20\,\sqrt{a-b\,x^4}\,\left(-c+d\,x^4\right)}x\left(-\frac{5\,\left(a-b\,x^4\right)}{c}-\left(75\,a^2\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\right.$$

$$\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)+$$

$$\left(9\,a\,b\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/$$

$$\left.\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)$$

Problem 188: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a-b x^4}\,\left(c-d x^4\right)^2}\,dx$$

Optimal (type 4, 310 leaves, 9 steps):

$$-\frac{d x \sqrt{a-b x^4}}{4 c (b c-a d) \left(c-d x^4\right)}-\frac{a^{1/4} b^{3/4} \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{4 c (b c-a d) \sqrt{a-b x^4}}+$$
$$\frac{a^{1/4} (5 b c-3 a d) \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 (b c-a d) \sqrt{a-b x^4}}+\frac{a^{1/4} (5 b c-3 a d) \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 (b c-a d) \sqrt{a-b x^4}}$$

Result (type 6, 349 leaves):

$$\frac{1}{20 \sqrt{a-b x^4}\,\left(-c+d x^4\right)} x \left(\frac{5 d \left(a-b x^4\right)}{c (b c-a d)}+\left(25 a (-4 b c+3 a d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left((b c-a d)\right.\right.$$
$$\left.\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4 \left(2 a d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)+$$
$$\left(9 a b d x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left((-b c+a d)\right.$$
$$\left.\left.\left(9 a c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4 \left(2 a d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 189: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^4\right)^{3/2} \left(c-d x^4\right)^2}\,dx$$

Optimal (type 4, 362 leaves, 10 steps):

$$\frac{b (2 b c+a d) x}{4 a c (b c-a d)^2 \sqrt{a-b x^4}}-\frac{d x}{4 c (b c-a d) \sqrt{a-b x^4}\,\left(c-d x^4\right)}+\frac{b^{3/4} (2 b c+a d) \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{4 a^{3/4} c (b c-a d)^2 \sqrt{a-b x^4}}-$$
$$\frac{3 a^{1/4} d (3 b c-a d) \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 (b c-a d)^2 \sqrt{a-b x^4}}-\frac{3 a^{1/4} d (3 b c-a d) \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 (b c-a d)^2 \sqrt{a-b x^4}}$$

Result (type 6, 465 leaves) :

$$\left(x\left(\left(25\left(2 b^2 c^2-8 a b c d+3 a^2 d^2\right) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\Big/\left(5 a c \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+2 x^4\left(2 a d \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right)+\left(9 a c\left(5 a^2 d^2-6 a b d^2 x^4+2 b^2 c\left(5 c-6 d x^4\right)\right) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]-10 x^4\left(-a^2 d^2+a b d^2 x^4-2 b^2 c\left(c-d x^4\right)\right)\left(2 a d \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right)\Big/\left(a c\left(9 a c \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+2 x^4\left(2 a d \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(20(b c-a d)^2 \sqrt{a-b x^4}\left(c-d x^4\right)\right)$$

- **Problem 190: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(a-b x^4\right)^{5/2}\left(c-d x^4\right)^2}\, dx$$

Optimal (type 4, 439 leaves, 11 steps) :

$$\frac{b(2 b c+3 a d) x}{12 a c(b c-a d)^2\left(a-b x^4\right)^{3/2}}+\frac{b\left(5 b^2 c^2-17 a b c d-3 a^2 d^2\right) x}{12 a^2 c(b c-a d)^3 \sqrt{a-b x^4}}-\frac{d x}{4 c(b c-a d)\left(a-b x^4\right)^{3/2}\left(c-d x^4\right)}+\frac{b^{3/4}\left(5 b^2 c^2-17 a b c d-3 a^2 d^2\right) \sqrt{1-\frac{b x^4}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{12 a^{7/4} c(b c-a d)^3 \sqrt{a-b x^4}}+\frac{a^{1/4} d^2(13 b c-3 a d) \sqrt{1-\frac{b x^4}{a}}\, \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2(b c-a d)^3 \sqrt{a-b x^4}}+\frac{a^{1/4} d^2(13 b c-3 a d) \sqrt{1-\frac{b x^4}{a}}\, \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2(b c-a d)^3 \sqrt{a-b x^4}}$$

Result (type 6, 617 leaves) :

$$\frac{1}{60 a^2 (-b c+a d)^3 \sqrt{a-b x^4}\, (c-d x^4)} \times \left(\left(25 a \left(-5 b^3 c^3+17 a b^2 c^2 d-36 a^2 b c d^2+9 a^3 d^3\right) \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right) \Big/ \left(5 a c\, \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4 \left(2 a d\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)+\left(9 a c \left(15 a^4 d^3-33 a^3 b d^3 x^4+5 b^4 c^2 x^4 (5 c-6 d x^4)+a^2 b^2 d (95 c^2-112 c d x^4+18 d^2 x^8)+a b^3 c (-35 c^2-45 c d x^4+102 d^2 x^8)\right) \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+10 x^4 \left(3 a^4 d^3-6 a^3 b d^3 x^4+5 b^4 c^2 x^4 (c-d x^4)+a^2 b^2 d (19 c^2-19 c d x^4+3 d^2 x^8)+a b^3 c (-7 c^2-10 c d x^4+17 d^2 x^8)\right) \left(2 a d\, \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right) \Big/ \left(c (a-b x^4) \left(9 a c\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4 \left(2 a d\, \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

- **Problem 191: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+b x^4}}{a c-b c x^4}\, dx$$

Optimal (type 3, 103 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\, a^{1/4} b^{1/4} x}{\sqrt{a+b x^4}}\right]}{2 \sqrt{2}\, a^{1/4} b^{1/4} c}+\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\, a^{1/4} b^{1/4} x}{\sqrt{a+b x^4}}\right]}{2 \sqrt{2}\, a^{1/4} b^{1/4} c}$$

Result (type 6, 155 leaves):

$$\left(5 a x \sqrt{a+b x^4}\, \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]\right) \Big/ \left(c (a-b x^4) \left(5 a\, \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]+2 b x^4 \left(2\, \text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]\right)\right)\right)$$

- **Problem 192: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a-b x^4}}{a c+b c x^4}\, dx$$

Optimal (type 3, 116 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{b^{1/4} x \left(\sqrt{a}+\sqrt{b}\, x^2\right)}{a^{1/4} \sqrt{a-b x^4}}\right]}{2 a^{1/4} b^{1/4} c}+\frac{\text{ArcTanh}\left[\frac{b^{1/4} x \left(\sqrt{a}-\sqrt{b}\, x^2\right)}{a^{1/4} \sqrt{a-b x^4}}\right]}{2 a^{1/4} b^{1/4} c}$$

Result (type 6, 155 leaves) :

$$\left(5\,a\,x\,\sqrt{a-b\,x^4}\;\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},-\frac{b\,x^4}{a}\right]\right)\Big/\left(c\,\left(a+b\,x^4\right)\left(5\,a\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},-\frac{b\,x^4}{a}\right]-2\,b\,x^4\left(2\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},-\frac{b\,x^4}{a}\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 193: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^4\right)^{7/4}}{c+d\,x^4}\,dx$$

Optimal (type 3, 211 leaves, 10 steps) :

$$\frac{b\,x\,\left(a+b\,x^4\right)^{3/4}}{4\,d}-\frac{b^{3/4}\,(4\,b\,c-7\,a\,d)\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{8\,d^2}+\frac{(b\,c-a\,d)^{7/4}\,\text{ArcTan}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{2\,c^{3/4}\,d^2}-\frac{b^{3/4}\,(4\,b\,c-7\,a\,d)\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{8\,d^2}+\frac{(b\,c-a\,d)^{7/4}\,\text{ArcTanh}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{2\,c^{3/4}\,d^2}$$

Result (type 6, 396 leaves) :

$$\frac{1}{80}\left(-\left(36\,a\,b\,c\,(-4\,b\,c+7\,a\,d)\,x^5\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(d\,\left(a+b\,x^4\right)^{1/4}\left(c+d\,x^4\right)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{5}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+\frac{1}{c^{3/4}\,d\,(b\,c-a\,d)^{1/4}}5\left(4\,b\,c^{3/4}\,(b\,c-a\,d)^{1/4}\,x\,\left(a+b\,x^4\right)^{3/4}+2\,a\,(-b\,c+4\,a\,d)\,\text{ArcTan}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,\left(b+a\,x^4\right)^{1/4}}\right]+a\,(b\,c-4\,a\,d)\,\text{Log}\left[c^{1/4}-\frac{(b\,c-a\,d)^{1/4}\,x}{\left(b+a\,x^4\right)^{1/4}}\right]-a\,b\,c\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{\left(b+a\,x^4\right)^{1/4}}\right]+4\,a^2\,d\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{\left(b+a\,x^4\right)^{1/4}}\right]\right)\right)$$

- **Problem 194: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^4\right)^{3/4}}{c+d\,x^4}\,dx$$

Optimal (type 3, 173 leaves, 9 steps) :

$$\frac{b^{3/4}\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right]}{2\,d}-\frac{(b\,c-a\,d)^{3/4}\,\text{ArcTan}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(a+b\,x^4)^{1/4}}\right]}{2\,c^{3/4}\,d}+\frac{b^{3/4}\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right]}{2\,d}-\frac{(b\,c-a\,d)^{3/4}\,\text{ArcTanh}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(a+b\,x^4)^{1/4}}\right]}{2\,c^{3/4}\,d}$$

Result (type 6, 161 leaves):

$$\left(5\,a\,c\,x\,(a+b\,x^4)^{3/4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((c+d\,x^4)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},-\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(-4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},-\frac{3}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

Problem 199: Result unnecessarily involves higher level functions.

$$\int\frac{(a+b\,x^4)^{9/4}}{c+d\,x^4}\,dx$$

Optimal (type 4, 316 leaves, 11 steps):

$$-\frac{b\,(6\,b\,c-11\,a\,d)\,x\,(a+b\,x^4)^{1/4}}{12\,d^2}+\frac{b\,x\,(a+b\,x^4)^{5/4}}{6\,d}+\frac{\sqrt{a}\,b^{3/2}\,(6\,b\,c-11\,a\,d)\left(1+\frac{a}{b\,x^4}\right)^{3/4}x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{12\,d^2\,(a+b\,x^4)^{3/4}}+$$
$$\frac{(b\,c-a\,d)^2\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^2}+$$
$$\frac{(b\,c-a\,d)^2\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^2}$$

Result (type 6, 396 leaves):

$$\frac{1}{60\,d^2\,(a+b\,x^4)^{3/4}}x\left(5\,b\,(a+b\,x^4)\,(-6\,b\,c+13\,a\,d+2\,b\,d\,x^4)-\left(25\,a^2\,c\,(6\,b^2\,c^2-13\,a\,b\,c\,d+12\,a^2\,d^2)\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((c+d\,x^4)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)-\left(9\,a\,b\,c\,(12\,b^2\,c^2-30\,a\,b\,c\,d+23\,a^2\,d^2)\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((c+d\,x^4)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

Problem 200: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{5/4}}{c+d x^4}\,dx$$

Optimal (type 4, 274 leaves, 10 steps):

$$\frac{b x \left(a+b x^4\right)^{1/4}}{2 d}-\frac{\sqrt{a}\, b^{3/2} \left(1+\frac{a}{b x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{2 d \left(a+b x^4\right)^{3/4}}-$$
$$\frac{(b c-a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]}{2 b^{1/4} c d}-$$
$$\frac{(b c-a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]}{2 b^{1/4} c d}$$

Result (type 6, 435 leaves):

$$\left(x \left(-\left(25 a^2 c\, (-b c+2 a d)\, \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/ \left(-5 a c\, \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+\right.\right.\right.$$
$$\left.x^4 \left(4 a d\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\, \text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)+$$
$$\left(b \left(-9 a c \left(5 a c+3 b c x^4+8 a d x^4+5 b d x^8\right) \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+5 x^4 \left(a+b x^4\right) \left(c+d x^4\right)\right.\right.$$
$$\left.\left.\left(4 a d\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\, \text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\Big/$$
$$\left(-9 a c\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4 \left(4 a d\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+\right.\right.$$
$$\left.\left.\left.\left.3 b c\, \text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\right)\Big/ \left(10 d \left(a+b x^4\right)^{3/4} \left(c+d x^4\right)\right)$$

Problem 201: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{1/4}}{c+d x^4}\,dx$$

Optimal (type 4, 166 leaves, 4 steps):

$$\frac{\sqrt{\frac{a}{a+b x^4}}\ \sqrt{a+b x^4}\ \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b}\ \sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],\ -1\right]}{2 b^{1/4} c}+\frac{\sqrt{\frac{a}{a+b x^4}}\ \sqrt{a+b x^4}\ \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b}\ \sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],\ -1\right]}{2 b^{1/4} c}$$

Result (type 6, 160 leaves):

$$\left(5 a c x \left(a+b x^4\right)^{1/4} \text{AppellF1}\left[\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(\left(c+d x^4\right)\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(-4 a d\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)$$

■ Problem 202: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)}\,dx$$

Optimal (type 4, 259 leaves, 9 steps):

$$-\frac{b^{3/2}\left(1+\frac{a}{b x^4}\right)^{3/4} x^3\,\text{EllipticF}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\ x^2}{\sqrt{a}}\right],2\right]}{\sqrt{a}\ (b c-a d)\left(a+b x^4\right)^{3/4}}-\frac{d\sqrt{\frac{a}{a+b x^4}}\ \sqrt{a+b x^4}\ \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b}\ \sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],\ -1\right]}{2 b^{1/4} c\,(b c-a d)}-\frac{d\sqrt{\frac{a}{a+b x^4}}\ \sqrt{a+b x^4}\ \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b}\ \sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],\ -1\right]}{2 b^{1/4} c\,(b c-a d)}$$

Result (type 6, 161 leaves):

$$-\left(5 a c x\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)\left(-5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)$$

■ Problem 203: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{7/4}\left(c+d x^4\right)}\,dx$$

Optimal (type 4, 304 leaves, 10 steps):

$$\frac{b\,x}{3\,a\,(b\,c-a\,d)\,\left(a+b\,x^4\right)^{3/4}}-\frac{b^{3/2}\,(2\,b\,c-5\,a\,d)\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{3\,a^{3/2}\,(b\,c-a\,d)^2\,\left(a+b\,x^4\right)^{3/4}}+$$

$$\frac{d^2\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^2}+\frac{d^2\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^2}$$

Result (type 6, 343 leaves):

$$\left(x\left(-\frac{5\,b}{a}+\left(25\,c\,(2\,b\,c-3\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+\left(18\,b\,c\,d\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(15\,(-b\,c+a\,d)\left(a+b\,x^4\right)^{3/4}\right)$$

- **Problem 204: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{\left(a+b\,x^4\right)^{11/4}\left(c+d\,x^4\right)}\,dx$$

Optimal (type 4, 357 leaves, 11 steps):

$$\frac{b\,x}{7\,a\,(b\,c-a\,d)\,\left(a+b\,x^4\right)^{7/4}}+\frac{b\,(6\,b\,c-13\,a\,d)\,x}{21\,a^2\,(b\,c-a\,d)^2\,\left(a+b\,x^4\right)^{3/4}}-\frac{b^{3/2}\,\left(12\,b^2\,c^2-38\,a\,b\,c\,d+47\,a^2\,d^2\right)\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{21\,a^{5/2}\,(b\,c-a\,d)^3\,\left(a+b\,x^4\right)^{3/4}}-$$

$$\frac{d^3\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^3}-\frac{d^3\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^3}$$

Result (type 6, 407 leaves):

$$\left(x\left(\frac{5\,b\,\left(-16\,a^2\,d+6\,b^2\,c\,x^4+a\,b\,\left(9\,c-13\,d\,x^4\right)\right)}{a+b\,x^4}+\right.\right.$$
$$\left(25\,a\,c\,\left(12\,b^2\,c^2-26\,a\,b\,c\,d+21\,a^2\,d^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]-\right.\right.$$
$$\left.\left.x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+$$
$$\left(18\,a\,b\,c\,d\,\left(-6\,b\,c+13\,a\,d\right)\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\right.\right.$$
$$\left.\left.\left.\left.\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(105\,a^2\,\left(b\,c-a\,d\right)^2\,\left(a+b\,x^4\right)^{3/4}\right)$$

- **Problem 205: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x^4\right)^{11/4}}{\left(c+d\,x^4\right)^2}\,dx$$

Optimal (type 3, 280 leaves, 11 steps):

$$\frac{b\,\left(2\,b\,c-a\,d\right)\,x\,\left(a+b\,x^4\right)^{3/4}}{4\,c\,d^2}-\frac{\left(b\,c-a\,d\right)\,x\,\left(a+b\,x^4\right)^{7/4}}{4\,c\,d\,\left(c+d\,x^4\right)}-\frac{b^{7/4}\,\left(8\,b\,c-11\,a\,d\right)\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{8\,d^3}+$$
$$\frac{\left(b\,c-a\,d\right)^{7/4}\,\left(8\,b\,c+3\,a\,d\right)\,\text{ArcTan}\left[\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{8\,c^{7/4}\,d^3}-\frac{b^{7/4}\,\left(8\,b\,c-11\,a\,d\right)\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{8\,d^3}+\frac{\left(b\,c-a\,d\right)^{7/4}\,\left(8\,b\,c+3\,a\,d\right)\,\text{ArcTanh}\left[\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{8\,c^{7/4}\,d^3}$$

Result (type 6, 735 leaves):

$$\frac{1}{80\,c^{7/4}\,d^2\,(c+d\,x^4)}\left(-\left(36\,a\,b^2\,c^{11/4}\,(-8\,b\,c+11\,a\,d)\,x^5\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((a+b\,x^4)^{1/4}\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{5}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+\frac{1}{(b\,c-a\,d)^{1/4}}\,5\left(8\,b^2\,c^{11/4}\,(b\,c-a\,d)^{1/4}\,x\,(a+b\,x^4)^{3/4}-8\,a\,b\,c^{7/4}\,d\,(b\,c-a\,d)^{1/4}\,x\,(a+b\,x^4)^{3/4}+4\,a^2\,c^{3/4}\,d^2\,(b\,c-a\,d)^{1/4}\,x\,(a+b\,x^4)^{3/4}+4\,b^2\,c^{7/4}\,d\,(b\,c-a\,d)^{1/4}\,x^5\,(a+b\,x^4)^{3/4}+2\,a\,(-2\,b^2\,c^2+2\,a\,b\,c\,d+3\,a^2\,d^2)\,(c+d\,x^4)\,\text{ArcTan}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(b+a\,x^4)^{1/4}}\right]-a\,(-2\,b^2\,c^2+2\,a\,b\,c\,d+3\,a^2\,d^2)\,(c+d\,x^4)\,\text{Log}\left[c^{1/4}-\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]-2\,a\,b^2\,c^3\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+2\,a^2\,b\,c^2\,d\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+3\,a^3\,c\,d^2\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]-2\,a\,b^2\,c^2\,d\,x^4\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+2\,a^2\,b\,c\,d^2\,x^4\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+3\,a^3\,d^3\,x^4\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]\right)\right)$$

- **Problem 206: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{(a+b\,x^4)^{7/4}}{(c+d\,x^4)^2}\,dx$$

Optimal (type 3, 230 leaves, 10 steps):

$$-\frac{(b\,c-a\,d)\,x\,(a+b\,x^4)^{3/4}}{4\,c\,d\,(c+d\,x^4)}+\frac{b^{7/4}\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right]}{2\,d^2}-\frac{(b\,c-a\,d)^{3/4}\,(4\,b\,c+3\,a\,d)\,\text{ArcTan}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(a+b\,x^4)^{1/4}}\right]}{8\,c^{7/4}\,d^2}+\frac{b^{7/4}\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right]}{2\,d^2}-\frac{(b\,c-a\,d)^{3/4}\,(4\,b\,c+3\,a\,d)\,\text{ArcTanh}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(a+b\,x^4)^{1/4}}\right]}{8\,c^{7/4}\,d^2}$$

Result (type 6, 462 leaves):

$$-\frac{(b c-a d) x \left(a+b x^4\right)^{3/4}}{4 c d \left(c+d x^4\right)}-\left(9 a b^2 c x^5 \text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(5 d \left(a+b x^4\right)^{1/4} \left(c+d x^4\right) \left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4 \left(4 a d \text{AppellF1}\left[\frac{9}{4},\frac{1}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4},\frac{5}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)+$$

$$\frac{3 a^2 \left(2 \text{ArcTan}\left[\frac{(b c-a d)^{1/4} x}{c^{1/4} \left(b+a x^4\right)^{1/4}}\right]-\text{Log}\left[c^{1/4}-\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]+\text{Log}\left[c^{1/4}+\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]\right)}{16 c^{7/4} (b c-a d)^{1/4}}+$$

$$\frac{a b \left(2 \text{ArcTan}\left[\frac{(b c-a d)^{1/4} x}{c^{1/4} \left(b+a x^4\right)^{1/4}}\right]-\text{Log}\left[c^{1/4}-\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]+\text{Log}\left[c^{1/4}+\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]\right)}{16 c^{3/4} d (b c-a d)^{1/4}}$$

■ Problem 211: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{9/4}}{\left(c+d x^4\right)^2} \, dx$$

Optimal (type 4, 353 leaves, 11 steps):

$$\frac{b (3 b c-a d) x \left(a+b x^4\right)^{1/4}}{4 c d^2}-\frac{(b c-a d) x \left(a+b x^4\right)^{5/4}}{4 c d \left(c+d x^4\right)}-\frac{\sqrt{a}\, b^{3/2} (3 b c-a d) \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{4 c d^2 \left(a+b x^4\right)^{3/4}}-$$

$$\frac{3 (b c-a d) (2 b c+a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d^2}-$$

$$\frac{3 (b c-a d) (2 b c+a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d^2}$$

Result (type 6, 506 leaves):

$$\left(x\left(-\left(25a^2\left(-3b^2c^2+2abcd+3a^2d^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\Big/\left(-5ac\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+x^4\left(4ad\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+3bc\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\right)+\left(-9ac\left(5a^3d^2+3ab^2c\left(5c+2dx^4\right)+a^2bd\left(-10c+7dx^4\right)+b^3cx^4\left(9c+10dx^4\right)\right)\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+5x^4\left(a+bx^4\right)\left(-2abcd+a^2d^2+b^2c\left(3c+2dx^4\right)\right)\left(4ad\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+3bc\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\right)\Big/\left(c\left(-9ac\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+x^4\left(4ad\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+3bc\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(20d^2\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)\right)$$

- **Problem 212: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+bx^4\right)^{5/4}}{\left(c+dx^4\right)^2}\,dx$$

Optimal (type 4, 298 leaves, 10 steps):

$$-\frac{(bc-ad)\,x\left(a+bx^4\right)^{1/4}}{4cd\left(c+dx^4\right)}+\frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{4cd\left(a+bx^4\right)^{3/4}}+\frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left[-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}x}{\left(a+bx^4\right)^{1/4}}\right],-1\right]}{8b^{1/4}c^2d}+\frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left[\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}x}{\left(a+bx^4\right)^{1/4}}\right],-1\right]}{8b^{1/4}c^2d}$$

Result (type 6, 440 leaves):

$$\left(x\left(-\left(25 a^2 (b c+3 a d) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right) \Big/ \left(-5 a c \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+\right.\right.\right.$$
$$x^4\left(4 a d \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+3 b c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\Big)+$$
$$\left(9 a c\left(5 a^2 d-3 b^2 c x^4+a b\left(-5 c+7 d x^4\right)\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+5(b c-a d) x^4\left(a+b x^4\right)\right.$$
$$\left.\left(4 a d \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+3 b c \operatorname{AppellF1}\left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right) \Big/$$
$$\left(c\left(9 a c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]-x^4\left(4 a d \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+\right.\right.\right.$$
$$\left.\left.\left.\left.3 b c \operatorname{AppellF1}\left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)\right)\right) \Big/ \left(20 d\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)\right)$$

- **Problem 213: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^4\right)^{1/4}}{\left(c+d x^4\right)^2} \, dx$$

Optimal (type 4, 308 leaves, 10 steps):

$$\frac{x\left(a+b x^4\right)^{1/4}}{4 c\left(c+d x^4\right)}-\frac{\sqrt{a}\, b^{3/2}\left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right], 2\right]}{4 c(b c-a d)\left(a+b x^4\right)^{3/4}}+$$
$$\frac{(2 b c-3 a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \operatorname{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \operatorname{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right], -1\right]}{8 b^{1/4} c^2(b c-a d)}+$$
$$\frac{(2 b c-3 a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \operatorname{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \operatorname{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right], -1\right]}{8 b^{1/4} c^2(b c-a d)}$$

Result (type 6, 322 leaves):

$$\frac{1}{20\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)}x\left(\frac{5\left(a+b x^4\right)}{c}-\left(75 a^2 \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)-\left(18 a b x^4 \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)$$

■ Problem 214: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)^2}\,dx$$

Optimal (type 4, 330 leaves, 10 steps):

$$-\frac{d x\left(a+b x^4\right)^{1/4}}{4 c (b c-a d)\left(c+d x^4\right)}-\frac{b^{3/2}(4 b c-a d)\left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]}{4\sqrt{a}\, c (b c-a d)^2\left(a+b x^4\right)^{3/4}}-\frac{3 d (2 b c-a d)\sqrt{\frac{a}{a+b x^4}}\sqrt{a+b x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 (b c-a d)^2}-\frac{3 d (2 b c-a d)\sqrt{\frac{a}{a+b x^4}}\sqrt{a+b x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 (b c-a d)^2}$$

Result (type 6, 341 leaves):

$$\left(x\left(-\frac{5 d\left(a+b x^4\right)}{c}+\left(25 a(-4 b c+3 a d)\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)+\left(18 a b d x^4 \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\right)\Big/\left(20 (b c-a d)\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)\right)$$

Problem 215: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{7/4} \left(c+d x^4\right)^2} \, dx$$

Optimal (type 4, 390 leaves, 11 steps):

$$\frac{b (4 b c+3 a d) x}{12 a c (b c-a d)^2 \left(a+b x^4\right)^{3/4}}-\frac{d x}{4 c (b c-a d) \left(a+b x^4\right)^{3/4} \left(c+d x^4\right)}-$$

$$\frac{b^{3/2} \left(8 b^2 c^2-32 a b c d+3 a^2 d^2\right) \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right], 2\right]}{12 a^{3/2} c (b c-a d)^3 \left(a+b x^4\right)^{3/4}}+$$

$$\frac{d^2 (10 b c-3 a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right], -1\right]}{8 b^{1/4} c^2 (b c-a d)^3}+$$

$$\frac{d^2 (10 b c-3 a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right], -1\right]}{8 b^{1/4} c^2 (b c-a d)^3}$$

Result (type 6, 485 leaves):

$$\left(x \left(-\left(25 \left(8 b^2 c^2-24 a b c d+9 a^2 d^2\right) \text{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right) \Big/ \left(-5 a c \text{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+\right.\right.\right.$$
$$\left. x^4 \left(4 a d \text{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)+$$
$$\left(9 a c \left(15 a^2 d^2+21 a b d^2 x^4+4 b^2 c \left(5 c+7 d x^4\right)\right) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]-5 x^4 \left(3 a^2 d^2+3 a b d^2 x^4+4 b^2 c \left(c+d x^4\right)\right)\right.$$
$$\left.\left(4 a d \text{AppellF1}\left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right) \Big/$$
$$\left(a c \left(9 a c \text{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]-x^4 \left(4 a d \text{AppellF1}\left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]+\right.\right.\right.$$
$$\left.\left.\left.\left.\left. 3 b c \text{AppellF1}\left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)\right)\right)\right) \Big/ \left(60 (b c-a d)^2 \left(a+b x^4\right)^{3/4} \left(c+d x^4\right)\right)$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \left(a+b x^4\right)^p \left(c+d x^4\right)^q \, dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x\left(a+b x^4\right)^p\left(1+\frac{b x^4}{a}\right)^{-p}\left(c+d x^4\right)^q\left(1+\frac{d x^4}{c}\right)^{-q}\text{AppellF1}\left[\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]$$

Result (type 6, 172 leaves):

$$\left(5 a c x\left(a+b x^4\right)^p\left(c+d x^4\right)^q\text{AppellF1}\left[\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+4 x^4\left(b c p\,\text{AppellF1}\left[\frac{5}{4},1-p,-q,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+a d q\,\text{AppellF1}\left[\frac{5}{4},-p,1-q,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)$$

- **Problem 221: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(c+d x^4\right)^q}{a+b x^4}\,dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x\left(c+d x^4\right)^q\left(1+\frac{d x^4}{c}\right)^{-q}\text{AppellF1}\left[\frac{1}{4},1,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]}{a}$$

Result (type 6, 162 leaves):

$$\left(5 a c x\left(c+d x^4\right)^q\text{AppellF1}\left[\frac{1}{4},-q,1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right)\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},-q,1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+4 x^4\left(a d q\,\text{AppellF1}\left[\frac{5}{4},1-q,1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]-b c\,\text{AppellF1}\left[\frac{5}{4},-q,2,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)$$

- **Problem 222: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(c+d x^4\right)^q}{\left(a+b x^4\right)^2}\,dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x\left(c+d x^4\right)^q\left(1+\frac{d x^4}{c}\right)^{-q}\text{AppellF1}\left[\frac{1}{4},2,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]}{a^2}$$

Result (type 6, 162 leaves):

$$\left(5 a c x\left(c+d x^4\right)^q\text{AppellF1}\left[\frac{1}{4},2,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(\left(a+b x^4\right)^2\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},2,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+4 x^4\left(a d q\,\text{AppellF1}\left[\frac{5}{4},2,1-q,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]-2 b c\,\text{AppellF1}\left[\frac{5}{4},3,-q,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)$$

Problem 229: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 147 leaves, 8 steps):

$$\frac{2 d \sqrt{a+\frac{b}{x}}}{c^2 \left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\; x}{c \left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}\; (3 b c-4 a d)\, \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3 \sqrt{b c-a d}}+\frac{(b c-4 a d)\, \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{\sqrt{a}\; c^3}$$

Result (type 3, 197 leaves):

$$\frac{1}{2 c^3}\left(\frac{2 c \sqrt{a+\frac{b}{x}}\; x\, (2 d+c x)}{d+c x}+\frac{(b c-4 a d)\, \text{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\; x\right]}{\sqrt{a}}+\frac{\mathrm{i} \sqrt{d}\; (3 b c-4 a d)\, \text{Log}\left[-\frac{2\, \mathrm{i}\, c^4 \left(-b d+b c x-2 a d x-2\, \mathrm{i} \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\; x\right)}{d^{3/2} (3 b c-4 a d) \sqrt{b c-a d}\; (d+c x)}\right]}{\sqrt{b c-a d}}\right)$$

Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

Optimal (type 3, 213 leaves, 9 steps):

$$\frac{3 d \sqrt{a+\frac{b}{x}}}{2 c^2 \left(c+\frac{d}{x}\right)^2}+\frac{d\, (11 b c-12 a d) \sqrt{a+\frac{b}{x}}}{4 c^3 (b c-a d) \left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\; x}{c \left(c+\frac{d}{x}\right)^2}+\frac{\sqrt{d}\; \left(15 b^2 c^2-40 a b c d+24 a^2 d^2\right) \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4 (b c-a d)^{3/2}}+\frac{(b c-6 a d)\, \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{\sqrt{a}\; c^4}$$

Result (type 3, 275 leaves):

$$\frac{1}{8\,c^4}\left(\frac{2\,c\,\sqrt{a+\frac{b}{x}}\;x\left(-2\,a\,d\left(6\,d^2+9\,c\,d\,x+2\,c^2\,x^2\right)+b\,c\left(11\,d^2+17\,c\,d\,x+4\,c^2\,x^2\right)\right)}{(b\,c-a\,d)\,(d+c\,x)^2}+\frac{4\,(b\,c-6\,a\,d)\,\text{Log}\left[b+2\,a\,x+2\,\sqrt{a}\,\sqrt{a+\frac{b}{x}}\;x\right]}{\sqrt{a}}+\frac{i\,\sqrt{d}\,\left(15\,b^2\,c^2-40\,a\,b\,c\,d+24\,a^2\,d^2\right)\,\text{Log}\left[-\frac{8\,i\,c^5\,\sqrt{b\,c-a\,d}\,\left(-b\,d+b\,c\,x-2\,a\,d\,x-2\,i\,\sqrt{d}\,\sqrt{b\,c-a\,d}\,\sqrt{a+\frac{b}{x}}\;x\right)}{d^{3/2}\,\left(15\,b^2\,c^2-40\,a\,b\,c\,d+24\,a^2\,d^2\right)\,(d+c\,x)}\right]}{(b\,c-a\,d)^{3/2}}\right)$$

Problem 236: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 156 leaves, 8 steps):

$$-\frac{(b\,c-2\,a\,d)\,\sqrt{a+\frac{b}{x}}}{c^2\,\left(c+\frac{d}{x}\right)}+\frac{a\,\sqrt{a+\frac{b}{x}}\;x}{c\,\left(c+\frac{d}{x}\right)}-\frac{(b\,c-4\,a\,d)\,\sqrt{b\,c-a\,d}\;\text{ArcTan}\left[\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b\,c-a\,d}}\right]}{c^3\,\sqrt{d}}+\frac{\sqrt{a}\,(3\,b\,c-4\,a\,d)\;\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^3}$$

Result (type 3, 231 leaves):

$$-\frac{1}{2\,c^3}\left(-\frac{2\,c\,\sqrt{a+\frac{b}{x}}\;x\,(-b\,c+2\,a\,d+a\,c\,x)}{d+c\,x}+\sqrt{a}\,(-3\,b\,c+4\,a\,d)\,\text{Log}\left[b+2\,a\,x+2\,\sqrt{a}\,\sqrt{a+\frac{b}{x}}\;x\right]+\frac{i\,\left(b^2\,c^2-5\,a\,b\,c\,d+4\,a^2\,d^2\right)\,\text{Log}\left[\frac{2\,c^4\,\left(-2\,i\,a\,d\,x+2\,\sqrt{d}\,\sqrt{b\,c-a\,d}\,\sqrt{a+\frac{b}{x}}\;x-i\,b\,(d-c\,x)\right)}{\sqrt{d}\,\sqrt{b\,c-a\,d}\,\left(b^2\,c^2-5\,a\,b\,c\,d+4\,a^2\,d^2\right)\,(d+c\,x)}\right]}{\sqrt{d}\,\sqrt{b\,c-a\,d}}\right)$$

Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

Optimal (type 3, 209 leaves, 9 steps):

$$-\frac{(b c-3 a d)\sqrt{a+\frac{b}{x}}}{2 c^2\left(c+\frac{d}{x}\right)^2}-\frac{3 (b c-4 a d)\sqrt{a+\frac{b}{x}}}{4 c^3\left(c+\frac{d}{x}\right)}+\frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2}-\frac{3\left(b^2 c^2-8 a b c d+8 a^2 d^2\right)\mathrm{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4\sqrt{d}\sqrt{b c-a d}}+\frac{3\sqrt{a}\,(b c-2 a d)\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^4}$$

Result (type 3, 256 leaves):

$$\frac{1}{8 c^4}\left(\frac{2 c\sqrt{a+\frac{b}{x}}\,x\left(-b c\,(3 d+5 c x)+2 a\left(6 d^2+9 c d x+2 c^2 x^2\right)\right)}{(d+c x)^2}-12\sqrt{a}\,(-b c+2 a d)\,\mathrm{Log}\left[b+2 a x+2\sqrt{a}\sqrt{a+\frac{b}{x}}\,x\right]+\frac{3\,i\left(b^2 c^2-8 a b c d+8 a^2 d^2\right)\mathrm{Log}\left[\frac{8 c^5\left(2\,i\,a d x+2\sqrt{d}\sqrt{b c-a d}\sqrt{a+\frac{b}{x}}\,x+i\,b\,(d-c x)\right)}{3\sqrt{d}\sqrt{b c-a d}\left(b^2 c^2-8 a b c d+8 a^2 d^2\right)(d+c x)}\right]}{\sqrt{d}\sqrt{b c-a d}}\right)$$

Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 166 leaves, 8 steps):

$$\frac{(b c-2 a d)(b c-a d)\sqrt{a+\frac{b}{x}}}{c^2 d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)}-\frac{(b c-a d)^{3/2}(b c+4 a d)\,\mathrm{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3 d^{3/2}}+\frac{a^{3/2}(5 b c-4 a d)\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^3}$$

Result (type 3, 219 leaves):

$$-\frac{1}{2c^3}\left(-\frac{2c\sqrt{a+\frac{b}{x}}\,x\left(b^2c^2-2abcd+a^2d\,(2d+cx)\right)}{d\,(d+cx)}+\right.$$

$$\left.a^{3/2}\,(-5bc+4ad)\,\mathrm{Log}\left[b+2ax+2\sqrt{a}\,\sqrt{a+\frac{b}{x}}\,x\right]+\frac{i\,(bc-ad)^{3/2}\,(bc+4ad)\,\mathrm{Log}\left[\frac{2c^4\left(-2\,i\,a\,d^{3/2}\,x+2d\sqrt{bc-ad}\,\sqrt{a+\frac{b}{x}}\,x-i\,b\sqrt{d}\,(d-cx)\right)}{(bc-ad)^{5/2}\,(bc+4ad)\,(d+cx)}\right]}{d^{3/2}}\right)$$

Problem 244: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

Optimal (type 3, 237 leaves, 9 steps) :

$$\frac{(bc-3ad)\,(bc-ad)\,\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2}-\frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2}-$$

$$\frac{\sqrt{bc-ad}\,\left(b^2c^2+8abcd-24a^2d^2\right)\mathrm{ArcTan}\left[\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right]}{4c^4d^{3/2}}+\frac{a^{3/2}\,(5bc-6ad)\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^4}$$

Result (type 3, 304 leaves) :

$$\frac{1}{8\,c^4}\left(\frac{2\,c\,\sqrt{a+\frac{b}{x}}\;x\left(b^2\,c^2\,(-d+c\,x)-a\,b\,c\,d\,(7\,d+11\,c\,x)+2\,a^2\,d\left(6\,d^2+9\,c\,d\,x+2\,c^2\,x^2\right)\right)}{d\,(d+c\,x)^2}-4\,a^{3/2}\,(-5\,b\,c+6\,a\,d)\,\mathrm{Log}\left[b+2\,a\,x+2\,\sqrt{a}\,\sqrt{a+\frac{b}{x}}\;x\right]-\right.$$

$$\left.\frac{i\left(b^3\,c^3+7\,a\,b^2\,c^2\,d-32\,a^2\,b\,c\,d^2+24\,a^3\,d^3\right)\mathrm{Log}\left[\frac{8\,c^5\left(-2\,i\,a\,d^{3/2}\,x+2\,d\,\sqrt{b\,c-a\,d}\,\sqrt{a+\frac{b}{x}}\;x-i\,b\,\sqrt{d}\,(d-c\,x)\right)}{\sqrt{b\,c-a\,d}\left(b^3\,c^3+7\,a\,b^2\,c^2\,d-32\,a^2\,b\,c\,d^2+24\,a^3\,d^3\right)(d+c\,x)}\right]}{d^{3/2}\,\sqrt{b\,c-a\,d}}\right)$$

- **Problem 250: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 172 leaves, 8 steps):

$$\frac{d\,(b\,c-2\,a\,d)\,\sqrt{a+\frac{b}{x}}}{a\,c^2\,(b\,c-a\,d)\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\;x}{a\,c\left(c+\frac{d}{x}\right)}-\frac{d^{3/2}\,(5\,b\,c-4\,a\,d)\,\mathrm{ArcTan}\left[\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b\,c-a\,d}}\right]}{c^3\,(b\,c-a\,d)^{3/2}}-\frac{(b\,c+4\,a\,d)\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{3/2}\,c^3}$$

Result (type 3, 224 leaves):

$$-\frac{1}{2\,c^3}\left(\frac{2\,c\,\sqrt{a+\frac{b}{x}}\;x\,(b\,c\,(d+c\,x)-a\,d\,(2\,d+c\,x))}{a\,(-b\,c+a\,d)\,(d+c\,x)}+\right.$$

$$\left.\frac{(b\,c+4\,a\,d)\,\mathrm{Log}\left[b+2\,a\,x+2\,\sqrt{a}\,\sqrt{a+\frac{b}{x}}\;x\right]}{a^{3/2}}+\frac{i\,d^{3/2}\,(5\,b\,c-4\,a\,d)\,\mathrm{Log}\left[\frac{2\,c^4\,\sqrt{b\,c-a\,d}\left(-2\,i\,a\,d\,x+2\,\sqrt{d}\,\sqrt{b\,c-a\,d}\,\sqrt{a+\frac{b}{x}}\;x-i\,b\,(d-c\,x)\right)}{d^{5/2}\,(5\,b\,c-4\,a\,d)\,(d+c\,x)}\right]}{(b\,c-a\,d)^{3/2}}\right)$$

Problem 251: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\,\left(c+\frac{d}{x}\right)^3}\,dx$$

Optimal (type 3, 250 leaves, 9 steps):

$$\frac{d\,(2\,b\,c-3\,a\,d)\,\sqrt{a+\frac{b}{x}}}{2\,a\,c^2\,(b\,c-a\,d)\,\left(c+\frac{d}{x}\right)^2}+\frac{d\,(b\,c-4\,a\,d)\,(4\,b\,c-3\,a\,d)\,\sqrt{a+\frac{b}{x}}}{4\,a\,c^3\,(b\,c-a\,d)^2\,\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{a\,c\,\left(c+\frac{d}{x}\right)^2}-$$
$$\frac{d^{3/2}\,\left(35\,b^2\,c^2-56\,a\,b\,c\,d+24\,a^2\,d^2\right)\,\text{ArcTan}\left[\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b\,c-a\,d}}\right]}{4\,c^4\,(b\,c-a\,d)^{5/2}}-\frac{(b\,c+6\,a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{3/2}\,c^4}$$

Result (type 3, 301 leaves):

$$\frac{1}{8\,c^4}\left(\frac{2\,c\,\sqrt{a+\frac{b}{x}}\,x\,\left(4\,b^2\,c^2\,(d+c\,x)^2+2\,a^2\,d^2\,\left(6\,d^2+9\,c\,d\,x+2\,c^2\,x^2\right)-a\,b\,c\,d\,\left(19\,d^2+29\,c\,d\,x+8\,c^2\,x^2\right)\right)}{a\,(b\,c-a\,d)^2\,(d+c\,x)^2}-\right.$$
$$\left.\frac{4\,(b\,c+6\,a\,d)\,\text{Log}\left[b+2\,a\,x+2\,\sqrt{a}\,\sqrt{a+\frac{b}{x}}\,x\right]}{a^{3/2}}-\frac{i\,d^{3/2}\,\left(35\,b^2\,c^2-56\,a\,b\,c\,d+24\,a^2\,d^2\right)\,\text{Log}\left[\frac{8\,c^5\,(b\,c-a\,d)^{3/2}\,\left(-2\,i\,a\,d\,x+2\,\sqrt{d}\,\sqrt{b\,c-a\,d}\,\sqrt{a+\frac{b}{x}}\,x-i\,b\,(d-c\,x)\right)}{d^{5/2}\,\left(35\,b^2\,c^2-56\,a\,b\,c\,d+24\,a^2\,d^2\right)\,(d+c\,x)}\right]}{(b\,c-a\,d)^{5/2}}\right)$$

Problem 257: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\,\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 224 leaves, 9 steps):

$$\frac{b\left(3 b^2 c^2-2 a b c d+2 a^2 d^2\right)}{a^2 c^2 (b c-a d)^2 \sqrt{a+\frac{b}{x}}}+\frac{d (b c-2 a d)}{a c^2 (b c-a d) \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)}+$$

$$\frac{x}{a c \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)}+\frac{d^{5/2} (7 b c-4 a d) \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3 (b c-a d)^{5/2}}-\frac{(3 b c+4 a d) \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{5/2} c^3}$$

Result (type 3, 290 leaves) :

$$\frac{1}{2 c^3}\left(\left(2 c \sqrt{a+\frac{b}{x}}\, x \left(3 b^3 c^2 (d+c x)+a^3 d^2 x (2 d+c x)+a^2 b d \left(2 d^2-c d x-2 c^2 x^2\right)+a b^2 c \left(-2 d^2-c d x+c^2 x^2\right)\right)\right)\Big/\right.$$

$$\left(a^2 (b c-a d)^2 (b+a x) (d+c x)\right)-\frac{(3 b c+4 a d) \text{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{a^{5/2}}+$$

$$\left.\frac{i\, d^{5/2} (7 b c-4 a d) \text{Log}\left[-\frac{2 i c^4 (b c-a d)^{3/2} \left(-b d+b c x-2 a d x-2 i \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x\right)}{d^{7/2} (7 b c-4 a d) (d+c x)}\right]}{(b c-a d)^{5/2}}\right)$$

Problem 258: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3}\, dx$$

Optimal (type 3, 320 leaves, 10 steps) :

$$\frac{3 b (2 b c-a d) \left(2 b^2 c^2-a b c d+4 a^2 d^2\right)}{4 a^2 c^3 (b c-a d)^3 \sqrt{a+\frac{b}{x}}}+\frac{d (2 b c-3 a d)}{2 a c^2 (b c-a d) \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2}+\frac{d \left(4 b^2 c^2-21 a b c d+12 a^2 d^2\right)}{4 a c^3 (b c-a d)^2 \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)}+$$

$$\frac{x}{a c \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2}+\frac{3 d^{5/2} \left(21 b^2 c^2-24 a b c d+8 a^2 d^2\right) \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4 (b c-a d)^{7/2}}-\frac{3 (b c+2 a d) \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{5/2} c^4}$$

Result (type 3, 385 leaves) :

$$\frac{1}{8 c^4}\left(\left(2 c \sqrt{a+\frac{b}{x}}\, x \left(-12 b^4 c^3 (d+c x)^2-4 a b^3 c^2 (-3 d+c x) (d+c x)^2+2 a^4 d^3 x \left(6 d^2+9 c d x+2 c^2 x^2\right)+a^3 b d^2 \left(12 d^3-9 c d^2 x-37 c^2 d x^2-12 c^3 x^3\right)+\right.\right.\right.$$

$$\left.\left. a^2 b^2 c d \left(-27 d^3-29 c d^2 x+12 c^2 d x^2+12 c^3 x^3\right)\right)\right) \Big/ \left(a^2 (-b c+a d)^3 (b+a x) (d+c x)^2\right)-$$

$$\left.\frac{12 (b c+2 a d) \text{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{a^{5/2}}+\frac{3 i d^{5/2} \left(21 b^2 c^2-24 a b c d+8 a^2 d^2\right) \text{Log}\left[-\frac{8 i c^5 (b c-a d)^{5/2} \left(-b d+b c x-2 a d x-2 i \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x\right)}{3 d^{7/2} \left(21 b^2 c^2-24 a b c d+8 a^2 d^2\right) (d+c x)}\right]}{(b c-a d)^{7/2}}\right)$$

- **Problem 264: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2}\, dx$$

Optimal (type 3, 287 leaves, 10 steps) :

$$\frac{b\left(5 b^2 c^2-6 a b c d+6 a^2 d^2\right)}{3 a^2 c^2 (b c-a d)^2 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{b (b c-2 a d) \left(5 b^2 c^2-a b c d+a^2 d^2\right)}{a^3 c^2 (b c-a d)^3 \sqrt{a+\frac{b}{x}}}+\frac{d (b c-2 a d)}{a c^2 (b c-a d) \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)}+$$

$$\frac{x}{a c \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)}-\frac{d^{7/2} (9 b c-4 a d) \operatorname{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3 (b c-a d)^{7/2}}-\frac{(5 b c+4 a d) \operatorname{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{7/2} c^3}$$

Result (type 3, 364 leaves) :

$$\frac{1}{6 c^3}\left(\left(2 \sqrt{a+\frac{b}{x}} \left(3 a^4 d^5 (b+a x)^2+2 b^5 c^3 (b c-a d) (d+c x)-4 b^4 c^3 (4 b c-7 a d) (b+a x) (d+c x)+14 b^4 c^4 (b+a x)^2 (d+c x)-\right.\right.\right.$$

$$\left.\left.26 a b^3 c^3 d (b+a x)^2 (d+c x)-3 a^4 d^4 (b+a x)^2 (d+c x)+3 a c (b c-a d)^3 x (b+a x)^2 (d+c x)\right)\right) \Big/ \left(a^4 (b c-a d)^3 (b+a x)^2 (d+c x)\right)-$$

$$\left.\frac{3 (5 b c+4 a d) \operatorname{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{a^{7/2}}+\frac{3 \, i \, d^{7/2} (-9 b c+4 a d) \operatorname{Log}\left[\frac{2 c^4 (b c-a d)^{5/2} \left(-2 i a d x+2 \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x-i b (d-c x)\right)}{d^{9/2} (9 b c-4 a d) (d+c x)}\right]}{(b c-a d)^{7/2}}\right)$$

- **Problem 265: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3} \, dx$$

Optimal (type 3, 409 leaves, 11 steps) :

$$\frac{b\left(20 b^3 c^3-36 a b^2 c^2 d+87 a^2 b c d^2-36 a^3 d^3\right)}{12 a^2 c^3 (b c-a d)^3 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{b\left(20 b^4 c^4-56 a b^3 c^3 d+24 a^2 b^2 c^2 d^2-35 a^3 b c d^3+12 a^4 d^4\right)}{4 a^3 c^3 (b c-a d)^4 \sqrt{a+\frac{b}{x}}}+$$

$$\frac{d (2 b c-3 a d)}{2 a c^2 (b c-a d) \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2}+\frac{d\left(4 b^2 c^2-23 a b c d+12 a^2 d^2\right)}{4 a c^3 (b c-a d)^2 \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)}+\frac{x}{a c \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2}-$$

$$\frac{d^{7/2}\left(99 b^2 c^2-88 a b c d+24 a^2 d^2\right) \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4 (b c-a d)^{9/2}}-\frac{(5 b c+6 a d) \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{7/2} c^4}$$

Result (type 3, 465 leaves) :

$$-\frac{1}{24 c^4}\left(\frac{1}{a^4 (b c-a d)^4 (b+a x)^2 (d+c x)^2}\right.$$

$$2 \sqrt{a+\frac{b}{x}} \left(6 a^4 d^6 (b c-a d) (b+a x)^2+3 a^4 d^5 (-23 b c+12 a d) (b+a x)^2 (d+c x)-8 b^6 c^4 (b c-a d) (d+c x)^2+\right.$$

$$8 b^5 c^4 (8 b c-17 a d) (b+a x) (d+c x)^2-56 b^5 c^5 (b+a x)^2 (d+c x)^2+128 a b^4 c^4 d (b+a x)^2 (d+c x)^2+63 a^4 b c d^4 (b+a x)^2 (d+c x)^2-$$

$$\left.30 a^5 d^5 (b+a x)^2 (d+c x)^2-12 a c (b c-a d)^4 x (b+a x)^2 (d+c x)^2\right)+\frac{12 (5 b c+6 a d) \text{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}} x\right]}{a^{7/2}}+$$

$$\left.\frac{3 i d^{7/2}\left(99 b^2 c^2-88 a b c d+24 a^2 d^2\right) \text{Log}\left[\frac{8 c^5 (b c-a d)^{7/2}\left(-2 i a d x+2 \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}} x-i b (d-c x)\right)}{d^{9/2}\left(99 b^2 c^2-88 a b c d+24 a^2 d^2\right) (d+c x)}\right]}{(b c-a d)^{9/2}}\right)$$

Problem 269: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q dx$$

Optimal (type 6, 96 leaves, 3 steps) :

$$-\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^{q}\left(\frac{b\left(c+\frac{d}{x}\right)}{b c-a d}\right)^{-q}\text{AppellF1}\left[1+p,-q,2,2+p,-\frac{d\left(a+\frac{b}{x}\right)}{b c-a d},\frac{a+\frac{b}{x}}{a}\right]}{a^{2}(1+p)}$$

Result (type 6, 206 leaves) :

$$\left(b d(-2+p+q)\left(a+\frac{b}{x}\right)^{p}\left(c+\frac{d}{x}\right)^{q} x\,\text{AppellF1}\left[1-p-q,-p,-q,2-p-q,-\frac{a x}{b},-\frac{c x}{d}\right]\right)\Big/\left((-1+p+q)\left(-b d(-2+p+q)\,\text{AppellF1}\left[1-p-q,-p,-q,2-p-q,-\frac{a x}{b},-\frac{c x}{d}\right]+x\left(a d p\,\text{AppellF1}\left[2-p-q,1-p,-q,3-p-q,-\frac{a x}{b},-\frac{c x}{d}\right]+b c q\,\text{AppellF1}\left[2-p-q,-p,1-q,3-p-q,-\frac{a x}{b},-\frac{c x}{d}\right]\right)\right)\right)$$

Problem 271: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{a+\frac{b}{x^{2}}}\sqrt{c+\frac{d}{x^{2}}}\,dx$$

Optimal (type 4, 233 leaves, 6 steps) :

$$-\frac{2 d\sqrt{a+\frac{b}{x^{2}}}}{\sqrt{c+\frac{d}{x^{2}}}\,x}+\sqrt{a+\frac{b}{x^{2}}}\sqrt{c+\frac{d}{x^{2}}}\,x+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^{2}}}\,\text{EllipticE}\left[\text{ArcCot}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right],1-\frac{b c}{a d}\right]}{\sqrt{\frac{c\left(a+\frac{b}{x^{2}}\right)}{a\left(c+\frac{d}{x^{2}}\right)}}\sqrt{c+\frac{d}{x^{2}}}}-\frac{\sqrt{c}(b c+a d)\sqrt{a+\frac{b}{x^{2}}}\,\text{EllipticF}\left[\text{ArcCot}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right],1-\frac{b c}{a d}\right]}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^{2}}\right)}{a\left(c+\frac{d}{x^{2}}\right)}}\sqrt{c+\frac{d}{x^{2}}}}$$

Result (type 4, 205 leaves) :

$$-\frac{1}{\sqrt{\frac{a}{b}}\left(b+a x^{2}\right)\left(d+c x^{2}\right)}\sqrt{a+\frac{b}{x^{2}}}\sqrt{c+\frac{d}{x^{2}}}\,x\left(\sqrt{\frac{a}{b}}\left(b+a x^{2}\right)\left(d+c x^{2}\right)+2\,i\,a d x\sqrt{1+\frac{a x^{2}}{b}}\sqrt{1+\frac{c x^{2}}{d}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{a}{b}}\,x\right],\frac{b c}{a d}\right]+i(b c-a d)\,x\sqrt{1+\frac{a x^{2}}{b}}\sqrt{1+\frac{c x^{2}}{d}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{a}{b}}\,x\right],\frac{b c}{a d}\right]\right)$$

Problem 273: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$$

Optimal (type 4, 262 leaves, 7 steps):

$$-\frac{2 d \sqrt{a+\frac{b}{x^2}}}{c^2 \sqrt{c+\frac{d}{x^2}}\, x}-\frac{\sqrt{a+\frac{b}{x^2}}\, x}{c \sqrt{c+\frac{d}{x^2}}}+\frac{2 \sqrt{a+\frac{b}{x^2}} \sqrt{c+\frac{d}{x^2}}\, x}{c^2}+$$

$$\frac{2 \sqrt{d} \sqrt{a+\frac{b}{x^2}}\, \text{EllipticE}\left[\text{ArcCot}\left[\frac{\sqrt{c}\, x}{\sqrt{d}}\right], 1-\frac{b c}{a d}\right]}{c^{3/2} \sqrt{\frac{c \left(a+\frac{b}{x^2}\right)}{a \left(c+\frac{d}{x^2}\right)}} \sqrt{c+\frac{d}{x^2}}}-\frac{b \sqrt{a+\frac{b}{x^2}}\, \text{EllipticF}\left[\text{ArcCot}\left[\frac{\sqrt{c}\, x}{\sqrt{d}}\right], 1-\frac{b c}{a d}\right]}{a \sqrt{c} \sqrt{d} \sqrt{\frac{c \left(a+\frac{b}{x^2}\right)}{a \left(c+\frac{d}{x^2}\right)}} \sqrt{c+\frac{d}{x^2}}}$$

Result (type 4, 191 leaves):

$$-\left(\sqrt{a+\frac{b}{x^2}} \left(\sqrt{\frac{a}{b}}\, c\, x \left(b+a x^2\right)+2 i a d \sqrt{1+\frac{a x^2}{b}} \sqrt{1+\frac{c x^2}{d}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{a}{b}}\, x\right], \frac{b c}{a d}\right]+\right.\right.$$

$$\left.\left. i (b c-2 a d) \sqrt{1+\frac{a x^2}{b}} \sqrt{1+\frac{c x^2}{d}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{a}{b}}\, x\right], \frac{b c}{a d}\right]\right)\right) \Big/ \left(\sqrt{\frac{a}{b}}\, c^2 \sqrt{c+\frac{d}{x^2}} \left(b+a x^2\right)\right)$$

Problem 274: Result more than twice size of optimal antiderivative.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Optimal (type 6, 79 leaves, 4 steps):

$$\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c x^2}\right)^{-q} x\, \text{AppellF1}\left[-\frac{1}{2}, -p, -q, \frac{1}{2}, -\frac{b}{a x^2}, -\frac{d}{c x^2}\right]$$

Result (type 6, 252 leaves):

$$\left(b\,d\,(-3+2\,p+2\,q)\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,x\,\text{AppellF1}\left[\frac{1}{2}-p-q,\,-p,\,-q,\,\frac{3}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-1+2\,p+2\,q)\,\left(b\,d\,(3-2\,p-2\,q)\,\text{AppellF1}\left[\frac{1}{2}-p-q,\,-p,\,-q,\,\frac{3}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.2\,x^2\,\left(a\,d\,p\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,1-p,\,-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,-p,\,1-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

- **Problem 312: Result more than twice size of optimal antiderivative.**

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

Optimal (type 6, 81 leaves, 3 steps):

$$x\,(a+b\,x^n)^p\,\left(1+\frac{b\,x^n}{a}\right)^{-p}\,(c+d\,x^n)^q\,\left(1+\frac{d\,x^n}{c}\right)^{-q}\,\text{AppellF1}\left[\frac{1}{n},\,-p,\,-q,\,1+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]$$

Result (type 6, 190 leaves):

$$\left(a\,c\,(1+n)\,x\,(a+b\,x^n)^p\,(c+d\,x^n)^q\,\text{AppellF1}\left[\frac{1}{n},\,-p,\,-q,\,1+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]\right)\Big/\left(b\,c\,n\,p\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},\,1-p,\,-q,\,2+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]+\right.$$
$$\left.a\,d\,n\,q\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},\,-p,\,1-q,\,2+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]+a\,c\,(1+n)\,\text{AppellF1}\left[\frac{1}{n},\,-p,\,-q,\,1+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]\right)$$

- **Problem 317: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b\,x^n)^p}{c+d\,x^n}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,(a+b\,x^n)^p\,\left(1+\frac{b\,x^n}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{n},\,-p,\,1,\,1+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]}{c}$$

Result (type 6, 180 leaves):

$$\left(a\,c\,(1+n)\,x\,(a+b\,x^n)^p\,\text{AppellF1}\left[\frac{1}{n},\,-p,\,1,\,1+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]\right)\Big/$$
$$\left((c+d\,x^n)\,\left(b\,c\,n\,p\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},\,1-p,\,1,\,2+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]-a\,d\,n\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},\,-p,\,2,\,2+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]+\right.\right.$$
$$\left.\left.a\,c\,(1+n)\,\text{AppellF1}\left[\frac{1}{n},\,-p,\,1,\,1+\frac{1}{n},\,-\frac{b\,x^n}{a},\,-\frac{d\,x^n}{c}\right]\right)\right)$$

- **Problem 318: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,(a+b\,x^n)^p\,\left(1+\frac{b\,x^n}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]}{c^2}$$

Result (type 6, 180 leaves):

$$\left(a\,c\,(1+n)\,x\,(a+b\,x^n)^p\,\text{AppellF1}\left[\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\Big/\left((c+d\,x^n)^2\,\left(b\,c\,n\,p\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},1-p,2,2+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]-2\,a\,d\,n\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},-p,3,2+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+a\,c\,(1+n)\,\text{AppellF1}\left[\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\right)$$

Problem 319: Result more than twice size of optimal antiderivative.

$$\int\frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,(a+b\,x^n)^p\,\left(1+\frac{b\,x^n}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]}{c^3}$$

Result (type 6, 180 leaves):

$$\left(a\,c\,(1+n)\,x\,(a+b\,x^n)^p\,\text{AppellF1}\left[\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\Big/\left((c+d\,x^n)^3\,\left(b\,c\,n\,p\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},1-p,3,2+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]-3\,a\,d\,n\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},-p,4,2+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+a\,c\,(1+n)\,\text{AppellF1}\left[\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\right)$$

Problem 321: Result unnecessarily involves higher level functions.

$$\int(a+b\,x^n)^3\,(c+d\,x^n)^{-4-\frac{1}{n}}\,dx$$

Optimal (type 3, 178 leaves, 4 steps):

$$\frac{x\,(a+b\,x^n)^3\,(c+d\,x^n)^{-3-\frac{1}{n}}}{c\,(1+3\,n)}+\frac{3\,a\,n\,x\,(a+b\,x^n)^2\,(c+d\,x^n)^{-2-\frac{1}{n}}}{c^2\,\left(1+5\,n+6\,n^2\right)}+\frac{6\,a^2\,n^2\,x\,(a+b\,x^n)\,(c+d\,x^n)^{-1-\frac{1}{n}}}{c^3\,(1+n)\,(1+2\,n)\,(1+3\,n)}+\frac{6\,a^3\,n^3\,x\,(c+d\,x^n)^{-1/n}}{c^4\,(1+n)\,(1+2\,n)\,(1+3\,n)}$$

Result (type 5, 198 leaves):

$$\frac{1}{c^4}x\,(c+d\,x^n)^{-1/n}\left(\frac{b^3\,c^3\,x^{3\,n}}{(1+3\,n)\,(c+d\,x^n)^3}+\frac{3\,a^2\,b\,x^n\,\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\,\text{Hypergeometric2F1}\left[1+\frac{1}{n},\,4+\frac{1}{n},\,2+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]}{1+n}+\frac{3\,a\,b^2\,x^{2\,n}\,\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\,\text{Hypergeometric2F1}\left[2+\frac{1}{n},\,4+\frac{1}{n},\,3+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]}{1+2\,n}+a^3\,\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\,\text{Hypergeometric2F1}\left[4+\frac{1}{n},\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]\right)$$

Problem 322: Result unnecessarily involves higher level functions.

$$\int (a+b\,x^n)^2\,(c+d\,x^n)^{-3-\frac{1}{n}}\,dx$$

Optimal (type 3, 116 leaves, 3 steps):

$$\frac{x\,(a+b\,x^n)^2\,(c+d\,x^n)^{-2-\frac{1}{n}}}{c\,(1+2\,n)}+\frac{2\,a\,n\,x\,(a+b\,x^n)\,(c+d\,x^n)^{-1-\frac{1}{n}}}{c^2\,(1+n)\,(1+2\,n)}+\frac{2\,a^2\,n^2\,x\,(c+d\,x^n)^{-1/n}}{c^3\,(1+n)\,(1+2\,n)}$$

Result (type 5, 139 leaves):

$$\frac{1}{c^3}x\,(c+d\,x^n)^{-1/n}\left(\frac{b^2\,c^2\,x^{2\,n}}{(1+2\,n)\,(c+d\,x^n)^2}+\frac{2\,a\,b\,x^n\,\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\,\text{Hypergeometric2F1}\left[1+\frac{1}{n},\,3+\frac{1}{n},\,2+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]}{1+n}+a^2\,\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\,\text{Hypergeometric2F1}\left[3+\frac{1}{n},\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]\right)$$

Problem 323: Result unnecessarily involves higher level functions.

$$\int (a+b\,x^n)\,(c+d\,x^n)^{-2-\frac{1}{n}}\,dx$$

Optimal (type 3, 58 leaves, 2 steps):

$$\frac{x\,(a+b\,x^n)\,(c+d\,x^n)^{-1-\frac{1}{n}}}{c\,(1+n)}+\frac{a\,n\,x\,(c+d\,x^n)^{-1/n}}{c^2\,(1+n)}$$

Result (type 5, 82 leaves):

$$\frac{x\,(c+d\,x^n)^{-\frac{1+n}{n}}\,\left(b\,c\,x^n+a\,(1+n)\,(c+d\,x^n)\,\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\,\text{Hypergeometric2F1}\left[2+\frac{1}{n},\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]\right)}{c^2\,(1+n)}$$

Problem 327: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

Optimal (type 5, 56 leaves, 1 step):

$$\frac{c^2\, x\, (c+d\, x^n)^{-1/n}\, \text{Hypergeometric2F1}\left[3, \frac{1}{n}, 1+\frac{1}{n}, -\frac{(b\, c-a\, d)\, x^n}{a\, (c+d\, x^n)}\right]}{a^3}$$

Result (type 1, 1 leaves):

???

Problem 328: Result more than twice size of optimal antiderivative.

$$\int (a+b\, x^n)^p\, (c+d\, x^n)^{-2-\frac{1}{n}-p}\, dx$$

Optimal (type 5, 193 leaves, 2 steps):

$$-\frac{b\, x\, (a+b\, x^n)^{1+p}\, (c+d\, x^n)^{-1-\frac{1}{n}-p}}{a\, (b\, c-a\, d)\, n\, (1+p)}+\frac{1}{a\, c\, (b\, c-a\, d)\, n\, (1+p)}$$
$$(b\, c+(b\, c-a\, d)\, n\, (1+p))\, x\, (a+b\, x^n)^{1+p}\left(\frac{c\, (a+b\, x^n)}{a\, (c+d\, x^n)}\right)^{-1-p} (c+d\, x^n)^{-1-\frac{1}{n}-p}\, \text{Hypergeometric2F1}\left[\frac{1}{n}, -1-p, 1+\frac{1}{n}, -\frac{(b\, c-a\, d)\, x^n}{a\, (c+d\, x^n)}\right]$$

Result (type 5, 1414 leaves):

$$\left(c^4\, (1+n)\, (1+2\, n)\, (1+3\, n)\, x\, (a+b\, x^n)^{3+p}\, (c+d\, x^n)^{-2-\frac{1}{n}-p}\left(1+\frac{d\, x^n}{c}\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\right.$$
$$\left(\text{Hypergeometric2F1}\left[1, -p, 1+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]+\frac{1}{c^2} d\, n\, x^n\left(\frac{c\, \text{Hypergeometric2F1}\left[1, -p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]}{1+n}+\right.\right.$$
$$\left.\left.\left.\frac{(b\, c-a\, d)\, x^n\, \text{Gamma}\left[1+\frac{1}{n}\right] \text{Gamma}[1-p]\, \text{Hypergeometric2F1}\left[2, 1-p, 3+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]}{(1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]}\right)\right)\right) \Big/$$
$$\left(-c\, d\, (1+3\, n)\, (1+n+n\, p)\, x^n\, (a+b\, x^n)^2\left(c^2\, (1+n)\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[1, -p, 1+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]+\right.\right.$$
$$d\, n\, x^n\left(c\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[1, -p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]+\right.$$
$$\left.\left.(b\, c-a\, d)\, (1+n)\, x^n\, \text{Gamma}\left[1+\frac{1}{n}\right] \text{Gamma}[1-p]\, \text{Hypergeometric2F1}\left[2, 1-p, 3+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]\right)\right)+$$
$$b\, c\, n\, (1+3\, n)\, p\, x^n\, (a+b\, x^n)\, (c+d\, x^n)\left(c^2\, (1+n)\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[1, -p, 1+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]+\right.$$
$$d\, n\, x^n\left(c\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[1, -p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]+\right.$$

$$(b\,c-a\,d)\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\Bigg)\Bigg)+$$

$$c\,(1+3\,n)\,(a+b\,x^n)^2\,(c+d\,x^n)\,\Bigg(c^2\,(1+n)\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Gamma}[-p]\,\text{Hypergeometric2F1}\left[1,\,-p,\,1+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+$$

$$d\,n\,x^n\,\Bigg(c\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Gamma}[-p]\,\text{Hypergeometric2F1}\left[1,\,-p,\,2+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+$$

$$(b\,c-a\,d)\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\Bigg)\Bigg)+$$

$$n^2\,x^n\,(c+d\,x^n)\,\Bigg(a\,c^2\,(-b\,c+a\,d)\,(1+2\,n)\,(1+3\,n)\,p\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Gamma}[-p]\,\text{Hypergeometric2F1}\left[2,\,1-p,\,2+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+$$

$$c\,d\,(1+3\,n)\,(a+b\,x^n)^2\,\Bigg(c\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Gamma}[-p]\,\text{Hypergeometric2F1}\left[1,\,-p,\,2+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+$$

$$(b\,c-a\,d)\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\Bigg)-$$

$$d\,(b\,c-a\,d)\,x^n\,\Bigg(b\,c\,(1+n)\,(1+3\,n)\,x^n\,(a+b\,x^n)\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]-$$

$$c\,(1+n)\,(1+3\,n)\,(a+b\,x^n)^2\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+$$

$$a\,c\,n\,(1+3\,n)\,p\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Gamma}[-p]\,\text{Hypergeometric2F1}\left[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]-$$

$$2\,a\,(-b\,c+a\,d)\,n\,(1+n)\,(-1+p)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\left[3,\,2-p,\,4+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\Bigg)\Bigg)\Bigg)$$

- **Problem 329: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+b\,x^n)^{\frac{a\,d\,n-b\,c\,(1+n)}{(b\,c-a\,d)\,n}}\,(c+d\,x^n)^{\frac{a\,d-b\,c\,n+a\,d\,n}{b\,c\,n-a\,d\,n}}\,dx$$

Optimal (type 3, 57 leaves, 1 step):

$$\frac{x\,(a+b\,x^n)^{-\frac{b\,c}{(b\,c-a\,d)\,n}}\,(c+d\,x^n)^{\frac{a\,d}{(b\,c-a\,d)\,n}}}{a\,c}$$

Result (type 6, 461 leaves):

$$\left(a c (-b c+a d) (1+n) x (a+b x^n)^{\frac{a d n-b c (1+n)}{(b c-a d) n}} (c+d x^n)^{\frac{a d-b c n+a d n}{b c n-a d n}} \text{AppellF1}\left[\frac{1}{n}, \frac{b c+b c n-a d n}{b c n-a d n}, \frac{b c n-a d (1+n)}{(b c-a d) n}, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right) \Big/ \left(b c (-a d n+b c (1+n)) x^n \text{AppellF1}\left[1+\frac{1}{n}, \frac{b c+2 b c n-2 a d n}{b c n-a d n}, \frac{b c n-a d (1+n)}{(b c-a d) n}, 2+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] - a \left(d (-b c n+a d (1+n)) x^n \text{AppellF1}\left[1+\frac{1}{n}, \frac{b c+b c n-a d n}{b c n-a d n}, -\frac{a d-2 b c n+2 a d n}{b c n-a d n}, 2+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] + c (b c-a d) (1+n) \text{AppellF1}\left[\frac{1}{n}, \frac{b c+b c n-a d n}{b c n-a d n}, \frac{b c n-a d (1+n)}{(b c-a d) n}, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right)\right)$$

Problem 330: Result unnecessarily involves higher level functions.

$$\int (a+b x^n)^2 (c+d x^n)^{-4-\frac{1}{n}} \, dx$$

Optimal (type 3, 327 leaves, 5 steps):

$$-\frac{b x (a+b x^n)^3 (c+d x^n)^{-3-\frac{1}{n}}}{3 a (b c-a d) n} - \frac{(3 a d n-b (c+3 c n)) x (a+b x^n)^3 (c+d x^n)^{-3-\frac{1}{n}}}{3 a c (b c-a d) n (1+3 n)} - \frac{(3 a d n-b (c+3 c n)) x (a+b x^n)^2 (c+d x^n)^{-2-\frac{1}{n}}}{c^2 (b c-a d) \left(1+5 n+6 n^2\right)} - \frac{2 a n (3 a d n-b (c+3 c n)) x (a+b x^n) (c+d x^n)^{-1-\frac{1}{n}}}{c^3 (b c-a d) (1+n) (1+2 n) (1+3 n)} - \frac{2 a^2 n^2 (3 a d n-b (c+3 c n)) x (c+d x^n)^{-1/n}}{c^4 (b c-a d) (1+n) (1+2 n) (1+3 n)}$$

Result (type 5, 153 leaves):

$$\frac{1}{c^4 (1+n) (1+2 n)} x (c+d x^n)^{-1/n} \left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}} \left(2 a b (1+2 n) x^n \text{Hypergeometric2F1}\left[1+\frac{1}{n}, 4+\frac{1}{n}, 2+\frac{1}{n}, -\frac{d x^n}{c}\right] + (1+n) \left(b^2 x^{2 n} \text{Hypergeometric2F1}\left[2+\frac{1}{n}, 4+\frac{1}{n}, 3+\frac{1}{n}, -\frac{d x^n}{c}\right] + a^2 (1+2 n) \text{Hypergeometric2F1}\left[4+\frac{1}{n}, \frac{1}{n}, 1+\frac{1}{n}, -\frac{d x^n}{c}\right]\right)\right)$$

Problem 331: Result unnecessarily involves higher level functions.

$$\int (a+b x^n) (c+d x^n)^{-3-\frac{1}{n}} \, dx$$

Optimal (type 3, 127 leaves, 3 steps):

$$-\frac{(b c-a d) x (c+d x^n)^{-2-\frac{1}{n}}}{c d (1+2 n)} + \frac{(b c+2 a d n) x (c+d x^n)^{-1-\frac{1}{n}}}{c^2 d (1+n) (1+2 n)} + \frac{n (b c+2 a d n) x (c+d x^n)^{-1/n}}{c^3 d (1+n) (1+2 n)}$$

Result (type 5, 96 leaves):

$$\frac{1}{c^3 (1+n)} x (c+d x^n)^{-1/n} \left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}} \left(b x^n \text{Hypergeometric2F1}\left[1+\frac{1}{n}, 3+\frac{1}{n}, 2+\frac{1}{n}, -\frac{d x^n}{c}\right] + a (1+n) \text{Hypergeometric2F1}\left[3+\frac{1}{n}, \frac{1}{n}, 1+\frac{1}{n}, -\frac{d x^n}{c}\right]\right)$$

- **Problem 332: Result unnecessarily involves higher level functions.**

$$\int (c+d\,x^n)^{-2-\frac{1}{n}}\,dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{x\,(c+d\,x^n)^{-1-\frac{1}{n}}}{c\,(1+n)}+\frac{n\,x\,(c+d\,x^n)^{-1/n}}{c^2\,(1+n)}$$

Result (type 5, 55 leaves):

$$\frac{x\,(c+d\,x^n)^{-1/n}\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[2+\frac{1}{n},\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]}{c^2}$$

- **Problem 334: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+d\,x^n)^{-1/n}}{(a+b\,x^n)^2}\,dx$$

Optimal (type 5, 127 leaves, 2 steps):

$$\frac{b\,x\,(c+d\,x^n)^{-\frac{1-n}{n}}}{a\,(b\,c-a\,d)\,n\,(a+b\,x^n)}-\frac{(b\,c\,(1-n)+a\,d\,n)\,x\,(c+d\,x^n)^{-1/n}\,\text{Hypergeometric2F1}\left[1,\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{(b\,c-a\,d)\,x^n}{a\,(c+d\,x^n)}\right]}{a^2\,(b\,c-a\,d)\,n}$$

Result (type 5, 1070 leaves):

$$\left(c^2 (1+2n) (1+3n) x (a+b x^n) (c+d x^n)^{-1/n} \left(1+\frac{d x^n}{c}\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}\left[3+\frac{1}{n}\right]\right.$$

$$\left.\left(\frac{c (c+c n+d n x^n) \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]}{\text{Gamma}\left[2+\frac{1}{n}\right]}+\frac{2 (b c-a d) n x^n (c+d x^n) \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]}{(a+b x^n) \text{Gamma}\left[3+\frac{1}{n}\right]}\right)\right)\Big/$$

$$\left(-c d (1-n) (1+2n) (1+3n) x^n (a+b x^n)^2 \left(c (a+b x^n) (c+c n+d n x^n) \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]+\right.\right.$$

$$\left.2 (b c-a d) n x^n (c+d x^n) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]\right)-$$

$$2 b c n (1+2n) (1+3n) x^n (a+b x^n) (c+d x^n) \left(c (a+b x^n) (c+c n+d n x^n) \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]+\right.$$

$$\left.2 (b c-a d) n x^n (c+d x^n) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]\right)+$$

$$c (1+2n) (1+3n) (a+b x^n)^2 (c+d x^n) \left(c (a+b x^n) (c+c n+d n x^n) \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]+\right.$$

$$\left.2 (b c-a d) n x^n (c+d x^n) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]\right)+$$

$$n^2 x^n (c+d x^n) \left(c^2 d (1+2n) (1+3n) (a+b x^n)^3 \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]+\right.$$

$$2 c d (b c-a d) (1+2n) (1+3n) x^n (a+b x^n)^2 \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]-$$

$$2 b c (b c-a d) (1+2n) (1+3n) x^n (a+b x^n) (c+d x^n) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]+$$

$$2 c (b c-a d) (1+2n) (1+3n) (a+b x^n)^2 (c+d x^n) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]+$$

$$2 a c (b c-a d) (1+3n) (a+b x^n) (c+c n+d n x^n) \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]+$$

$$\left.\left.12 a (b c-a d)^2 n (1+2n) x^n (c+d x^n) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[3,4,4+\frac{1}{n},\frac{(b c-a d) x^n}{c (a+b x^n)}\right]\right)\right)$$

- **Problem 335: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+d x^n)^{1-\frac{1}{n}}}{(a+b x^n)^3} \, dx$$

Optimal (type 5, 131 leaves, 2 steps) :

$$\frac{b\,x\,(c+d\,x^n)^{2-\frac{1}{n}}}{2\,a\,(b\,c-a\,d)\,n\,(a+b\,x^n)^2}-\frac{c\,(b\,c\,(1-2\,n)+2\,a\,d\,n)\,x\,(c+d\,x^n)^{-1/n}\,\text{Hypergeometric2F1}\left[2,\frac{1}{n},1+\frac{1}{n},-\frac{(b\,c-a\,d)\,x^n}{a\,(c+d\,x^n)}\right]}{2\,a^3\,(b\,c-a\,d)\,n}$$

Result (type 5, 1251 leaves) :

$$-\left(\left(c^4\,(1+n)\,(1+2\,n)\,(1+3\,n)\,x\,(c+d\,x^n)^{\frac{-1+n}{n}}\left(1+\frac{d\,x^n}{c}\right)\text{Gamma}\left[2+\frac{1}{n}\right]\left(\text{Hypergeometric2F1}\left[1,3,1+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.\right.\right.$$

$$\left.\left.\frac{d\,n\,x^n\left(\frac{c\,\text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]}{1+n}+\frac{3\,(b\,c-a\,d)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]}{(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]}\right)}{c^2}\right)\right)\Bigg/$$

$$\left(c\,d\,(1-2\,n)\,(1+3\,n)\,x^n\,(a+b\,x^n)^2\left(c^2\,(1+n)\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[1,3,1+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.\right.$$

$$d\,n\,x^n\left(c\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.$$

$$\left.\left.3\,(b\,c-a\,d)\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\right)\right)+$$

$$3\,b\,c\,n\,(1+3\,n)\,x^n\,(a+b\,x^n)\,(c+d\,x^n)\left(c^2\,(1+n)\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[1,3,1+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.$$

$$d\,n\,x^n\left(c\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.$$

$$\left.\left.3\,(b\,c-a\,d)\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\right)\right)-$$

$$c\,(1+3\,n)\,(a+b\,x^n)^2\,(c+d\,x^n)\left(c^2\,(1+n)\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[1,3,1+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.$$

$$d\,n\,x^n\left(c\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.$$

$$\left.\left.3\,(b\,c-a\,d)\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\right)\right)+$$

$$n^2\,x^n\,(c+d\,x^n)\left(3\,a\,c^2\,(-b\,c+a\,d)\,(1+2\,n)\,(1+3\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,2+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]-\right.$$

$$c\,d\,(1+3\,n)\,(a+b\,x^n)^2\left(c\,(1+2\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]+\right.$$

$$3\,(b\,c-a\,d)\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\Bigg)+$$

$$3\,d\,(b\,c-a\,d)\,x^n\,\Bigg(b\,c\,(1+n)\,(1+3\,n)\,x^n\,(a+b\,x^n)\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]-c\,(1+n)\,(1+3\,n)$$

$$(a+b\,x^n)^2\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]-a\,c\,n\,(1+3\,n)\,(a+b\,x^n)\,\text{Gamma}\left[2+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\Bigg[$$

$$2,4,3+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\Bigg]+8\,a\,(-b\,c+a\,d)\,n\,(1+n)\,x^n\,\text{Gamma}\left[1+\frac{1}{n}\right]\,\text{Hypergeometric2F1}\left[3,5,4+\frac{1}{n},\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\Bigg)\Bigg)\Bigg)\Bigg)$$

- **Problem 336: Attempted integration timed out after 120 seconds.**

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}\,dx$$

Optimal (type 5, 133 leaves, 2 steps):

$$\frac{b\,x\,(c+d\,x^n)^{3-\frac{1}{n}}}{3\,a\,(b\,c-a\,d)\,n\,(a+b\,x^n)^3}-\frac{c^2\,(b\,c\,(1-3\,n)+3\,a\,d\,n)\,x\,(c+d\,x^n)^{-1/n}\,\text{Hypergeometric2F1}\left[3,\frac{1}{n},1+\frac{1}{n},-\frac{(b\,c-a\,d)\,x^n}{a\,(c+d\,x^n)}\right]}{3\,a^4\,(b\,c-a\,d)\,n}$$

Result (type 1, 1 leaves):

???

- **Problem 341: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}\,\left(a+b\,x^2\right)}{x^3}\,dx$$

Optimal (type 3, 96 leaves, 5 steps):

$$b\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}-\frac{a\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{2\,x^2}-\frac{\left(2\,b\,c^2-a\,d^2\right)\,\text{ArcTan}\left[\frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{c}\right]}{2\,c}$$

Result (type 3, 105 leaves):

$$\frac{1}{2}\left(\frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}\,\left(-a+2\,b\,x^2\right)}{x^2}+\left(2\,i\,b\,c-\frac{i\,a\,d^2}{c}\right)\,\text{Log}\left[\frac{4\,i\,c-4\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{2\,b\,c^2\,x-a\,d^2\,x}\right]\right)$$

- **Problem 342: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}\,\left(a+b\,x^2\right)}{x^5}\,dx$$

Optimal (type 3, 121 leaves, 5 steps):

$$-\frac{\left(4 b c^2+a d^2\right) \sqrt{-c+d x} \sqrt{c+d x}}{8 c^2 x^2}+\frac{a (-c+d x)^{3/2} (c+d x)^{3/2}}{4 c^2 x^4}+\frac{d^2 \left(4 b c^2+a d^2\right) \text{ArcTan}\left[\frac{\sqrt{-c+d x} \sqrt{c+d x}}{c}\right]}{8 c^3}$$

Result (type 3, 132 leaves):

$$\frac{c \sqrt{-c+d x} \sqrt{c+d x} \left(-2 a c^2-4 b c^2 x^2+a d^2 x^2\right)-i d^2 \left(4 b c^2+a d^2\right) x^4 \text{Log}\left[\frac{16 c^2 \left(-i c+\sqrt{-c+d x} \sqrt{c+d x}\right)}{d^2 \left(4 b c^2+a d^2\right) x}\right]}{8 c^3 x^4}$$

- **Problem 365: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b x^2}{x^3 \sqrt{-c+d x} \sqrt{c+d x}} \, dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{a \sqrt{-c+d x} \sqrt{c+d x}}{2 c^2 x^2}+\frac{\left(2 b c^2+a d^2\right) \text{ArcTan}\left[\frac{\sqrt{-c+d x} \sqrt{c+d x}}{c}\right]}{2 c^3}$$

Result (type 3, 103 leaves):

$$\frac{a c \sqrt{-c+d x} \sqrt{c+d x}-i \left(2 b c^2+a d^2\right) x^2 \text{Log}\left[\frac{4 c^2 \left(-i c+\sqrt{-c+d x} \sqrt{c+d x}\right)}{\left(2 b c^2+a d^2\right) x}\right]}{2 c^3 x^2}$$

- **Problem 367: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b x^2}{x^5 \sqrt{-c+d x} \sqrt{c+d x}} \, dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$\frac{a \sqrt{-c+d x} \sqrt{c+d x}}{4 c^2 x^4}+\frac{\left(4 b c^2+3 a d^2\right) \sqrt{-c+d x} \sqrt{c+d x}}{8 c^4 x^2}+\frac{d^2 \left(4 b c^2+3 a d^2\right) \text{ArcTan}\left[\frac{\sqrt{-c+d x} \sqrt{c+d x}}{c}\right]}{8 c^5}$$

Result (type 3, 135 leaves):

$$\frac{c \sqrt{-c+d x} \sqrt{c+d x} \left(2 a c^2+4 b c^2 x^2+3 a d^2 x^2\right)-i d^2 \left(4 b c^2+3 a d^2\right) x^4 \text{Log}\left[\frac{16 c^4 \left(-i c+\sqrt{-c+d x} \sqrt{c+d x}\right)}{d^2 \left(4 b c^2+3 a d^2\right) x}\right]}{8 c^5 x^4}$$

- **Problem 375: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b x^2}{x^3 (-c+d x)^{3/2} (c+d x)^{3/2}} \, dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$-\frac{2\,b\,c^2+3\,a\,d^2}{2\,c^4\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}+\frac{a}{2\,c^2\,x^2\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}-\frac{\left(2\,b\,c^2+3\,a\,d^2\right)\,\text{ArcTan}\left[\frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{c}\right]}{2\,c^5}$$

Result (type 3, 126 leaves):

$$\frac{\frac{-2\,b\,c^3\,x^2+a\,\left(c^3-3\,c\,d^2\,x^2\right)}{x^2\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}+i\,\left(2\,b\,c^2+3\,a\,d^2\right)\,\text{Log}\left[\frac{4\,i\,c^5-4\,c^4\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{2\,b\,c^2\,x+3\,a\,d^2\,x}\right]}{2\,c^5}$$

- **Problem 377: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a+b\,x^2}{x^5\,\left(-c+d\,x\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

Optimal (type 3, 166 leaves, 7 steps):

$$-\frac{3\,d^2\,\left(4\,b\,c^2+5\,a\,d^2\right)}{8\,c^6\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}+\frac{a}{4\,c^2\,x^4\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}+\frac{4\,b\,c^2+5\,a\,d^2}{8\,c^4\,x^2\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}-\frac{3\,d^2\,\left(4\,b\,c^2+5\,a\,d^2\right)\,\text{ArcTan}\left[\frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{c}\right]}{8\,c^7}$$

Result (type 3, 157 leaves):

$$\frac{\frac{4\,b\,c^3\,x^2\,\left(c^2-3\,d^2\,x^2\right)+a\,\left(2\,c^5+5\,c^3\,d^2\,x^2-15\,c\,d^4\,x^4\right)}{x^4\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}+3\,i\,\left(4\,b\,c^2\,d^2+5\,a\,d^4\right)\,\text{Log}\left[\frac{16\,i\,c^7-16\,c^6\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{12\,b\,c^2\,d^2\,x+15\,a\,d^4\,x}\right]}{8\,c^7}$$

- **Problem 379: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{x^{-\frac{2\,b^2\,c+a^2\,d}{b^2\,c+a^2\,d}}\,\left(c+d\,x^2\right)}{\sqrt{-a+b\,x}\,\sqrt{a+b\,x}}\,dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\left(\frac{c}{a^2}+\frac{d}{b^2}\right)\,x^{-\frac{b^2\,c}{b^2\,c+a^2\,d}}\,\sqrt{-a+b\,x}\,\sqrt{a+b\,x}$$

Result (type 6, 1424 leaves):

$$-\frac{1}{b^4\,\sqrt{-a+b\,x}\,\sqrt{a+b\,x}\,\sqrt{1-\frac{b^2\,x^2}{a^2}}}\,d\,\left(b^2\,c+a^2\,d\right)\,x^{-\frac{b^2\,c}{b^2\,c+a^2\,d}}\,\left(-\frac{\left(a-b\,x\right)\,\left(a+b\,x\right)\,\text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{b^2\,c}{2\,\left(b^2\,c+a^2\,d\right)},1-\frac{b^2\,c}{2\,\left(b^2\,c+a^2\,d\right)},\frac{b^2\,x^2}{a^2}\right]}{c}+\right.$$

$$\left(a\,b^2\,\left(a-b\,x\right)^2\,\sqrt{1+\frac{b\,x}{a}}\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},-\frac{1}{2},\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]\right)\Big/$$

$$\left(\sqrt{1-\frac{b\,x}{a}}\left(2\,a^3\,d\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},-\frac{1}{2},\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]-b\left(b^2\,c+a^2\,d\right)x\left(\text{AppellF1}\left[\frac{a^2\,d}{b^2\,c+a^2\,d},-\frac{1}{2},\frac{3}{2},\frac{b^2\,c+2\,a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\left\{\frac{b^2\,c}{b^2\,c+a^2\,d}+\frac{3\,a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\frac{b^2\,x^2}{a^2}\right]\right)\right)\right)+$$

$$\left(a^3\,d\,(a-b\,x)^2\sqrt{1+\frac{b\,x}{a}}\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},-\frac{1}{2},\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]\right)\Big/$$

$$\left(c\sqrt{1-\frac{b\,x}{a}}\left(2\,a^3\,d\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},-\frac{1}{2},\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]-b\left(b^2\,c+a^2\,d\right)x\left(\text{AppellF1}\left[\frac{a^2\,d}{b^2\,c+a^2\,d},-\frac{1}{2},\frac{3}{2},\frac{b^2\,c+2\,a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\left\{\frac{b^2\,c}{b^2\,c+a^2\,d}+\frac{3\,a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\frac{b^2\,x^2}{a^2}\right]\right)\right)\right)+$$

$$\left(a\,b^2\,(a+b\,x)^2\sqrt{1-\frac{b\,x}{a}}\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},\frac{1}{2},-\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]\right)\Big/$$

$$\left(\sqrt{1+\frac{b\,x}{a}}\left(2\,a^3\,d\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},\frac{1}{2},-\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]+b\left(b^2\,c+a^2\,d\right)x\left(\text{AppellF1}\left[\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{3}{2},-\frac{1}{2},\frac{b^2\,c+2\,a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\left\{\frac{b^2\,c}{b^2\,c+a^2\,d}+\frac{3\,a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\frac{b^2\,x^2}{a^2}\right]\right)\right)\right)+$$

$$\left(a^3\,d\,(a+b\,x)^2\sqrt{1-\frac{b\,x}{a}}\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},\frac{1}{2},-\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]\right)\Big/$$

$$\left(c\sqrt{1+\frac{b\,x}{a}}\left(2\,a^3\,d\,\text{AppellF1}\left[-\frac{b^2\,c}{b^2\,c+a^2\,d},\frac{1}{2},-\frac{1}{2},\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]+b\left(b^2\,c+a^2\,d\right)x\left(\text{AppellF1}\left[\frac{a^2\,d}{b^2\,c+a^2\,d},\frac{3}{2},-\frac{1}{2},\frac{b^2\,c+2\,a^2\,d}{b^2\,c+a^2\,d},\frac{b\,x}{a},-\frac{b\,x}{a}\right]+\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\left\{\frac{b^2\,c}{b^2\,c+a^2\,d}+\frac{3\,a^2\,d}{2\left(b^2\,c+a^2\,d\right)}\right\},\frac{b^2\,x^2}{a^2}\right]\right)\right)\right)\Bigg)$$

Problem 380: Unable to integrate problem.

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\ \sqrt{-1+\sqrt{x}}\ \sqrt{1+x}}\, dx$$

Optimal (type 3, 36 leaves, 3 steps):

$$\frac{\sqrt{1-x}\ \mathrm{ArcSin}[x]}{\sqrt{-1-\sqrt{x}}\ \sqrt{-1+\sqrt{x}}}$$

Result (type 8, 34 leaves):

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\ \sqrt{-1+\sqrt{x}}\ \sqrt{1+x}}\, dx$$

Problem 381: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\ \sqrt{a+b\sqrt{x}}\ \sqrt{a^2+b^2 x}}\, dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$-\frac{2\sqrt{a^2-b^2 x}\ \mathrm{ArcTan}\left[\frac{\sqrt{a^2-b^2 x}}{\sqrt{a^2+b^2 x}}\right]}{b^2\sqrt{a-b\sqrt{x}}\ \sqrt{a+b\sqrt{x}}}$$

Result (type 8, 43 leaves):

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\ \sqrt{a+b\sqrt{x}}\ \sqrt{a^2+b^2 x}}\, dx$$

Problem 382: Unable to integrate problem.

$$\int (a-b x^n)^p\ (a+b x^n)^p\ \left(c+d x^{2n}\right)^q\, dx$$

Optimal (type 6, 113 leaves, 4 steps):

$$x\ (a-b x^n)^p\ (a+b x^n)^p\ \left(1-\frac{b^2 x^{2n}}{a^2}\right)^{-p}\ \left(c+d x^{2n}\right)^q\ \left(1+\frac{d x^{2n}}{c}\right)^{-q}\ \mathrm{AppellF1}\left[\frac{1}{2n},\ -p,\ -q,\ \frac{1}{2}\left(2+\frac{1}{n}\right),\ \frac{b^2 x^{2n}}{a^2},\ -\frac{d x^{2n}}{c}\right]$$

Result (type 8, 33 leaves):

$$\int (a-b x^n)^p\ (a+b x^n)^p\ \left(c+d x^{2n}\right)^q\, dx$$

- **Problem 383: Unable to integrate problem.**

$$\int (a-b x^n)^p (a+b x^n)^p \left(a^2+b^2 x^{2n}\right)^p \, dx$$

Optimal (type 5, 87 leaves, 4 steps):

$$x (a-b x^n)^p (a+b x^n)^p \left(a^2+b^2 x^{2n}\right)^p \left(1-\frac{b^4 x^{4n}}{a^4}\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{4n}, -p, \frac{1}{4}\left(4+\frac{1}{n}\right), \frac{b^4 x^{4n}}{a^4}\right]$$

Result (type 8, 37 leaves):

$$\int (a-b x^n)^p (a+b x^n)^p \left(a^2+b^2 x^{2n}\right)^p \, dx$$

- **Problem 384: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+d x^{2n}\right)^p}{(a-b x^n)(a+b x^n)} \, dx$$

Optimal (type 6, 76 leaves, 3 steps):

$$\frac{x \left(c+d x^{2n}\right)^p \left(1+\frac{d x^{2n}}{c}\right)^{-p} \text{AppellF1}\left[\frac{1}{2n}, 1, -p, \frac{1}{2}\left(2+\frac{1}{n}\right), \frac{b^2 x^{2n}}{a^2}, -\frac{d x^{2n}}{c}\right]}{a^2}$$

Result (type 6, 258 leaves):

$$\left(a^2 c (1+2n) x \left(c+d x^{2n}\right)^p \text{AppellF1}\left[\frac{1}{2n}, -p, 1, 1+\frac{1}{2n}, -\frac{d x^{2n}}{c}, \frac{b^2 x^{2n}}{a^2}\right]\right) \Big/$$
$$\left(\left(a^2-b^2 x^{2n}\right) \left(2 a^2 d n p x^{2n} \text{AppellF1}\left[1+\frac{1}{2n}, 1-p, 1, 2+\frac{1}{2n}, -\frac{d x^{2n}}{c}, \frac{b^2 x^{2n}}{a^2}\right] + \right.\right.$$
$$\left.\left. 2 b^2 c n x^{2n} \text{AppellF1}\left[1+\frac{1}{2n}, -p, 2, 2+\frac{1}{2n}, -\frac{d x^{2n}}{c}, \frac{b^2 x^{2n}}{a^2}\right] + a^2 c (1+2n) \text{AppellF1}\left[\frac{1}{2n}, -p, 1, 1+\frac{1}{2n}, -\frac{d x^{2n}}{c}, \frac{b^2 x^{2n}}{a^2}\right]\right)\right)$$

- **Problem 385: Unable to integrate problem.**

$$\int \left(a-b x^{n/2}\right)^p \left(a+b x^{n/2}\right)^p \left(\frac{a^2 d (1+p)}{b^2 \left(1+\frac{-1-2n-n p}{n}\right)}+d x^n\right)^{\frac{-1-2n-n p}{n}} dx$$

Optimal (type 3, 96 leaves, 2 steps):

$$-\frac{b^2 (1+n+n p) x \left(a-b x^{n/2}\right)^{1+p} \left(a+b x^{n/2}\right)^{1+p} \left(-\frac{a^2 d n (1+p)}{b^2 (1+n+n p)}+d x^n\right)^{-\frac{1+n+n p}{n}}}{a^4 d n (1+p)}$$

Result (type 8, 78 leaves):

$$\int \left(a-b\,x^{n/2}\right)^p \left(a+b\,x^{n/2}\right)^p \left(\frac{a^2\,d\,(1+p)}{b^2\left(1+\frac{-1-2\,n-n\,p}{n}\right)}+d\,x^n\right)^{\frac{-1-2\,n-n\,p}{n}} dx$$

Test results for the 1081 problems in "1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q.m"

- **Problem 30: Result more than twice size of optimal antiderivative.**

$$\int x^2\left(a+b\,x^3\right)^5\left(A+B\,x^3\right)dx$$

Optimal (type 1, 42 leaves, 3 steps):

$$\frac{(A\,b-a\,B)\left(a+b\,x^3\right)^6}{18\,b^2}+\frac{B\left(a+b\,x^3\right)^7}{21\,b^2}$$

Result (type 1, 107 leaves):

$$\frac{1}{126}x^3\left(42\,a^5\,A+21\,a^4\,(5\,A\,b+a\,B)\,x^3+70\,a^3\,b\,(2\,A\,b+a\,B)\,x^6+105\,a^2\,b^2\,(A\,b+a\,B)\,x^9+42\,a\,b^3\,(A\,b+2\,a\,B)\,x^{12}+7\,b^4\,(A\,b+5\,a\,B)\,x^{15}+6\,b^5\,B\,x^{18}\right)$$

- **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b\,x^3\right)^5\left(A+B\,x^3\right)}{x^{22}}dx$$

Optimal (type 1, 48 leaves, 3 steps):

$$-\frac{A\left(a+b\,x^3\right)^6}{21\,a\,x^{21}}+\frac{(A\,b-7\,a\,B)\left(a+b\,x^3\right)^6}{126\,a^2\,x^{18}}$$

Result (type 1, 118 leaves):

$$-\frac{1}{126\,x^{21}}\left(21\,b^5\,x^{15}\left(A+2\,B\,x^3\right)+35\,a\,b^4\,x^{12}\left(2\,A+3\,B\,x^3\right)+35\,a^2\,b^3\,x^9\left(3\,A+4\,B\,x^3\right)+21\,a^3\,b^2\,x^6\left(4\,A+5\,B\,x^3\right)+7\,a^4\,b\,x^3\left(5\,A+6\,B\,x^3\right)+a^5\left(6\,A+7\,B\,x^3\right)\right)$$

- **Problem 155: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^{7/2}\left(A+B\,x^3\right)}{a+b\,x^3}dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$\frac{2\,(A\,b-a\,B)\,x^{3/2}}{3\,b^2}+\frac{2\,B\,x^{9/2}}{9\,b}-\frac{2\sqrt{a}\,(A\,b-a\,B)\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right]}{3\,b^{5/2}}$$

Result (type 3, 180 leaves):

$$\frac{2\,(A\,b-a\,B)\,x^{3/2}}{3\,b^2}+\frac{2\,B\,x^{9/2}}{9\,b}+\frac{2\,\sqrt{a}\,(-A\,b+a\,B)\,\text{ArcTan}\left[\frac{-\sqrt{3}\,a^{1/6}+2\,b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]}{3\,b^{5/2}}+\frac{2\,\sqrt{a}\,(-A\,b+a\,B)\,\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}+2\,b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]}{3\,b^{5/2}}-\frac{2\,\sqrt{a}\,(-A\,b+a\,B)\,\text{ArcTan}\left[\frac{b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]}{3\,b^{5/2}}$$

- **Problem 158: Result more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{x}\,\left(A+B\,x^3\right)}{a+b\,x^3}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{2\,B\,x^{3/2}}{3\,b}+\frac{2\,(A\,b-a\,B)\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right]}{3\,\sqrt{a}\,b^{3/2}}$$

Result (type 3, 139 leaves):

$$\frac{1}{3\,\sqrt{a}\,b^{3/2}}$$

$$2\left(\sqrt{a}\,\sqrt{b}\,B\,x^{3/2}+(-A\,b+a\,B)\,\text{ArcTan}\left[\sqrt{3}-\frac{2\,b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]+(A\,b-a\,B)\,\text{ArcTan}\left[\sqrt{3}+\frac{2\,b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]-A\,b\,\text{ArcTan}\left[\frac{b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]+a\,B\,\text{ArcTan}\left[\frac{b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]\right)$$

- **Problem 161: Result more than twice size of optimal antiderivative.**

$$\int\frac{A+B\,x^3}{x^{5/2}\,\left(a+b\,x^3\right)}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{2\,A}{3\,a\,x^{3/2}}-\frac{2\,(A\,b-a\,B)\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right]}{3\,a^{3/2}\,\sqrt{b}}$$

Result (type 3, 160 leaves):

$$-\frac{2\,A}{3\,a\,x^{3/2}}+\frac{2\,(-A\,b+a\,B)\,\text{ArcTan}\left[\frac{-\sqrt{3}\,a^{1/6}+2\,b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]}{3\,a^{3/2}\,\sqrt{b}}+\frac{2\,(-A\,b+a\,B)\,\text{ArcTan}\left[\frac{\sqrt{3}\,a^{1/6}+2\,b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]}{3\,a^{3/2}\,\sqrt{b}}-\frac{2\,(-A\,b+a\,B)\,\text{ArcTan}\left[\frac{b^{1/6}\,\sqrt{x}}{a^{1/6}}\right]}{3\,a^{3/2}\,\sqrt{b}}$$

- **Problem 185: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3\,\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)\,dx$$

Optimal (type 4, 303 leaves, 4 steps):

$$\frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2}+\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}-\left(4\times3^{3/4}\sqrt{2+\sqrt{3}}\,a^2(17Ab-8aB)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(935b^{7/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 209 leaves):

$$\sqrt{a+bx^3}\left(-\frac{6a(-17Ab+8aB)x}{935b^2}+\frac{2(17Ab+3aB)x^4}{187b}+\frac{2Bx^7}{17}\right)-\frac{1}{935(-b)^{1/3}b^2\sqrt{a+bx^3}}$$

$$4\,\mathrm{i}\,3^{3/4}a^{7/3}(17Ab-8aB)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

- **Problem 186: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{a+bx^3}\left(A+Bx^3\right)dx$$

Optimal (type 4, 268 leaves, 3 steps):

$$\frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}+\left(2\times3^{3/4}\sqrt{2+\sqrt{3}}\,a(11Ab-2aB)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(55b^{4/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 182 leaves):

$$-\frac{1}{55\,(-b)^{4/3}\sqrt{a+b x^3}}2\left((-b)^{1/3} x\,(a+b x^3)\,(11 A b+3 a B+5 b B x^3)+\right.$$

$$\left.i\,3^{3/4} a^{4/3}\,(11 A b-2 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 187: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b x^3}\,(A+B x^3)}{x^3}\,dx$$

Optimal (type 4, 269 leaves, 3 steps):

$$\frac{(5 A b+4 a B)\,x\sqrt{a+b x^3}}{10 a}-\frac{A\,(a+b x^3)^{3/2}}{2 a x^2}+$$

$$\left(3^{3/4}\sqrt{2+\sqrt{3}}\,(5 A b+4 a B)\,(a^{1/3}+b^{1/3} x)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left((1+\sqrt{3})\,a^{1/3}+b^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})\,a^{1/3}+b^{1/3} x}{(1+\sqrt{3})\,a^{1/3}+b^{1/3} x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(10\,b^{1/3}\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3} x)}{\left((1+\sqrt{3})\,a^{1/3}+b^{1/3} x\right)^2}}\,\sqrt{a+b x^3}\right)$$

Result (type 4, 175 leaves):

$$\left(-\frac{A}{2 x^2}+\frac{2 B x}{5}\right)\sqrt{a+b x^3}+\frac{1}{10\,(-b)^{1/3}\sqrt{a+b x^3}}$$

$$i\,3^{3/4} a^{1/3}\,(5 A b+4 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]$$

Problem 188: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b x^3}\,(A+B x^3)}{x^6}\,dx$$

Optimal (type 4, 272 leaves, 3 steps):

$$\frac{(A b-10 a B)\sqrt{a+b x^3}}{20 a x^2}-\frac{A\left(a+b x^3\right)^{3/2}}{5 a x^5}-\left(3^{3/4}\sqrt{2+\sqrt{3}}\, b^{2/3}(A b-10 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(20 a\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 189 leaves):

$$\left(-\frac{A}{5 x^5}+\frac{-3 A b-10 a B}{20 a x^2}\right)\sqrt{a+b x^3}+\frac{1}{20 a^{2/3}(-b)^{1/3}\sqrt{a+b x^3}}\,\mathrm{i}\, 3^{3/4} b(-A b+10 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

■ Problem 189: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b x^3}\left(A+B x^3\right)}{x^9}\,dx$$

Optimal (type 4, 305 leaves, 4 steps):

$$\frac{(7 A b-16 a B)\sqrt{a+b x^3}}{80 a x^5}+\frac{3 b(7 A b-16 a B)\sqrt{a+b x^3}}{320 a^2 x^2}-\frac{A\left(a+b x^3\right)^{3/2}}{8 a x^8}+\left(3^{3/4}\sqrt{2+\sqrt{3}}\, b^{5/3}(7 A b-16 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(320 a^2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 206 leaves):

$$-\frac{\sqrt{a+b x^3}\,\left(40 a^2 A+4 a (3 A b+16 a B) x^3-3 b (7 A b-16 a B) x^6\right)}{320 a^2 x^8}+\frac{1}{320 a^{5/3}\sqrt{a+b x^3}}$$

$$i\,3^{3/4}(-b)^{5/3}(7 A b-16 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

■ Problem 190: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\sqrt{a+b x^3}\,(A+B x^3)\,dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$\frac{6 a (19 A b-10 a B) x^2\sqrt{a+b x^3}}{1729 b^2}+\frac{2 (19 A b-10 a B) x^5\sqrt{a+b x^3}}{247 b}-\frac{24 a^2 (19 A b-10 a B)\sqrt{a+b x^3}}{1729 b^{8/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+\frac{2 B x^5 (a+b x^3)^{3/2}}{19 b}+$$

$$\left(12\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{7/3}(19 A b-10 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)-$$

$$\left(8\sqrt{2}\,3^{3/4}a^{7/3}(19 A b-10 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 263 leaves):

$$\frac{1}{1729\,(-b)^{8/3}\sqrt{a+b\,x^3}}\,2\left((-b)^{2/3}\left(a+b\,x^3\right)\left(3\,a\,(19\,A\,b-10\,a\,B)\,x^2+7\,b\,(19\,A\,b+3\,a\,B)\,x^5+91\,b^2\,B\,x^8\right)+\right.$$
$$4\,(-1)^{2/3}\,3^{3/4}\,a^{8/3}\,(19\,A\,b-10\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$
$$\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

Problem 191: Result unnecessarily involves imaginary or complex numbers.

$$\int x\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)\,dx$$

Optimal (type 4, 548 leaves, 5 steps):

$$\frac{2\,(13\,A\,b-4\,a\,B)\,x^2\sqrt{a+b\,x^3}}{91\,b}+\frac{6\,a\,(13\,A\,b-4\,a\,B)\sqrt{a+b\,x^3}}{91\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{2\,B\,x^2\left(a+b\,x^3\right)^{3/2}}{13\,b}-$$
$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\,(13\,A\,b-4\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(91\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)+$$
$$\left(2\sqrt{2}\;3^{3/4}\,a^{4/3}\,(13\,A\,b-4\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(91\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 246 leaves):

$$\frac{2\,x^2\,\sqrt{a+b\,x^3}\,\left(13\,A\,b+3\,a\,B+7\,b\,B\,x^3\right)}{91\,b}-\frac{1}{91\,(-b)^{5/3}\,\sqrt{a+b\,x^3}}$$

$$2\,(-1)^{1/6}\,3^{3/4}\,a^{5/3}\,(13\,A\,b-4\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(-i\,\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 192: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{x^2}\,dx$$

Optimal (type 4, 545 leaves, 5 steps):

$$\frac{(7\,A\,b+2\,a\,B)\,x^2\,\sqrt{a+b\,x^3}}{7\,a}+\frac{3\,(7\,A\,b+2\,a\,B)\,\sqrt{a+b\,x^3}}{7\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{A\,\left(a+b\,x^3\right)^{3/2}}{a\,x}-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,(7\,A\,b+2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(14\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(\sqrt{2}\,3^{3/4}\,a^{1/3}\,(7\,A\,b+2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(7\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 236 leaves):

$$\left(-\frac{A}{x}+\frac{2\,B\,x^2}{7}\right)\sqrt{a+b\,x^3}+\frac{1}{7\,(-b)^{2/3}\sqrt{a+b\,x^3}}(-1)^{1/6}\,3^{3/4}\,a^{2/3}\,(7\,A\,b+2\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(-\mathrm{i}\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 193: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{x^5}\,dx$$

Optimal (type 4, 546 leaves, 5 steps) :

$$-\frac{(A\,b+8\,a\,B)\sqrt{a+b\,x^3}}{8\,a\,x}+\frac{3\,b^{1/3}\,(A\,b+8\,a\,B)\sqrt{a+b\,x^3}}{8\,a\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-\frac{A\,\left(a+b\,x^3\right)^{3/2}}{4\,a\,x^4}-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;b^{1/3}\,(A\,b+8\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(16\,a^{2/3}\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(3^{3/4}\,b^{1/3}\,(A\,b+8\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(4\sqrt{2}\;a^{2/3}\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 249 leaves) :

$$\left(-\frac{A}{4x^4}+\frac{-3Ab-8aB}{8ax}\right)\sqrt{a+bx^3}+\frac{1}{8a^{1/3}(-b)^{2/3}\sqrt{a+bx^3}}(-1)^{1/6}3^{3/4}b(Ab+8aB)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}$$
$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 194: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^8}\,dx$$

Optimal (type 4, 581 leaves, 6 steps) :

$$\frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4}+\frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x}-\frac{3b^{4/3}(5Ab-14aB)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{A(a+bx^3)^{3/2}}{7ax^7}+$$
$$\left(3\times3^{1/4}\sqrt{2-\sqrt{3}}\,b^{4/3}(5Ab-14aB)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(224a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)-$$
$$\left(3^{3/4}b^{4/3}(5Ab-14aB)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(56\sqrt{2}\,a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 272 leaves) :

$$\left(-\frac{A}{7 x^7}+\frac{-3 A b-14 a B}{56 a x^4}-\frac{3 b (-5 A b+14 a B)}{112 a^2 x}\right) \sqrt{a+b x^3}+$$

$$\left((-1)^{1/6} 3^{3/4} b^2 (-5 A b+14 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left.\left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Big/ \left(112\right.$$

$$\left. a^{4/3} (-b)^{2/3} \sqrt{a+b x^3}\right)$$

- **Problem 195: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+b x^3} \left(A+B x^3\right)}{x^{11}} \, dx$$

Optimal (type 4, 614 leaves, 7 steps):

$$\frac{(11 A b-20 a B) \sqrt{a+b x^3}}{140 a x^7}+\frac{3 b (11 A b-20 a B) \sqrt{a+b x^3}}{1120 a^2 x^4}-\frac{3 b^2 (11 A b-20 a B) \sqrt{a+b x^3}}{448 a^3 x}+\frac{3 b^{7/3} (11 A b-20 a B) \sqrt{a+b x^3}}{448 a^3 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{A \left(a+b x^3\right)^{3/2}}{10 a x^{10}}-$$

$$\left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}}\, b^{7/3} (11 A b-20 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(896 a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(3^{3/4} b^{7/3} (11 A b-20 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(224 \sqrt{2}\, a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 284 leaves):

$$-\frac{\sqrt{a+b x^3}\,\left(224 a^3 A+16 a^2 (3 A b+20 a B) x^3+6 a b (-11 A b+20 a B) x^6+15 b^2 (11 A b-20 a B) x^9\right)}{2240 a^3 x^{10}}+$$
$$\frac{1}{448 a^{7/3} \sqrt{a+b x^3}} (-1)^{2/3} 3^{3/4} (-b)^{7/3} (11 A b-20 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$
$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 202: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) dx$$

Optimal (type 4, 336 leaves, 5 steps):

$$\frac{54 a^2 (23 A b-8 a B) x \sqrt{a+b x^3}}{21505 b^2}+\frac{18 a (23 A b-8 a B) x^4 \sqrt{a+b x^3}}{4301 b}+\frac{2 (23 A b-8 a B) x^4 \left(a+b x^3\right)^{3/2}}{391 b}+\frac{2 B x^4 \left(a+b x^3\right)^{5/2}}{23 b}-$$
$$\left(36\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^3 (23 A b-8 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(21505 b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 229 leaves):

$$\sqrt{a+b x^3} \left(-\frac{54 a^2 (-23 A b+8 a B) x}{21505 b^2}+\frac{2 a (460 A b+27 a B) x^4}{4301 b}+\frac{2}{391} (23 A b+26 a B) x^7+\frac{2}{23} b B x^{10}\right)-$$
$$\left(36\, i\, 3^{3/4} a^{10/3} (23 A b-8 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/$$
$$\left(21505 (-b)^{1/3} b^2 \sqrt{a+b x^3}\right)$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) \, dx$$

Optimal (type 4, 299 leaves, 4 steps):

$$\frac{18 a (17 A b-2 a B) x \sqrt{a+b x^3}}{935 b}+\frac{2 (17 A b-2 a B) x \left(a+b x^3\right)^{3/2}}{187 b}+\frac{2 B x \left(a+b x^3\right)^{5/2}}{17 b}+$$
$$\left(18\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^2 (17 A b-2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(935 b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 202 leaves):

$$-\frac{1}{935 (-b)^{4/3} \sqrt{a+b x^3}} 2 \left((-b)^{1/3} \left(a+b x^3\right) \left(a (238 A b+27 a B) x+5 b (17 A b+20 a B) x^4+55 b^2 B x^7\right)+\right.$$
$$\left.9 i\, 3^{3/4} a^{7/3} (17 A b-2 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 204: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{x^3} \, dx$$

Optimal (type 4, 295 leaves, 4 steps):

$$\frac{9}{110}\,(11\,A\,b+4\,a\,B)\,x\,\sqrt{a+b\,x^3}+\frac{(11\,A\,b+4\,a\,B)\,x\,\left(a+b\,x^3\right)^{3/2}}{22\,a}-\frac{A\,\left(a+b\,x^3\right)^{5/2}}{2\,a\,x^2}+$$

$$\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,\,a\,(11\,A\,b+4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(110\,b^{1/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 193 leaves) :

$$\sqrt{a+b\,x^3}\,\left(-\frac{a\,A}{2\,x^2}+\frac{2}{55}\,(11\,A\,b+14\,a\,B)\,x+\frac{2}{11}\,b\,B\,x^4\right)+\frac{1}{110\,(-b)^{1/3}\,\sqrt{a+b\,x^3}}$$

$$9\,\mathrm{i}\,3^{3/4}\,a^{4/3}\,(11\,A\,b+4\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]$$

Problem 205: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(A+B\,x^3\right)}{x^6}\,dx$$

Optimal (type 4, 297 leaves, 4 steps) :

$$\frac{9\,b\,(A\,b+2\,a\,B)\,x\,\sqrt{a+b\,x^3}}{20\,a}-\frac{(A\,b+2\,a\,B)\,\left(a+b\,x^3\right)^{3/2}}{4\,a\,x^2}-\frac{A\,\left(a+b\,x^3\right)^{5/2}}{5\,a\,x^5}+$$

$$\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,\,b^{2/3}\,(A\,b+2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(20\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 193 leaves) :

$$\left(-\frac{a A}{5 x^5}+\frac{-13 A b-10 a B}{20 x^2}+\frac{2 b B x}{5}\right) \sqrt{a+b x^3}+\frac{1}{20 (-b)^{1/3} \sqrt{a+b x^3}}$$

$$9 i 3^{3/4} a^{1/3} b (A b+2 a B) \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

- **Problem 206: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{x^9} \, dx$$

Optimal (type 4, 302 leaves, 4 steps):

$$\frac{9 b (A b-16 a B) \sqrt{a+b x^3}}{320 a x^2}+\frac{(A b-16 a B) \left(a+b x^3\right)^{3/2}}{80 a x^5}-\frac{A \left(a+b x^3\right)^{5/2}}{8 a x^8}-$$

$$\left(9 \times 3^{3/4} \sqrt{2+\sqrt{3}}\, b^{5/3} (A b-16 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(320 a \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 209 leaves):

$$\left(-\frac{a A}{8 x^8}+\frac{-19 A b-16 a B}{80 x^5}-\frac{b (27 A b+208 a B)}{320 a x^2}\right) \sqrt{a+b x^3}+$$

$$\left(9 i 3^{3/4} b^2 (-A b+16 a B) \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right) \Big/$$

$$\left(320 a^{2/3} (-b)^{1/3} \sqrt{a+b x^3}\right)$$

- **Problem 207: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4 \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) dx$$

Optimal (type 4, 614 leaves, 7 steps):

$$\frac{54\,a^2\,(5\,A\,b-2\,a\,B)\,x^2\,\sqrt{a+b\,x^3}}{8645\,b^2}+\frac{18\,a\,(5\,A\,b-2\,a\,B)\,x^5\,\sqrt{a+b\,x^3}}{1235\,b}-$$

$$\frac{216\,a^3\,(5\,A\,b-2\,a\,B)\,\sqrt{a+b\,x^3}}{8645\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{2\,(5\,A\,b-2\,a\,B)\,x^5\,\left(a+b\,x^3\right)^{3/2}}{95\,b}+\frac{2\,B\,x^5\,\left(a+b\,x^3\right)^{5/2}}{25\,b}+$$

$$\left(108\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\;a^{10/3}\,(5\,A\,b-2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(8645\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)-$$

$$\left(72\,\sqrt{2}\;3^{3/4}\,a^{10/3}\,(5\,A\,b-2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(8645\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 283 leaves) :

$$\frac{1}{43\,225\,(-b)^{8/3}\,\sqrt{a+b\,x^3}}\,2\left((-b)^{2/3}\,\left(a+b\,x^3\right)\,\left(135\,a^2\,(5\,A\,b-2\,a\,B)\,x^2+7\,a\,b\,(550\,A\,b+27\,a\,B)\,x^5+91\,b^2\,(25\,A\,b+28\,a\,B)\,x^8+1729\,b^3\,B\,x^{11}\right)+\right.$$

$$180\,(-1)^{2/3}\,3^{3/4}\,a^{11/3}\,(5\,A\,b-2\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

■ Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int x \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) \, dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$\frac{18 a (19 A b-4 a B) x^2 \sqrt{a+b x^3}}{1729 b}+\frac{54 a^2 (19 A b-4 a B) \sqrt{a+b x^3}}{1729 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 (19 A b-4 a B) x^2 \left(a+b x^3\right)^{3/2}}{247 b}+\frac{2 B x^2 \left(a+b x^3\right)^{5/2}}{19 b}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{7/3} (19 A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(18 \sqrt{2}\, 3^{3/4} a^{7/3} (19 A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 262 leaves):

$$-\frac{1}{1729 (-b)^{5/3} \sqrt{a+b x^3}} 2 \left((-b)^{2/3} \left(a+b x^3\right) \left(a (304 A b+27 a B) x^2+7 b (19 A b+22 a B) x^5+91 b^2 B x^8\right)-\right.$$

$$9 (-1)^{2/3} 3^{3/4} a^{8/3} (19 A b-4 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left.\left(\sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

Problem 209: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{x^2} \, dx$$

Optimal (type 4, 573 leaves, 6 steps):

$$\frac{9}{91} (13 A b+2 a B) x^2 \sqrt{a+b x^3}+\frac{27 a (13 A b+2 a B) \sqrt{a+b x^3}}{91 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{(13 A b+2 a B) x^2 \left(a+b x^3\right)^{3/2}}{13 a}-\frac{A \left(a+b x^3\right)^{5/2}}{a x}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{4/3} (13 A b+2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(182 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(9 \sqrt{2}\, 3^{3/4} a^{4/3} (13 A b+2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 254 leaves):

$$\sqrt{a+b x^3} \left(-\frac{a A}{x}+\frac{2}{91} (13 A b+16 a B) x^2+\frac{2}{13} b B x^5\right)+\frac{1}{91 (-b)^{2/3} \sqrt{a+b x^3}}$$

$$9 (-1)^{1/6} 3^{3/4} a^{5/3} (13 A b+2 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 210: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{x^5} \, dx$$

Optimal (type 4, 578 leaves, 6 steps):

$$\frac{9 b (7 A b+8 a B) x^2 \sqrt{a+b x^3}}{56 a}+\frac{27 b^{1/3} (7 A b+8 a B) \sqrt{a+b x^3}}{56 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{(7 A b+8 a B) \left(a+b x^3\right)^{3/2}}{8 a x}-\frac{A \left(a+b x^3\right)^{5/2}}{4 a x^4}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}} \, a^{1/3} b^{1/3} (7 A b+8 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(112 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(9\times 3^{3/4} a^{1/3} b^{1/3} (7 A b+8 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(28 \sqrt{2} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 254 leaves):

$$-\frac{\sqrt{a+b x^3} \left(b x^3 \left(77 A-16 B x^3\right)+14 a \left(A+4 B x^3\right)\right)}{56 x^4}-\frac{1}{56 \sqrt{a+b x^3}}$$

$$9 (-1)^{1/6} 3^{3/4} a^{2/3} (-b)^{1/3} (7 A b+8 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(-i \sqrt{3} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 211: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{x^8} \, dx$$

Optimal (type 4, 576 leaves, 6 steps):

$$-\frac{9 b (A b+14 a B) \sqrt{a+b x^3}}{112 a x}+\frac{27 b^{4/3} (A b+14 a B) \sqrt{a+b x^3}}{112 a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{(A b+14 a B) \left(a+b x^3\right)^{3/2}}{56 a x^4}-\frac{A \left(a+b x^3\right)^{5/2}}{7 a x^7}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, b^{4/3} (A b+14 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(224 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(9\times 3^{3/4} b^{4/3} (A b+14 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(56 \sqrt{2}\, a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 269 leaves):

$$\left(-\frac{a A}{7 x^7}+\frac{-17 A b-14 a B}{56 x^4}-\frac{b (27 A b+154 a B)}{112 a x}\right) \sqrt{a+b x^3}+\left(9 (-1)^{1/6} 3^{3/4} b^2 (A b+14 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left.\left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(112\right.$$

$$\left.a^{1/3} (-b)^{2/3} \sqrt{a+b x^3}\right)$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{x^{11}} \, dx$$

Optimal (type 4, 608 leaves, 7 steps):

$$\frac{9 b (A b-4 a B) \sqrt{a+b x^3}}{224 a x^4}+\frac{27 b^2 (A b-4 a B) \sqrt{a+b x^3}}{448 a^2 x}-\frac{27 b^{7/3} (A b-4 a B) \sqrt{a+b x^3}}{448 a^2 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{(A b-4 a B) \left(a+b x^3\right)^{3/2}}{28 a x^7}-\frac{A \left(a+b x^3\right)^{5/2}}{10 a x^{10}}+$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, b^{7/3} (A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(896 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(9\times 3^{3/4} b^{7/3} (A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(224 \sqrt{2}\, a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 282 leaves):

$$-\frac{\sqrt{a+b x^3} \left(224 a^3 A+16 a^2 (23 A b+20 a B) x^3+2 a b (27 A b+340 a B) x^6-135 b^2 (A b-4 a B) x^9\right)}{2240 a^2 x^{10}}-$$

$$\frac{1}{448 a^{4/3} \sqrt{a+b x^3}} 9 (-1)^{2/3} 3^{3/4} (-b)^{7/3} (A b-4 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left(\sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(A+B x^3\right)}{\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 270 leaves, 3 steps):

$$\frac{2\,(11 A b-8 a B)\,x\sqrt{a+b x^3}}{55 b^2}+\frac{2 B x^4\sqrt{a+b x^3}}{11 b}-$$

$$\left(4\sqrt{2+\sqrt{3}}\;a\,(11 A b-8 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(55\times 3^{1/4} b^{7/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\sqrt{a+b x^3}\right)$$

Result (type 4, 189 leaves):

$$\frac{1}{165\,(-b)^{7/3}\sqrt{a+b x^3}}\left(6\,(-b)^{1/3} x\left(a+b x^3\right)\left(11 A b-8 a B+5 b B x^3\right)-\right.$$

$$\left.4\,\mathrm{i}\,3^{3/4} a^{4/3}\,(11 A b-8 a B)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 220: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 239 leaves, 2 steps):

$$\frac{2 B x \sqrt{a+b x^3}}{5 b}+\left(2 \sqrt{2+\sqrt{3}}\ (5 A b-2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(5\times 3^{1/4} b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 168 leaves):

$$\frac{2 B x \sqrt{a+b x^3}}{5 b}-\frac{1}{5\times 3^{1/4} (-b)^{4/3} \sqrt{a+b x^3}}$$
$$2 i a^{1/3} (5 A b-2 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]$$

- **Problem 221: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x^3}{x^3 \sqrt{a+b x^3}}\, dx$$

Optimal (type 4, 243 leaves, 2 steps):

$$-\frac{A \sqrt{a+b x^3}}{2 a x^2}-\left(\sqrt{2+\sqrt{3}}\ (A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(2\times 3^{1/4} a\, b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 170 leaves):

$$-\frac{A \sqrt{a+b x^3}}{2 a x^2}+\left(i (-A b+4 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/$$
$$\left(2\times 3^{1/4} a^{2/3} (-b)^{1/3} \sqrt{a+b x^3}\right)$$

- **Problem 222: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x^3}{x^6 \sqrt{a+b x^3}} \, dx$$

Optimal (type 4, 274 leaves, 3 steps):

$$-\frac{A \sqrt{a+b x^3}}{5 a x^5}+\frac{(7 A b-10 a B) \sqrt{a+b x^3}}{20 a^2 x^2}+$$

$$\left(\sqrt{2+\sqrt{3}}\, b^{2/3} (7 A b-10 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(20 \times 3^{1/4} a^2 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 188 leaves):

$$-\frac{\sqrt{a+b x^3} \left(4 a A-7 A b x^3+10 a B x^3\right)}{20 a^2 x^5}+\frac{1}{20 \times 3^{1/4} a^{5/3} \sqrt{a+b x^3}}$$

$$i (-b)^{2/3} (-7 A b+10 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

- **Problem 223: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4 \left(A+B x^3\right)}{\sqrt{a+b x^3}} \, dx$$

Optimal (type 4, 548 leaves, 5 steps):

$$\frac{2\,(13\,A\,b-10\,a\,B)\,x^2\sqrt{a+b\,x^3}}{91\,b^2}+\frac{2\,B\,x^5\sqrt{a+b\,x^3}}{13\,b}-\frac{8\,a\,(13\,A\,b-10\,a\,B)\sqrt{a+b\,x^3}}{91\,b^{8/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(4\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\,(13\,A\,b-10\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91\,b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)-$$

$$\left(8\sqrt{2}\;a^{4/3}\,(13\,A\,b-10\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91\times 3^{1/4}\,b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 243 leaves):

$$\frac{1}{273\,(-b)^{8/3}\sqrt{a+b\,x^3}}$$

$$2\left(3\,(-b)^{2/3}\,x^2\left(a+b\,x^3\right)\left(13\,A\,b-10\,a\,B+7\,b\,B\,x^3\right)+4\,(-1)^{2/3}\,3^{3/4}\,a^{5/3}\,(13\,A\,b-10\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 224: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x\left(A+B\,x^3\right)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 517 leaves, 4 steps):

$$\frac{2\,B\,x^2\,\sqrt{a+b\,x^3}}{7\,b}+\frac{2\,(7\,A\,b-4\,a\,B)\,\sqrt{a+b\,x^3}}{7\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{1/3}\,(7\,A\,b-4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(7\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)+$$

$$\left(2\,\sqrt{2}\,\,a^{1/3}\,(7\,A\,b-4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{1/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 231 leaves) :

$$\frac{2\,B\,x^2\,\sqrt{a+b\,x^3}}{7\,b}-\frac{1}{7\times 3^{1/4}\,(-b)^{5/3}\,\sqrt{a+b\,x^3}}2\,(-1)^{1/6}\,a^{2/3}\,(7\,A\,b-4\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(-i\,\sqrt{3}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 225: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{x^2\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 509 leaves, 4 steps) :

$$-\frac{A\sqrt{a+b x^3}}{a x}+\frac{(A b+2 a B)\sqrt{a+b x^3}}{a b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,(A b+2 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(2 a^{2/3}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)+$$

$$\frac{\sqrt{2}\,(A b+2 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3^{1/4}a^{2/3}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}}$$

Result (type 4, 225 leaves) :

$$-\frac{A\sqrt{a+b x^3}}{a x}+$$

$$\left((-1)^{1/6}(A b+2 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.\right.$$

$$\left.\left.(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{1/4}a^{1/3}(-b)^{2/3}\sqrt{a+b x^3}\right)$$

- **Problem 226: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B x^3}{x^5\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 550 leaves, 5 steps) :

$$-\frac{A\sqrt{a+b x^3}}{4 a x^4}+\frac{(5 A b-8 a B)\sqrt{a+b x^3}}{8 a^2 x}-\frac{b^{1/3}(5 A b-8 a B)\sqrt{a+b x^3}}{8 a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,b^{1/3}(5 A b-8 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(16 a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)-$$

$$\left(b^{1/3}(5 A b-8 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(4\sqrt{2}\,3^{1/4}a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 249 leaves) :

$$\frac{\sqrt{a+b x^3}\left(5 A b x^3-2 a\left(A+4 B x^3\right)\right)}{8 a^2 x^4}-\frac{1}{8\times 3^{1/4}a^{4/3}\sqrt{a+b x^3}}$$

$$(-1)^{1/6}(-b)^{1/3}(-5 A b+8 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}$$

$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 227: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B x^3}{x^8\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 581 leaves, 6 steps) :

$$-\frac{A\sqrt{a+b x^3}}{7 a x^7}+\frac{(11 A b-14 a B)\sqrt{a+b x^3}}{56 a^2 x^4}-\frac{5 b (11 A b-14 a B)\sqrt{a+b x^3}}{112 a^3 x}+\frac{5 b^{4/3}(11 A b-14 a B)\sqrt{a+b x^3}}{112 a^3\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(5\times 3^{1/4}\sqrt{2-\sqrt{3}}\, b^{4/3}(11 A b-14 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(224 a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)+$$

$$\left(5 b^{4/3}(11 A b-14 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(56\sqrt{2}\, 3^{1/4} a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 269 leaves) :

$$\frac{1}{336 a^3\sqrt{a+b x^3}}\left(-\frac{3\left(a+b x^3\right)\left(16 a^2 A+2 a(-11 A b+14 a B) x^3+5 b(11 A b-14 a B) x^6\right)}{x^7}+\right.$$

$$5(-1)^{1/6} 3^{3/4} a^{2/3}(-b)^{4/3}(11 A b-14 a B)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left.\left(-i\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 234: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^6\left(A+B x^3\right)}{\left(a+b x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 300 leaves, 4 steps) :

$$-\frac{2\,(11\,A\,b-14\,a\,B)\,x^4}{33\,b^2\,\sqrt{a+b\,x^3}}+\frac{2\,B\,x^7}{11\,b\,\sqrt{a+b\,x^3}}+\frac{16\,(11\,A\,b-14\,a\,B)\,x\,\sqrt{a+b\,x^3}}{165\,b^3}-$$

$$\left(32\,\sqrt{2+\sqrt{3}}\,a\,(11\,A\,b-14\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(165\times 3^{1/4}\,b^{10/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 205 leaves) :

$$\frac{1}{495\,(-b)^{10/3}\,\sqrt{a+b\,x^3}}\left(-6\,(-b)^{1/3}\,x\,\left(-112\,a^2\,B+3\,b^2\,x^3\,\left(11\,A+5\,B\,x^3\right)+a\,\left(88\,A\,b-42\,b\,B\,x^3\right)\right)+\right.$$

$$\left.32\,\mathrm{i}\,3^{3/4}\,a^{4/3}\,(11\,A\,b-14\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 235: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 269 leaves, 3 steps) :

$$-\frac{2\,(5\,A\,b-8\,a\,B)\,x}{15\,b^2\,\sqrt{a+b\,x^3}}+\frac{2\,B\,x^4}{5\,b\,\sqrt{a+b\,x^3}}+$$

$$\left(4\,\sqrt{2+\sqrt{3}}\,(5\,A\,b-8\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(15\times 3^{1/4}\,b^{7/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 182 leaves) :

$$\frac{1}{45\,(-b)^{7/3}\sqrt{a+b\,x^3}}\left(6\,(-b)^{1/3}\,x\,\left(-5\,A\,b+8\,a\,B+3\,b\,B\,x^3\right)+\right.$$
$$\left.4\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,(5\,A\,b-8\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 236: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 251 leaves, 2 steps) :

$$\frac{2\,(A\,b-a\,B)\,x}{3\,a\,b\,\sqrt{a+b\,x^3}}+\left(2\,\sqrt{2+\sqrt{3}}\,(A\,b+2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$
$$\left(3\times3^{1/4}\,a\,b^{4/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 176 leaves) :

$$-\frac{1}{9\,a\,(-b)^{4/3}\sqrt{a+b\,x^3}}\left(6\,(-b)^{1/3}\,(A\,b-a\,B)\,x+\right.$$
$$\left.2\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,(A\,b+2\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{x^3 \left(a+b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 272 leaves, 3 steps):

$$-\frac{A}{2 a x^2 \sqrt{a+b x^3}}-\frac{(7 A b-4 a B) x}{6 a^2 \sqrt{a+b x^3}}-$$

$$\left(\sqrt{2+\sqrt{3}} \, (7 A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(6 \times 3^{1/4} a^2 b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 193 leaves):

$$\frac{1}{18 a^2 (-b)^{1/3} x^2 \sqrt{a+b x^3}}\left(-3 (-b)^{1/3} \left(3 a A+7 A b x^3-4 a B x^3\right)-\right.$$

$$\left. i\, 3^{3/4} a^{1/3} (7 A b-4 a B) x^2 \sqrt{\frac{(-1)^{5/6} \left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{x^6 \left(a+b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 304 leaves, 4 steps):

$$-\frac{A}{5\,a\,x^5\sqrt{a+b\,x^3}}-\frac{13\,A\,b-10\,a\,B}{15\,a^2\,x^2\sqrt{a+b\,x^3}}+\frac{7\,(13\,A\,b-10\,a\,B)\sqrt{a+b\,x^3}}{60\,a^3\,x^2}+$$

$$\left(7\sqrt{2+\sqrt{3}}\,b^{2/3}\,(13\,A\,b-10\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(60\times 3^{1/4}\,a^3\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 218 leaves) :

$$\sqrt{a+b\,x^3}\left(-\frac{A}{5\,a^2\,x^5}+\frac{17\,A\,b-10\,a\,B}{20\,a^3\,x^2}-\frac{2\,b\,(-A\,b+a\,B)\,x}{3\,a^3\left(a+b\,x^3\right)}\right)-$$

$$\left(7\,\mathrm{i}\,b\,(-13\,A\,b+10\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(60\times 3^{1/4}\,a^{8/3}\,(-b)^{1/3}\sqrt{a+b\,x^3}\right)$$

- **Problem 239: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 547 leaves, 5 steps) :

$$-\frac{2\,(7\,A\,b-10\,a\,B)\,x^2}{21\,b^2\,\sqrt{a+b\,x^3}}+\frac{2\,B\,x^5}{7\,b\,\sqrt{a+b\,x^3}}+\frac{8\,(7\,A\,b-10\,a\,B)\,\sqrt{a+b\,x^3}}{21\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(4\,\sqrt{2-\sqrt{3}}\;a^{1/3}\,(7\,A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{3/4}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(8\,\sqrt{2}\;a^{1/3}\,(7\,A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(21\times 3^{1/4}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 236 leaves):

$$-\frac{1}{63\,(-b)^{8/3}\,\sqrt{a+b\,x^3}}$$

$$2\left(-3\,(-b)^{2/3}\,x^2\,\left(-7\,A\,b+10\,a\,B+3\,b\,B\,x^3\right)+4\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,(7\,A\,b-10\,a\,B)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 240: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 524 leaves, 4 steps):

$$\frac{2\,(A\,b-a\,B)\,x^2}{3\,a\,b\,\sqrt{a+b\,x^3}}-\frac{2\,(A\,b-4\,a\,B)\,\sqrt{a+b\,x^3}}{3\,a\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(\sqrt{2-\sqrt{3}}\,\,(A\,b-4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(3^{3/4}\,a^{2/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)-$$

$$\left(2\,\sqrt{2}\,\,(A\,b-4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(3\times 3^{1/4}\,a^{2/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 235 leaves) :

$$\frac{1}{9\,a\,b\,\sqrt{a+b\,x^3}}2\left(3\,(A\,b-a\,B)\,x^2+1\Big/(-b)^{5/3}\,(-1)^{1/6}\,3^{3/4}\,a^{2/3}\,b\,(A\,b-4\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(-i\,\sqrt{3}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 241: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{x^2\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 548 leaves, 5 steps) :

$$-\frac{A}{a x \sqrt{a+b x^3}}-\frac{(5 A b-2 a B) x^2}{3 a^2 \sqrt{a+b x^3}}+\frac{(5 A b-2 a B) \sqrt{a+b x^3}}{3 a^2 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(\sqrt{2-\sqrt{3}}\ (5 A b-2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(2\times 3^{3/4} a^{5/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(\sqrt{2}\ (5 A b-2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(3\times 3^{1/4} a^{5/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 243 leaves) :

$$\frac{1}{9 a^2 (-b)^{2/3} x \sqrt{a+b x^3}}$$

$$\left(-3 (-b)^{2/3} \left(3 a A+5 A b x^3-2 a B x^3\right)-(-1)^{2/3} 3^{3/4} a^{2/3} (5 A b-2 a B) x \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left.\left(\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{5/6}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)$$

- **Problem 242: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x^3}{x^5 \left(a+b x^3\right)^{3/2}}\, dx$$

Optimal (type 4, 580 leaves, 6 steps) :

$$-\frac{A}{4\,a\,x^4\sqrt{a+b\,x^3}}-\frac{11\,A\,b-8\,a\,B}{12\,a^2\,x\sqrt{a+b\,x^3}}+\frac{5\,(11\,A\,b-8\,a\,B)\sqrt{a+b\,x^3}}{24\,a^3\,x}-\frac{5\,b^{1/3}\,(11\,A\,b-8\,a\,B)\sqrt{a+b\,x^3}}{24\,a^3\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(5\sqrt{2-\sqrt{3}}\;b^{1/3}\,(11\,A\,b-8\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(16\times 3^{3/4}\,a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)-$$

$$\left(5\,b^{1/3}\,(11\,A\,b-8\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(12\sqrt{2}\;3^{1/4}\,a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 266 leaves) :

$$\frac{1}{72\,a^3\,(-b)^{2/3}\,x^4\sqrt{a+b\,x^3}}\left(3\,(-b)^{2/3}\left(55\,A\,b^2\,x^6+a\,b\,x^3\left(33\,A-40\,B\,x^3\right)-6\,a^2\left(A+4\,B\,x^3\right)\right)+\right.$$

$$5\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,b\,(11\,A\,b-8\,a\,B)\,x^4\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 243: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{x^8\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 611 leaves, 7 steps) :

$$-\frac{A}{7\,a\,x^7\sqrt{a+b\,x^3}}-\frac{17\,A\,b-14\,a\,B}{21\,a^2\,x^4\sqrt{a+b\,x^3}}+\frac{11\,(17\,A\,b-14\,a\,B)\sqrt{a+b\,x^3}}{168\,a^3\,x^4}-\frac{55\,b\,(17\,A\,b-14\,a\,B)\sqrt{a+b\,x^3}}{336\,a^4\,x}+\frac{55\,b^{4/3}\,(17\,A\,b-14\,a\,B)\sqrt{a+b\,x^3}}{336\,a^4\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(55\sqrt{2-\sqrt{3}}\;b^{4/3}\,(17\,A\,b-14\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(224\times3^{3/4}\,a^{11/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(55\,b^{4/3}\,(17\,A\,b-14\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(168\sqrt{2}\;3^{1/4}\,a^{11/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 292 leaves) :

$$\frac{1}{1008\,a^4\,(-b)^{2/3}\,x^7\sqrt{a+b\,x^3}}\left(-3\,(-b)^{2/3}\left(935\,A\,b^3\,x^9+11\,a\,b^2\,x^6\left(51\,A-70\,B\,x^3\right)+12\,a^3\left(4\,A+7\,B\,x^3\right)-6\,a^2\,b\,x^3\left(17\,A+77\,B\,x^3\right)\right)-\right.$$

$$55\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,b^2\,(17\,A\,b-14\,a\,B)\,x^7\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left.\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 249: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^6\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 299 leaves, 4 steps) :

$$-\frac{2\,(5\,A\,b-14\,a\,B)\,x^4}{45\,b^2\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,B\,x^7}{5\,b\,\left(a+b\,x^3\right)^{3/2}}-\frac{16\,(5\,A\,b-14\,a\,B)\,x}{135\,b^3\,\sqrt{a+b\,x^3}}+$$

$$\left(32\,\sqrt{2+\sqrt{3}}\,(5\,A\,b-14\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(135\times3^{1/4}\,b^{10/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 205 leaves) :

$$-\frac{1}{405\,(-b)^{10/3}\,\left(a+b\,x^3\right)^{3/2}}$$

$$2\left(3\,(-b)^{1/3}\,x\,\left(112\,a^2\,B+b^2\,x^3\,\left(-55\,A+27\,B\,x^3\right)+a\,\left(-40\,A\,b+154\,b\,B\,x^3\right)\right)+16\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,(5\,A\,b-14\,a\,B)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 250: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 283 leaves, 3 steps) :

$$\frac{2\,(A\,b-a\,B)\,x^4}{9\,a\,b\,\left(a+b\,x^3\right)^{3/2}}-\frac{2\,(A\,b+8\,a\,B)\,x}{27\,a\,b^2\,\sqrt{a+b\,x^3}}+$$

$$\left(4\,\sqrt{2+\sqrt{3}}\,\,(A\,b+8\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Bigg/$$

$$\left(27\times3^{1/4}\,a\,b^{7/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\,\right)$$

Result (type 4, 199 leaves) :

$$\frac{1}{81\,a\,(-b)^{7/3}\,\left(a+b\,x^3\right)^{3/2}}$$

$$2\,i\,\left(-3\,i\,(-b)^{1/3}\,x\,\left(-8\,a^2\,B+2\,A\,b^2\,x^3-a\,b\,\left(A+11\,B\,x^3\right)\right)+2\times3^{3/4}\,a^{1/3}\,(A\,b+8\,a\,B)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(a+b\,x^3\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 251: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 283 leaves, 3 steps) :

$$\frac{2\,(A\,b-a\,B)\,x}{9\,a\,b\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,(7\,A\,b+2\,a\,B)\,x}{27\,a^2\,b\,\sqrt{a+b\,x^3}}+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\,\,(7\,A\,b+2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(27\times 3^{1/4}\,a^2\,b^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\,\right)$$

Result (type 4, 199 leaves) :

$$-\frac{1}{81\,a^2\,(-b)^{4/3}\,\left(a+b\,x^3\right)^{3/2}}$$

$$2\left(3\,(-b)^{1/3}\,x\,\left(-a^2\,B+7\,A\,b^2\,x^3+2\,a\,b\,\left(5\,A+B\,x^3\right)\right)+\mathrm{i}\,3^{3/4}\,a^{1/3}\,(7\,A\,b+2\,a\,B)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(a+b\,x^3\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 252: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{x^3\,\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 300 leaves, 4 steps) :

$$-\frac{A}{2\,a\,x^2\,(a+b\,x^3)^{3/2}}-\frac{(13\,A\,b-4\,a\,B)\,x}{18\,a^2\,(a+b\,x^3)^{3/2}}-\frac{7\,(13\,A\,b-4\,a\,B)\,x}{54\,a^3\,\sqrt{a+b\,x^3}}-$$

$$\left(7\,\sqrt{2+\sqrt{3}}\,(13\,A\,b-4\,a\,B)\,(a^{1/3}+b^{1/3}\,x)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(54\times 3^{1/4}\,a^3\,b^{1/3}\,\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 210 leaves) :

$$\frac{-91\,A\,b^2\,x^6+a^2\,(-27\,A+40\,B\,x^3)+a\,(-130\,A\,b\,x^3+28\,b\,B\,x^6)}{54\,a^3\,x^2\,(a+b\,x^3)^{3/2}}+$$

$$\left(7\,\mathrm{i}\,(-13\,A\,b+4\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(54\times 3^{1/4}\,a^{8/3}\,(-b)^{1/3}\,\sqrt{a+b\,x^3}\right)$$

Problem 253: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{x^6\,(a+b\,x^3)^{5/2}}\,dx$$

Optimal (type 4, 334 leaves, 5 steps) :

$$-\frac{A}{5\,a\,x^5\,(a+b\,x^3)^{3/2}}-\frac{19\,A\,b-10\,a\,B}{45\,a^2\,x^2\,(a+b\,x^3)^{3/2}}-\frac{13\,(19\,A\,b-10\,a\,B)}{135\,a^3\,x^2\,\sqrt{a+b\,x^3}}+\frac{91\,(19\,A\,b-10\,a\,B)\,\sqrt{a+b\,x^3}}{540\,a^4\,x^2}+$$

$$\left(91\,\sqrt{2+\sqrt{3}}\,b^{2/3}\,(19\,A\,b-10\,a\,B)\,(a^{1/3}+b^{1/3}\,x)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(540\times 3^{1/4}\,a^4\,\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 228 leaves) :

$$\frac{1}{1620\,a^4\,x^5\,\left(a+b\,x^3\right)^{3/2}}\left(5187\,A\,b^3\,x^9+3\,a^2\,b\,x^3\,\left(513\,A-1300\,B\,x^3\right)+390\,a\,b^2\,x^6\,\left(19\,A-7\,B\,x^3\right)-162\,a^3\,\left(2\,A+5\,B\,x^3\right)-91\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,(-b)^{2/3}\,(19\,A\,b-10\,a\,B)\,x^5\right.$$
$$\left.\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 254: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^7\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 577 leaves, 6 steps) :

$$-\frac{2\,(7\,A\,b-16\,a\,B)\,x^5}{63\,b^2\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,B\,x^8}{7\,b\,\left(a+b\,x^3\right)^{3/2}}-\frac{20\,(7\,A\,b-16\,a\,B)\,x^2}{189\,b^3\,\sqrt{a+b\,x^3}}+\frac{80\,(7\,A\,b-16\,a\,B)\,\sqrt{a+b\,x^3}}{189\,b^{11/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$
$$\left(40\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,(7\,A\,b-16\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$
$$\left(63\times 3^{3/4}\,b^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$
$$\left(80\,\sqrt{2}\,a^{1/3}\,(7\,A\,b-16\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$
$$\left(189\times 3^{1/4}\,b^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 265 leaves) :

$$-\frac{1}{567\,(-b)^{11/3}\,\left(a+b\,x^3\right)^{3/2}}\,2\,\Bigg(3\,(-b)^{2/3}\,x^2\,\left(160\,a^2\,B+b^2\,x^3\,\left(-91\,A+27\,B\,x^3\right)+a\,\left(-70\,A\,b+208\,b\,B\,x^3\right)\right)-$$

$$40\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,(7\,A\,b-16\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)$$

$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Bigg)$$

- **Problem 255: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 559 leaves, 5 steps) :

$$\frac{2\,(A\,b-a\,B)\,x^5}{9\,a\,b\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,(A\,b-10\,a\,B)\,x^2}{27\,a\,b^2\,\sqrt{a+b\,x^3}}-\frac{8\,(A\,b-10\,a\,B)\,\sqrt{a+b\,x^3}}{27\,a\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(4\,\sqrt{2-\sqrt{3}}\,(A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(9\times3^{3/4}\,a^{2/3}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(8\,\sqrt{2}\,(A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(27\times3^{1/4}\,a^{2/3}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 256 leaves) :

$$\frac{1}{81\,a\,(-b)^{8/3}\left(a+b\,x^3\right)^{3/2}}\,2\left(3\,(-b)^{2/3}\,x^2\left(-10\,a^2\,B+4\,A\,b^2\,x^3+a\,b\left(A-13\,B\,x^3\right)\right)+4\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,(A\,b-10\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$
$$\left.\left(a+b\,x^3\right)\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 256: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 563 leaves, 5 steps) :

$$\frac{2\,(A\,b-a\,B)\,x^2}{9\,a\,b\left(a+b\,x^3\right)^{3/2}}+\frac{2\,(5\,A\,b+4\,a\,B)\,x^2}{27\,a^2\,b\sqrt{a+b\,x^3}}-\frac{2\,(5\,A\,b+4\,a\,B)\sqrt{a+b\,x^3}}{27\,a^2\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$
$$\left(\sqrt{2-\sqrt{3}}\,(5\,A\,b+4\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$
$$\left(9\times3^{3/4}\,a^{5/3}\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)-$$
$$\left(2\sqrt{2}\,(5\,A\,b+4\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$
$$\left(27\times3^{1/4}\,a^{5/3}\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 257 leaves):

$$-\frac{1}{81\,a^2\,(-b)^{5/3}\,\left(a+b\,x^3\right)^{3/2}}\,2\left(3\,(-b)^{2/3}\,x^2\,\left(a^2\,B+5\,A\,b^2\,x^3+4\,a\,b\,\left(2\,A+B\,x^3\right)\right)+(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,(5\,A\,b+4\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\left(a+b\,x^3\right)\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 257: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{x^2\,\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 578 leaves, 6 steps):

$$-\frac{A}{a\,x\,\left(a+b\,x^3\right)^{3/2}}-\frac{(11\,A\,b-2\,a\,B)\,x^2}{9\,a^2\,\left(a+b\,x^3\right)^{3/2}}-\frac{5\,(11\,A\,b-2\,a\,B)\,x^2}{27\,a^3\,\sqrt{a+b\,x^3}}+\frac{5\,(11\,A\,b-2\,a\,B)\,\sqrt{a+b\,x^3}}{27\,a^3\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(5\,\sqrt{2-\sqrt{3}}\,(11\,A\,b-2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(18\times 3^{3/4}\,a^{8/3}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(5\,\sqrt{2}\,(11\,A\,b-2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(27\times 3^{1/4}\,a^{8/3}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 273 leaves):

$$\frac{1}{81\,a^3\,\left(a+b\,x^3\right)^{3/2}}\left(-\frac{3\,\left(55\,A\,b^2\,x^6+a^2\,\left(27\,A-16\,B\,x^3\right)+2\,a\,b\,x^3\,\left(44\,A-5\,B\,x^3\right)\right)}{x}+\right.$$

$$1\Big/\left(-b\right)^{2/3}5\,\left(-1\right)^{1/6}\,3^{3/4}\,a^{2/3}\,\left(11\,A\,b-2\,a\,B\right)\,\sqrt{\frac{\left(-1\right)^{5/6}\,\left(-a^{1/3}+\left(-b\right)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)$$

$$\left.\left(-i\,\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]+\left(-1\right)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]\right)\right)$$

Problem 258: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{x^5\,\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 610 leaves, 7 steps):

$$-\frac{A}{4\,a\,x^4\,\left(a+b\,x^3\right)^{3/2}}-\frac{17\,A\,b-8\,a\,B}{36\,a^2\,x\,\left(a+b\,x^3\right)^{3/2}}-\frac{11\,\left(17\,A\,b-8\,a\,B\right)}{108\,a^3\,x\,\sqrt{a+b\,x^3}}+\frac{55\,\left(17\,A\,b-8\,a\,B\right)\,\sqrt{a+b\,x^3}}{216\,a^4\,x}-\frac{55\,b^{1/3}\,\left(17\,A\,b-8\,a\,B\right)\,\sqrt{a+b\,x^3}}{216\,a^4\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(55\,\sqrt{2-\sqrt{3}}\,b^{1/3}\,\left(17\,A\,b-8\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(144\times3^{3/4}\,a^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(55\,b^{1/3}\,\left(17\,A\,b-8\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(108\,\sqrt{2}\,3^{1/4}\,a^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 293 leaves):

$$\frac{1}{648\, a^4 \left(a+b x^3\right)^{3/2}}\left(-\frac{3\left(-935\, A\, b^3 x^9+54\, a^3\left(A+4\, B x^3\right)+88\, a\, b^2 x^6\left(-17\, A+5\, B x^3\right)+a^2\left(-459\, A\, b\, x^3+704\, b\, B\, x^6\right)\right)}{x^4}+\right.$$

$$55\,(-1)^{1/6}\, 3^{3/4}\, a^{2/3}\,(-b)^{1/3}\,(17\, A\, b-8\, a\, B)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\left(a+b x^3\right)$$

$$\left.\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 262: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x\left(4\, c+d x^3\right)}\, dx$$

Optimal (type 3, 65 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\,\sqrt{c}}\right]}{2\sqrt{3}\,\sqrt{c}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{6\sqrt{c}}$$

Result (type 6, 158 leaves):

$$-\left(2\, d\, x^3\sqrt{c+d x^3}\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},-\frac{4\, c}{d x^3}\right]\right)\Big/\left(\left(4\, c+d x^3\right)\right.$$

$$\left.\left(3\, d\, x^3\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},-\frac{4\, c}{d x^3}\right]+c\left(-8\,\text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},-\frac{c}{d x^3},-\frac{4\, c}{d x^3}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{4\, c}{d x^3}\right]\right)\right)\right)$$

- **Problem 263: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x^4\left(4\, c+d x^3\right)}\, dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{12\, c\, x^3}-\frac{d\,\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\,\sqrt{c}}\right]}{8\sqrt{3}\, c^{3/2}}-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{24\, c^{3/2}}$$

Result (type 6, 319 leaves):

$$\frac{1}{36 x^3 \sqrt{c+d x^3}} \left(-3-\frac{3 d x^3}{c}+\left(12 d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right) \Big/ \left(\left(4 c+d x^3\right) \left(-8 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+d x^3 \left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)+\left(10 d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]\right) \Big/ \left(\left(4 c+d x^3\right) \left(-5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]+c \left(8 \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]+\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]\right)\right)\right)\right)$$

Problem 264: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \sqrt{c+d x^3}}{4 c+d x^3} \, dx$$

Optimal (type 4, 689 leaves, 7 steps):

$$\frac{2 x^2 \sqrt{c+d x^3}}{7 d}-\frac{50 c \sqrt{c+d x^3}}{7 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{2\times 2^{1/3} c^{7/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{\sqrt{3} d^{5/3}}+$$

$$\frac{2\times 2^{1/3} c^{7/6} \text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3} \sqrt{c}}\right]}{\sqrt{3} d^{5/3}}-\frac{2\times 2^{1/3} c^{7/6} \text{ArcTanh}\left[\frac{c^{1/6} \left(c^{1/3}-2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{5/3}}+\frac{2\times 2^{1/3} c^{7/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 d^{5/3}}+$$

$$\left(25\times 3^{1/4} \sqrt{2-\sqrt{3}} c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(7 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\frac{50 \sqrt{2} c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{7\times 3^{1/4} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 343 leaves):

$$\frac{1}{7\sqrt{c+d x^3}}2 x^2\left(\frac{c}{d}+x^3+\left(80 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(d\left(4 c+d x^3\right)\left(-20 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)-\left(80 c^2 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(32 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)\right)$$

- **Problem 265: Result unnecessarily involves higher level functions.**

$$\int \frac{x\sqrt{c+d x^3}}{4 c+d x^3}\,dx$$

Optimal (type 4, 659 leaves, 5 steps) :

$$\frac{2\sqrt{c+d x^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{c^{1/6} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{2^{2/3}\sqrt{3}\, d^{2/3}}-\frac{c^{1/6} \text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\sqrt{c}}\right]}{2^{2/3}\sqrt{3}\, d^{2/3}}+\frac{c^{1/6} \text{ArcTanh}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{2^{2/3} d^{2/3}}-\frac{c^{1/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3\times 2^{2/3} d^{2/3}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\, c^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)+\frac{2\sqrt{2}\, c^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{3^{1/4} d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 167 leaves) :

$$\left(10 c x^2\sqrt{c+d x^3}\, \text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(20 c \text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-2 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

Problem 266: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x^2 \left(4 c+d x^3\right)} \, dx$$

Optimal (type 4, 697 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{4 c x}+\frac{d^{1/3} \sqrt{c+d x^3}}{4 c \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{4\times 2^{2/3} \sqrt{3} c^{5/6}}+\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3} \sqrt{c}}\right]}{4\times 2^{2/3} \sqrt{3} c^{5/6}}-\frac{d^{1/3} \text{ArcTanh}\left[\frac{c^{1/6} \left(c^{1/3}-2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{4\times 2^{2/3} c^{5/6}}+$$

$$\frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{12\times 2^{2/3} c^{5/6}}-\left(3^{1/4} \sqrt{2-\sqrt{3}}\, d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(8 c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\frac{d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{2 \sqrt{2}\, 3^{1/4} c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 344 leaves):

$$\frac{1}{20 x \sqrt{c+d x^3}}\left(-5-\frac{5 d x^3}{c}+\left(250 c d x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\right.\right.$$

$$\left(20 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\Big)+$$

$$\left(16 d^2 x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right) \left(32 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-\right.\right.$$

$$\left.\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)\right)$$

Problem 267: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 \sqrt{c+d x^3}}{4 c+d x^3} \, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4 \sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{16 c \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 344 leaves):

$$\frac{1}{5 \sqrt{c+d x^3}} x \left(2 \left(\frac{c}{d}+x^3\right)+\left(128 c^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(d \left(4 c+d x^3\right) \left(-16 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)-\left(119 c^2 x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right) \left(28 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)\right)$$

- **Problem 268: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{4 c+d x^3}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x \sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{4 c \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 165 leaves):

$$\left(16 c x \sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right) \left(16 c \text{AppellF1}\left[\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-2 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

- **Problem 269: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x^3 \left(4 c+d x^3\right)}\, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{8 c x^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 344 leaves) :

$$\frac{1}{16 x^2 \sqrt{c+d x^3}}\left(-2-\frac{2 d x^3}{c}+\left(128 c d x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(16 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)+\left(7 d^2 x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(-28 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)\right)$$

- **Problem 273: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x \sqrt{c+d x^3}\ \left(4 c+d x^3\right)}\, dx$$

Optimal (type 3, 65 leaves, 6 steps) :

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\ \sqrt{c}}\right]}{6 \sqrt{3}\ c^{3/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{6 c^{3/2}}$$

Result (type 6, 160 leaves) :

$$\left(10 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]\right)\Big/\left(9 \sqrt{c+d x^3}\ \left(4 c+d x^3\right)\left(-5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]+c\left(8 \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]+\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 274: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4 \sqrt{c+d x^3}\ \left(4 c+d x^3\right)}\, dx$$

Optimal (type 3, 88 leaves, 7 steps) :

$$-\frac{\sqrt{c+d x^3}}{12 c^2 x^3}+\frac{d \text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\ \sqrt{c}}\right]}{24 \sqrt{3}\ c^{5/2}}+\frac{d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{8 c^{5/2}}$$

Result (type 6, 324 leaves) :

$$\frac{1}{12\,c^2\,x^3\,\sqrt{c+d\,x^3}}\left(-c-d\,x^3-\left(4\,c\,d^2\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(8\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)-\left(10\,c\,d^2\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(-5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]+8\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]+c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]\right)\right)\right)$$

Problem 275: Result unnecessarily involves higher level functions.

$$\int\frac{x^4}{\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)}\,dx$$

Optimal (type 4, 667 leaves, 5 steps) :

$$\frac{2\,\sqrt{c+d\,x^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}+\frac{2\times 2^{1/3}\,c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+2^{1/3}\,d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{3\,\sqrt{3}\,d^{5/3}}-\frac{2\times 2^{1/3}\,c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{3}\,\sqrt{c}}\right]}{3\,\sqrt{3}\,d^{5/3}}+\frac{2\times 2^{1/3}\,c^{1/6}\,\text{ArcTanh}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3}\,d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{3\,d^{5/3}}-\frac{2\times 2^{1/3}\,c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{9\,d^{5/3}}-\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(d^{5/3}\,\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)+\frac{2\,\sqrt{2}\,c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]}{3^{1/4}\,d^{5/3}\,\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}}$$

Result (type 6, 169 leaves) :

$$\left(32\,c\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(5\,\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)\right.$$
$$\left.\left(32\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)$$

- **Problem 276: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)}\,dx$$

Optimal (type 3, 206 leaves, 1 step) :

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+2^{1/3}\,d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{3\times2^{2/3}\,\sqrt{3}\,c^{5/6}\,d^{2/3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{3}\,\sqrt{c}}\right]}{3\times2^{2/3}\,\sqrt{3}\,c^{5/6}\,d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{c^{1/6}\,\left(c^{1/3}-2^{1/3}\,d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{3\times2^{2/3}\,c^{5/6}\,d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{9\times2^{2/3}\,c^{5/6}\,d^{2/3}}$$

Result (type 6, 167 leaves) :

$$\left(10\,c\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)\right.$$
$$\left.\left(20\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)$$

- **Problem 277: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^2\,\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)}\,dx$$

Optimal (type 4, 697 leaves, 7 steps) :

$$-\frac{\sqrt{c+d x^3}}{4 c^2 x}+\frac{d^{1/3}\sqrt{c+d x^3}}{4 c^2\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+2^{1/3}d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{12\times 2^{2/3}\sqrt{3}\,c^{11/6}}-\frac{d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\sqrt{c}}\right]}{12\times 2^{2/3}\sqrt{3}\,c^{11/6}}+\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3}d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{12\times 2^{2/3}c^{11/6}}-$$

$$\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{36\times 2^{2/3}c^{11/6}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\;d^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(8\,c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\frac{d^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{2\sqrt{2}\,3^{1/4}c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 348 leaves):

$$\frac{1}{20\,x\sqrt{c+d x^3}}\left(\left(50\,d\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\right.\right.$$

$$\left.\left(20\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)+$$

$$1\Big/c^2\left(-5\left(c+d x^3\right)+\left(16\,c\,d^2x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(32\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-\right.\right.\right.$$

$$\left.\left.\left.\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)\right)\right)$$

- **Problem 278: Result more than twice size of optimal antiderivative.**

$$\int\frac{x^3}{\sqrt{c+d x^3}\left(4 c+d x^3\right)}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4\sqrt{1+\frac{d x^3}{c}}\;\text{AppellF1}\left[\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{16\,c\sqrt{c+d x^3}}$$

Result (type 6, 167 leaves):

$$\left(7\,c\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\bigg/\left(\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)\right.$$
$$\left.\left(28\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)$$

- **Problem 279: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{d\,x^3}{4\,c},-\frac{d\,x^3}{c}\right]}{4\,c\,\sqrt{c+d\,x^3}}$$

Result (type 6, 165 leaves):

$$\left(16\,c\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\bigg/\left(\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)\right.$$
$$\left.\left(16\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)$$

- **Problem 280: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^3\,\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{d\,x^3}{4\,c},-\frac{d\,x^3}{c}\right]}{8\,c\,x^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 348 leaves):

$$\frac{1}{16\,x^2\sqrt{c+d\,x^3}}\left(\left(128\,d\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(-16\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)+1\Big/c^2\left(-2\left(c+d\,x^3\right)-\left(7\,c\,d^2\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(28\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)\right)\right)$$

- **Problem 281: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\sqrt{1-x^3}\,\left(4-x^3\right)}\,dx$$

Optimal (type 3, 127 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,\left(1-2^{1/3}\,x\right)}{\sqrt{1-x^3}}\right]}{3\times 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{1-x^3}}{\sqrt{3}}\right]}{3\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{1+2^{1/3}\,x}{\sqrt{1-x^3}}\right]}{3\times 2^{2/3}}+\frac{\text{ArcTanh}\left[\sqrt{1-x^3}\right]}{9\times 2^{2/3}}$$

Result (type 6, 120 leaves):

$$-\left(10\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]\right)\Big/\left(\sqrt{1-x^3}\,\left(-4+x^3\right)\left(20\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]+3\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,\frac{x^3}{4}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,\frac{x^3}{4}\right]\right)\right)\right)$$

- **Problem 286: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{c+d\,x^3}}{x\,\left(8\,c-d\,x^3\right)}\,dx$$

Optimal (type 3, 58 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{4\,\sqrt{c}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{12\,\sqrt{c}}$$

Result (type 6, 158 leaves):

$$\left(2\,d\,x^3\sqrt{c+d\,x^3}\ \text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(\left(-8\,c+d\,x^3\right)\left(3\,d\,x^3\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+c\left(16\,\text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 287: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{c+d\,x^3}}{x^4\left(8\,c-d\,x^3\right)}\,dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{\sqrt{c+d\,x^3}}{24\,c\,x^3}+\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{32\,c^{3/2}}-\frac{5\,d\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{96\,c^{3/2}}$$

Result (type 6, 321 leaves):

$$\frac{1}{72\,x^3\sqrt{c+d\,x^3}}\left(-3-\frac{3\,d\,x^3}{c}+\left(24\,d^2\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\left(50\,d^2\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(\left(-8\,c+d\,x^3\right)\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 288: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{c+d\,x^3}}{x^7\left(8\,c-d\,x^3\right)}\,dx$$

Optimal (type 3, 107 leaves, 8 steps):

$$-\frac{\sqrt{c+d\,x^3}}{48\,c\,x^6}-\frac{d\sqrt{c+d\,x^3}}{64\,c^2\,x^3}+\frac{d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{256\,c^{5/2}}+\frac{d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{256\,c^{5/2}}$$

Result (type 6, 341 leaves):

$$\frac{1}{96\sqrt{c+d x^3}}\left(-\frac{3 d^2}{2 c^2}-\frac{2}{x^6}-\frac{7 d}{2 c x^3}+\left(12 d^3 x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\right.\right.$$

$$\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\Big)+$$

$$\left(5 d^3 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\right.$$

$$\left.\left.\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 289: Result unnecessarily involves higher level functions.**

$$\int\frac{x^7\sqrt{c+d x^3}}{8 c-d x^3}\,dx$$

Optimal (type 4, 648 leaves, 15 steps):

$$-\frac{214 c x^2\sqrt{c+d x^3}}{91 d^2}-\frac{2 x^5\sqrt{c+d x^3}}{13 d}-\frac{12248 c^2\sqrt{c+d x^3}}{91 d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-$$

$$\frac{32\sqrt{3}\,c^{13/6}\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{d^{8/3}}+\frac{32 c^{13/6}\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{d^{8/3}}-\frac{32 c^{13/6}\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{d^{8/3}}+$$

$$\left(6124\times 3^{1/4}\sqrt{2-\sqrt{3}}\,c^{7/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91 d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)-$$

$$\frac{12248\sqrt{2}\,c^{7/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{91\times 3^{1/4}d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 361 leaves):

$$\frac{1}{455\, d^2 \sqrt{c+d x^3}} 2 x^2 \left(-5 \left(107 c^2+114 c d x^3+7 d^2 x^6\right)+\left(171200 c^4 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(195968 c^3 d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 290: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4 \sqrt{c+d x^3}}{8 c-d x^3} \, dx$$

Optimal (type 4, 624 leaves, 14 steps):

$$-\frac{2 x^2 \sqrt{c+d x^3}}{7 d}-\frac{118 c \sqrt{c+d x^3}}{7 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{4 \sqrt{3}\, c^{7/6} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{5/3}}+\frac{4 c^{7/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{d^{5/3}}-\frac{4 c^{7/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{d^{5/3}}+$$

$$\left(59 \times 3^{1/4} \sqrt{2-\sqrt{3}}\, c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\frac{118 \sqrt{2}\, c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{7 \times 3^{1/4} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 349 leaves):

$$\frac{1}{35\sqrt{c+d x^3}}2 x^2 \left(-\frac{5\left(c+d x^3\right)}{d}+\left(1600 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d\left(8 c-d x^3\right)\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(1888 c^2 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 291: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{c+d x^3}}{8 c-d x^3}\,dx$$

Optimal (type 4, 601 leaves, 12 steps):

$$-\frac{2\sqrt{c+d x^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{\sqrt{3}\,c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{2 d^{2/3}}+\frac{c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{2 d^{2/3}}-\frac{c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{2 d^{2/3}}+\left(3^{1/4}\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)-\frac{2\sqrt{2}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{3^{1/4}d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 168 leaves):

$$\left(20 c x^2\sqrt{c+d x^3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+4\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)$$

- **Problem 292: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c+d x^3}}{x^2 \left(8 c-d x^3\right)} \, dx$$

Optimal (type 4, 632 leaves, 14 steps):

$$-\frac{\sqrt{c+d x^3}}{8 c x}+\frac{d^{1/3} \sqrt{c+d x^3}}{8 c \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\sqrt{3}\, d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{16 c^{5/6}}+\frac{d^{1/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{16 c^{5/6}}-\frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{16 c^{5/6}}-$$
$$\left(3^{1/4} \sqrt{2-\sqrt{3}}\, d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$
$$\left(16 c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\frac{d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{4 \sqrt{2}\, 3^{1/4} c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 345 leaves):

$$\frac{1}{40 x \sqrt{c+d x^3}}\left(-5-\frac{5 d x^3}{c}+\left(1300 c d x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(32 d^2 x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(-8 c+d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 293: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c+d x^3}}{x^5 \left(8 c-d x^3\right)} \, dx$$

Optimal (type 4, 654 leaves, 15 steps):

$$-\frac{\sqrt{c+d x^3}}{32 c x^4}-\frac{d \sqrt{c+d x^3}}{16 c^2 x}+\frac{d^{4/3} \sqrt{c+d x^3}}{16 c^2 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\sqrt{3}\, d^{4/3}\, \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{128 c^{11/6}}+\frac{d^{4/3}\, \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{128 c^{11/6}}-$$

$$\frac{d^{4/3}\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{128 c^{11/6}}-\left(3^{1/4} \sqrt{2-\sqrt{3}}\, d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(32 c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\frac{d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{8 \sqrt{2}\, 3^{1/4} c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 367 leaves) :

$$\frac{1}{80 \sqrt{c+d x^3}}\left(\left(625 d^2 x^2\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$

$$\left.\left(40 c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$

$$1\Big/\left(2 c^2 x^4\right) \left(5 \left(c^2+3 c d x^3+2 d^2 x^6\right)+\left(64 c d^3 x^9\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(64 c\right.\right.\right.$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

- **Problem 294: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c+d x^3}}{x^8 \left(8 c-d x^3\right)}\, dx$$

Optimal (type 4, 678 leaves, 16 steps) :

$$-\frac{\sqrt{c+d x^3}}{56 c x^7}-\frac{19 d \sqrt{c+d x^3}}{1792 c^2 x^4}+\frac{d^2 \sqrt{c+d x^3}}{112 c^3 x}-\frac{d^{7/3} \sqrt{c+d x^3}}{112 c^3 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\sqrt{3}\, d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{1024 c^{17/6}}+\frac{d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{1024 c^{17/6}}-$$

$$\frac{d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{1024 c^{17/6}}+\left(3^{1/4} \sqrt{2-\sqrt{3}}\, d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(224 c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\frac{d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]}{56 \sqrt{2}\, 3^{1/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 378 leaves) :

$$\frac{1}{8960 c^3 x^7 \sqrt{c+d x^3}}\left(-5 \left(32 c^3+51 c^2 d x^3+3 c d^2 x^6-16 d^3 x^9\right)-\left(3250 c^2 d^3 x^9 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.\right.$$

$$\left.\left(40 c\, \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)+$$

$$\left(512 c d^4 x^{12} \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.$$

$$\left.\left.\left(64 c\, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 299: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x \left(8 c-d x^3\right)} \, dx$$

Optimal (type 3, 73 leaves, 7 steps) :

$$-\frac{2}{3} \sqrt{c+d x^3}+\frac{9}{4} \sqrt{c}\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]-\frac{1}{12} \sqrt{c}\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]$$

Result (type 6, 319 leaves) :

$$\frac{1}{9\sqrt{c+d x^3}}2\left(-3\left(c+d x^3\right)+\left(240 c^2 d x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(5 c^2 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

■ **Problem 300: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^4\left(8 c-d x^3\right)}\,dx$$

Optimal (type 3, 78 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{24 x^3}+\frac{9 d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{32\sqrt{c}}-\frac{13 d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96\sqrt{c}}$$

Result (type 6, 322 leaves):

$$\frac{1}{72 x^3\sqrt{c+d x^3}}\left(-3\left(c+d x^3\right)+\left(408 c d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(130 c d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

■ **Problem 301: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^7\left(8 c-d x^3\right)}\,dx$$

Optimal (type 3, 104 leaves, 8 steps):

$$-\frac{\sqrt{c+d x^3}}{48 x^6}-\frac{11 d\sqrt{c+d x^3}}{192 c x^3}+\frac{9 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{256 c^{3/2}}-\frac{37 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{768 c^{3/2}}$$

Result (type 6, 332 leaves):

$$\frac{1}{288\sqrt{c+d x^3}}\left(-\frac{33 d^2}{2 c}-\frac{6 c}{x^6}-\frac{45 d}{2 x^3}+\left(132 d^3 x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(185 d^3 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\right.$$
$$\left.\left.\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{x^7\left(c+d x^3\right)^{3/2}}{8 c-d x^3}\,dx$$

Optimal (type 4, 669 leaves, 16 steps):

$$-\frac{36534 c^2 x^2\sqrt{c+d x^3}}{1729 d^2}-\frac{348 c x^5\sqrt{c+d x^3}}{247 d}-\frac{2}{19}x^8\sqrt{c+d x^3}-\frac{2094648 c^3\sqrt{c+d x^3}}{1729 d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-$$
$$\frac{288\sqrt{3}\,c^{19/6}\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{d^{8/3}}+\frac{288 c^{19/6}\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{d^{8/3}}-\frac{288 c^{19/6}\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{d^{8/3}}+$$
$$\left(1047324\times 3^{1/4}\sqrt{2-\sqrt{3}}\,c^{10/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(1729 d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)-$$
$$\left(698216\sqrt{2}\,3^{3/4}c^{10/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(1729 d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 371 leaves):

$$\frac{1}{8645\sqrt{c+d x^3}}2 x^2 \left(-\frac{91335 c^3}{d^2}-\frac{97425 c^2 x^3}{d}-6545 c x^6-455 d x^9+\left(29227200 c^5 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d^2 \left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(33514368 c^4 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d \left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 303: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$$

Optimal (type 4, 645 leaves, 15 steps):

$$-\frac{240 c x^2 \sqrt{c+d x^3}}{91 d}-\frac{2}{13} x^5 \sqrt{c+d x^3}-\frac{13782 c^2 \sqrt{c+d x^3}}{91 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{36 \sqrt{3} c^{13/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{5/3}}+\frac{36 c^{13/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{d^{5/3}}-\frac{36 c^{13/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{d^{5/3}}+\left(6891 \times 3^{1/4} \sqrt{2-\sqrt{3}}\, c^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(91 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(4594 \sqrt{2}\, 3^{3/4} c^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(91 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 357 leaves):

$$\frac{1}{455\sqrt{c+d x^3}} 2 x^2 \left(-\frac{600 c^2}{d} - 635 c x^3 - 35 d x^6 + \left(192000 c^4 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(d \left(8 c - d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right) +$$
$$\left(220512 c^3 x^3 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 304: Result unnecessarily involves higher level functions.**

$$\int \frac{x \left(c + d x^3\right)^{3/2}}{8 c - d x^3} \, dx$$

Optimal (type 4, 627 leaves, 14 steps):

$$-\frac{2}{7} x^2 \sqrt{c+d x^3} - \frac{132 c \sqrt{c+d x^3}}{7 d^{2/3} \left(\left(1+\sqrt{3}\right) c^{1/3} + d^{1/3} x\right)} - \frac{9 \sqrt{3} c^{7/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{2 d^{2/3}} + \frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{2 d^{2/3}} - \frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{2 d^{2/3}} +$$
$$\left(66 \times 3^{1/4} \sqrt{2-\sqrt{3}} c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3} + d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3} + d^{1/3} x}\right], -7-4\sqrt{3}\right]\right) \Big/$$
$$\left(7 d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right) -$$
$$\frac{44 \sqrt{2}\, 3^{3/4} c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4\sqrt{3}\right]}{7 d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 344 leaves):

$$\frac{1}{35\sqrt{c+d x^3}}2 x^2 \left(-5 \left(c+d x^3\right)+\left(1950 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(2112 c^2 d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 305: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^2 \left(8 c-d x^3\right)} \, dx$$

Optimal (type 4, 626 leaves, 14 steps):

$$-\frac{\sqrt{c+d x^3}}{8 x}-\frac{15 d^{1/3} \sqrt{c+d x^3}}{8 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{9}{16} \sqrt{3} c^{1/6} d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]+$$
$$\frac{9}{16} c^{1/6} d^{1/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]-\frac{9}{16} c^{1/6} d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]+$$
$$\left(15\times 3^{1/4} \sqrt{2-\sqrt{3}} c^{1/3} d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(16 \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\frac{5\times 3^{3/4} c^{1/3} d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{4 \sqrt{2} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 348 leaves):

$$\frac{1}{8x\sqrt{c+dx^3}}\left(-c-dx^3+\left(420c^2dx^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(\left(8c-dx^3\right)\left(40c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)+\left(96cd^2x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(\left(8c-dx^3\right)\left(64c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)\right)$$

- **Problem 306: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)}\,dx$$

Optimal (type 4, 651 leaves, 15 steps):

$$-\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{9\sqrt{3}\,d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+dx^3}}\right]}{128c^{5/6}}+\frac{9d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3c^{1/6}\sqrt{c+dx^3}}\right]}{128c^{5/6}}-\frac{9d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right]}{128c^{5/6}}-\left(3\times3^{1/4}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(32c^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}\right)+\frac{3^{3/4}d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{8\sqrt{2}\,c^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}}$$

Result (type 6, 363 leaves):

$$\frac{1}{80\sqrt{c+d x^3}}\left(-\frac{5\left(c^2+7 c d x^3+6 d^2 x^6\right)}{2 c x^4}+\left(3225 c d^2 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(96 d^3 x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 307: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^8\left(8 c-d x^3\right)}\,dx$$

Optimal (type 4, 675 leaves, 16 steps):

$$-\frac{\sqrt{c+d x^3}}{56 x^7}-\frac{75 d\sqrt{c+d x^3}}{1792 c x^4}-\frac{3 d^2\sqrt{c+d x^3}}{56 c^2 x}+\frac{3 d^{7/3}\sqrt{c+d x^3}}{56 c^2\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)}-$$
$$\frac{9\sqrt{3}\, d^{7/3}\text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{1024 c^{11/6}}+\frac{9 d^{7/3}\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{1024 c^{11/6}}-\frac{9 d^{7/3}\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{1024 c^{11/6}}-$$
$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\, d^{7/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(112 c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)+\frac{3^{3/4} d^{7/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{28\sqrt{2}\, c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 379 leaves):

$$\frac{1}{4480\sqrt{c+d x^3}}\left(-\frac{5\left(32 c^3+107 c^2 d x^3+171 c d^2 x^6+96 d^3 x^9\right)}{2 c^2 x^7}+\left(33375 d^3 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(1536 d^4 x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\left(64 c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 312: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\left(8 c-d x^3\right)\sqrt{c+d x^3}}\,dx$$

Optimal (type 3, 58 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{36 c^{3/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{12 c^{3/2}}$$

Result (type 6, 161 leaves):

$$\left(10 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(9\left(-8 c+d x^3\right)\sqrt{c+d x^3}\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\, \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\, \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)$$

- **Problem 313: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4\left(8 c-d x^3\right)\sqrt{c+d x^3}}\,dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{24 c^2 x^3}+\frac{d\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{288 c^{5/2}}+\frac{d\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{32 c^{5/2}}$$

Result (type 6, 326 leaves):

$$\frac{1}{24\,c^2\,x^3\,\sqrt{c+d\,x^3}}\left(-c-d\,x^3+\left(8\,c\,d^2\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\left(10\,c\,d^2\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 314: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{1}{x^7\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 3, 107 leaves, 8 steps):

$$-\frac{\sqrt{c+d\,x^3}}{48\,c^2\,x^6}+\frac{5\,d\,\sqrt{c+d\,x^3}}{192\,c^3\,x^3}+\frac{d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{2304\,c^{7/2}}-\frac{7\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{256\,c^{7/2}}$$

Result (type 6, 332 leaves):

$$\frac{1}{192\,c^3\,\sqrt{c+d\,x^3}}\left(5\,d^2-\frac{4\,c^2}{x^6}+\frac{c\,d}{x^3}-\left(40\,c\,d^3\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\left(70\,c\,d^3\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(\left(-8\,c+d\,x^3\right)\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 315: Result unnecessarily involves higher level functions.**

$$\int\frac{x^7}{\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 4, 630 leaves, 14 steps):

$$-\frac{2x^2\sqrt{c+dx^3}}{7d^2}-\frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{32c^{7/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+dx^3}}\right]}{3\sqrt{3}\,d^{8/3}}+\frac{32c^{7/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3c^{1/6}\sqrt{c+dx^3}}\right]}{9d^{8/3}}-\frac{32c^{7/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right]}{9d^{8/3}}+$$

$$\left(52\times3^{1/4}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}\right)-\frac{104\sqrt{2}\,c^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{7\times3^{1/4}d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}}$$

Result (type 6, 347 leaves) :

$$\frac{1}{35d^2\sqrt{c+dx^3}}2x^2\left(-5\left(c+dx^3\right)+\left(1600c^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(\left(8c-dx^3\right)\right.\right.$$

$$\left.\left(40c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)+$$

$$\left(1664c^2dx^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(\left(8c-dx^3\right)\right.$$

$$\left.\left.\left(64c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)\right)$$

- **Problem 316: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

Optimal (type 4, 601 leaves, 12 steps) :

$$-\frac{2\sqrt{c+d x^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{4 c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{3\sqrt{3}\,d^{5/3}}+\frac{4 c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{9 d^{5/3}}-\frac{4 c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{9 d^{5/3}}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)-\frac{2\sqrt{2}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{3^{1/4}d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 170 leaves) :

$$\left(64\,c\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(5\left(8 c-d x^3\right)\sqrt{c+d x^3}\right.$$
$$\left.\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)$$

- **Problem 317: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\left(8 c-d x^3\right)\sqrt{c+d x^3}}\,dx$$

Optimal (type 3, 141 leaves, 8 steps) :

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{18 c^{5/6}d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{18 c^{5/6}d^{2/3}}$$

Result (type 6, 168 leaves) :

$$\left(20\,c\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\sqrt{c+d x^3}\right.$$
$$\left.\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)$$

Problem 318: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(8 c-d x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 4, 632 leaves, 14 steps):

$$-\frac{\sqrt{c+d x^3}}{8 c^2 x}+\frac{d^{1/3} \sqrt{c+d x^3}}{8 c^2 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{48 \sqrt{3} c^{11/6}}+\frac{d^{1/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{144 c^{11/6}}-\frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{144 c^{11/6}}-$$
$$\left(3^{1/4} \sqrt{2-\sqrt{3}} d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$
$$\left(16 c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\frac{d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]}{4 \sqrt{2} 3^{1/4} c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 350 leaves):

$$\frac{1}{40 x \sqrt{c+d x^3}}\left(\left(500 d x^3 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$1 / c^2 \left(-5 \left(c+d x^3\right)-\left(32 c d^2 x^6 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+\right.\right.\right.$$
$$\left.\left.\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

Problem 319: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(8 c-d x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 4, 654 leaves, 15 steps):

$$-\frac{\sqrt{c+d x^3}}{32 c^2 x^4}+\frac{d \sqrt{c+d x^3}}{16 c^3 x}-\frac{d^{4/3} \sqrt{c+d x^3}}{16 c^3 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{d^{4/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{384 \sqrt{3}\, c^{17/6}}+\frac{d^{4/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{1152 c^{17/6}}-\frac{d^{4/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{1152 c^{17/6}}+$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}}\; d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(32 c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \sqrt{c+d x^3}\right)-\frac{d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]}{8 \sqrt{2}\; 3^{1/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \sqrt{c+d x^3}}$$

Result (type 6, 364 leaves) :

$$\frac{1}{160 c^3 x^4 \sqrt{c+d x^3}} \left(-5 c^2+5 c d x^3+10 d^2 x^6-\left(750 c^2 d^2 x^6 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.\right.$$

$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)+$$

$$\left(64 c d^3 x^9 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.$$

$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 320: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^8 \left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 4, 678 leaves, 16 steps) :

$$-\frac{\sqrt{c+d x^3}}{56 c^2 x^7}+\frac{37 d \sqrt{c+d x^3}}{1792 c^3 x^4}-\frac{3 d^2 \sqrt{c+d x^3}}{56 c^4 x}+\frac{3 d^{7/3} \sqrt{c+d x^3}}{56 c^4 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-$$

$$\frac{d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3072 \sqrt{3} c^{23/6}}+\frac{d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{9216 c^{23/6}}-\frac{d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{9216 c^{23/6}}-$$

$$\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\, d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(112 c^{11/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\frac{3^{3/4} d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{28 \sqrt{2} c^{11/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 378 leaves) :

$$\frac{1}{8960 c^4 x^7 \sqrt{c+d x^3}}\left(-5 \left(32 c^3-5 c^2 d x^3+59 c d^2 x^6+96 d^3 x^9\right)+\left(38750 c^2 d^3 x^9 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$

$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$

$$\left(3072 c d^4 x^{12} \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$

$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 321: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(8 c-d x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 66 leaves, 2 steps) :

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}}\, \text{AppellF1}\left[\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{32 c \sqrt{c+d x^3}}$$

Result (type 6, 168 leaves):

$$\left(14\,c\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}\right.$$
$$\left.\left(56\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)$$

Problem 322: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{d\,x^3}{8\,c},-\frac{d\,x^3}{c}\right]}{8\,c\,\sqrt{c+d\,x^3}}$$

Result (type 6, 166 leaves):

$$\left(32\,c\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}\right.$$
$$\left.\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)$$

Problem 323: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^3\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{d\,x^3}{8\,c},-\frac{d\,x^3}{c}\right]}{16\,c\,x^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 347 leaves):

$$\frac{1}{16 x^2 \sqrt{c+d x^3}}\left(\left(64 d x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\right.\right.$$
$$\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\Big)-$$
$$1\Big/c^2\left(c+d x^3-\left(7 c d^2 x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.\right.$$
$$\left.\left.\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

- **Problem 324: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^6 \left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{5}{3},1,\frac{1}{2},-\frac{2}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{40 c x^5 \sqrt{c+d x^3}}$$

Result (type 6, 364 leaves):

$$\frac{1}{640 c^3 x^5 \sqrt{c+d x^3}}\left(-16 c^2+7 c d x^3+23 d^2 x^6+\left(3264 c^2 d^2 x^6 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$
$$\left(161 c d^3 x^9 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$
$$\left.\left.\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 329: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 3, 76 leaves, 7 steps):

$$\frac{2}{27 c^2 \sqrt{c+d x^3}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{324 c^{5/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{12 c^{5/2}}$$

Result (type 6, 310 leaves):

$$\frac{1}{27 c^2 \sqrt{c+d x^3}} 2 \left(1-\left(8 c d x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3 \left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(15 c d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\right.$$
$$\left.\left.\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 330: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4 \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 100 leaves, 8 steps):

$$-\frac{25 d}{216 c^3 \sqrt{c+d x^3}}-\frac{1}{24 c^2 x^3 \sqrt{c+d x^3}}+\frac{d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{2592 c^{7/2}}+\frac{11 d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 c^{7/2}}$$

Result (type 6, 326 leaves):

$$\frac{1}{216 c^3 x^3 \sqrt{c+d x^3}} \left(-9 c-25 d x^3+\left(200 c d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(\left(8 c-d x^3\right) \left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3 \left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(330 c d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$
$$\left.\left.\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 331: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^7 \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 128 leaves, 9 steps) :

$$\frac{245\,d^2}{1728\,c^4\,\sqrt{c+d\,x^3}}-\frac{1}{48\,c^2\,x^6\,\sqrt{c+d\,x^3}}+\frac{3\,d}{64\,c^3\,x^3\,\sqrt{c+d\,x^3}}+\frac{d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{20\,736\,c^{9/2}}-\frac{109\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{768\,c^{9/2}}$$

Result (type 6, 336 leaves) :

$$\frac{1}{1728\,c^4\,x^6\,\sqrt{c+d\,x^3}}\left(-36\,c^2+81\,c\,d\,x^3+245\,d^2\,x^6-\left(1960\,c\,d^3\,x^9\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.$$
$$\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+$$
$$\left(3270\,c\,d^3\,x^9\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(\left(-8\,c+d\,x^3\right)\right.$$
$$\left.\left.\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 332: Result unnecessarily involves higher level functions.**

$$\int\frac{x^7}{\left(8\,c-d\,x^3\right)\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 629 leaves, 14 steps) :

$$\frac{2x^2}{27 d^2\sqrt{c+d x^3}}-\frac{56\sqrt{c+d x^3}}{27 d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{32 c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{27\sqrt{3}\,d^{8/3}}+\frac{32 c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{81 d^{8/3}}-\frac{32 c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{81 d^{8/3}}+$$

$$\left(28\sqrt{2-\sqrt{3}}\;c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(9\times 3^{3/4}d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\sqrt{c+d x^3}\right)-$$

$$\frac{56\sqrt{2}\;c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{27\times 3^{1/4}d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\sqrt{c+d x^3}}$$

Result (type 6, 337 leaves) :

$$\frac{1}{135 d^2\sqrt{c+d x^3}}2x^2\left(5-\left(1600 c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8c-d x^3\right)\right.\right.$$

$$\left.\left.\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$

$$\left(896 c d x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8c-d x^3\right)\right.$$

$$\left.\left.\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 333: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4}{\left(8c-d x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 635 leaves, 14 steps) :

$$-\frac{2x^2}{27cd\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{4\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+dx^3}}\right]}{27\sqrt{3}\,c^{5/6}d^{5/3}}+\frac{4\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3c^{1/6}\sqrt{c+dx^3}}\right]}{81c^{5/6}d^{5/3}}-$$

$$\frac{4\,\text{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right]}{81c^{5/6}d^{5/3}}-\frac{\sqrt{2-\sqrt{3}}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{9\times 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}}+$$

$$\frac{2\sqrt{2}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{27\times 3^{1/4}c^{2/3}d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}}$$

Result (type 6, 340 leaves) :

$$\frac{1}{135\sqrt{c+dx^3}}2x^2\left(-\frac{5}{cd}+\left(1600\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(d\left(8c-dx^3\right)\right.\right.$$
$$\left.\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)-$$
$$\left(32x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(\left(8c-dx^3\right)\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+\right.\right.$$
$$\left.\left.\left.3dx^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)\right)$$

- **Problem 334: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 632 leaves, 14 steps) :

$$\frac{2 x^2}{27 c^2 \sqrt{c+d x^3}}-\frac{2 \sqrt{c+d x^3}}{27 c^2 d^{2/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{54 \sqrt{3} c^{11/6} d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{162 c^{11/6} d^{2/3}}-$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{162 c^{11/6} d^{2/3}}+\frac{\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{9\times 3^{3/4} c^{5/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}-$$

$$\frac{2 \sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{27\times 3^{1/4} c^{5/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 336 leaves) :

$$\frac{1}{135 \sqrt{c+d x^3}} 2 x^2 \left(-\left(250 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.\right.$$

$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$

$$\left.\frac{5+\frac{32 c d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]}{\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)}}{c^2}\right)$$

- **Problem 335: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 653 leaves, 15 steps) :

$$\frac{2}{27 c^2 x \sqrt{c+d x^3}}-\frac{43 \sqrt{c+d x^3}}{216 c^3 x}+\frac{43 d^{1/3} \sqrt{c+d x^3}}{216 c^3 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{432 \sqrt{3} c^{17/6}}+\frac{d^{1/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{1296 c^{17/6}}-$$

$$\frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{1296 c^{17/6}}-\left(43 \sqrt{2-\sqrt{3}}\, d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(144\times 3^{3/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+$$

$$\frac{43 d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{108 \sqrt{2}\, 3^{1/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 356 leaves) :

$$\frac{1}{270 \sqrt{c+d x^3}}\left(\left(4375 d x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c \left(8 c-d x^3\right)\right.\right.$$

$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$

$$1/\left(4 c^3 x\right) \left(135 c+215 d x^3+\left(1376 c d^2 x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.\right.$$

$$\left.\left.\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

- **Problem 336: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^5 \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 4, 675 leaves, 16 steps) :

$$\frac{2}{27\,c^2\,x^4\,\sqrt{c+d\,x^3}}-\frac{91\,\sqrt{c+d\,x^3}}{864\,c^3\,x^4}+\frac{113\,d\,\sqrt{c+d\,x^3}}{432\,c^4\,x}-\frac{113\,d^{4/3}\,\sqrt{c+d\,x^3}}{432\,c^4\,\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)}-\frac{d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{3456\,\sqrt{3}\,c^{23/6}}+\frac{d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\,\sqrt{c+d\,x^3}}\right]}{10\,368\,c^{23/6}}-$$

$$\frac{d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{10\,368\,c^{23/6}}+\left(113\,\sqrt{2-\sqrt{3}}\,d^{4/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\bigg/$$

$$\left(288\times 3^{3/4}\,c^{11/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)-$$

$$\frac{113\,d^{4/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{216\,\sqrt{2}\,3^{1/4}\,c^{11/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}}$$

Result (type 6, 364 leaves) :

$$\frac{1}{4320\,c^4\,x^4\,\sqrt{c+d\,x^3}}\left(-135\,c^2+675\,c\,d\,x^3+1130\,d^2\,x^6-\left(90\,250\,c^2\,d^2\,x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\bigg/\left(\left(8\,c-d\,x^3\right)\right.\right.$$

$$\left.\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+$$

$$\left(7232\,c\,d^3\,x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\bigg/\left(\left(8\,c-d\,x^3\right)\right.$$

$$\left.\left.\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

- **Problem 337: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^8\,\left(8\,c-d\,x^3\right)\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 699 leaves, 17 steps) :

$$\frac{2}{27\,c^2\,x^7\,\sqrt{c+d\,x^3}}-\frac{139\,\sqrt{c+d\,x^3}}{1512\,c^3\,x^7}+\frac{6095\,d\,\sqrt{c+d\,x^3}}{48\,384\,c^4\,x^4}-\frac{953\,d^2\,\sqrt{c+d\,x^3}}{3024\,c^5\,x}+$$

$$\frac{953\,d^{7/3}\,\sqrt{c+d\,x^3}}{3024\,c^5\,\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}-\frac{d^{7/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{27\,648\,\sqrt{3}\,c^{29/6}}+\frac{d^{7/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\,\sqrt{c+d\,x^3}}\right]}{82\,944\,c^{29/6}}-\frac{d^{7/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{82\,944\,c^{29/6}}-$$

$$\left(953\,\sqrt{2-\sqrt{3}}\,d^{7/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(2016\times 3^{3/4}\,c^{14/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)+$$

$$\frac{953\,d^{7/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]}{1512\,\sqrt{2}\,3^{1/4}\,c^{14/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}}$$

Result (type 6, 378 leaves) :

$$\frac{1}{241\,920\,c^5\,x^7\,\sqrt{c+d\,x^3}}$$

$$\left(-5\,\left(864\,c^3-1647\,c^2\,d\,x^3+9153\,c\,d^2\,x^6+15\,248\,d^3\,x^9\right)+\left(6\,100\,250\,c^2\,d^3\,x^9\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\right.\right.$$

$$\left.\left.\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)-$$

$$\left(487\,936\,c\,d^4\,x^{12}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\right.$$

$$\left.\left.\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

- **Problem 338: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(8\,c-d\,x^3\right)\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps) :

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{32 c^2 \sqrt{c+d x^3}}$$

Result (type 6, 338 leaves) :

$$\frac{1}{27 \sqrt{c+d x^3}} 2 x \left(-\frac{1}{c d}+\left(256 c\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d \left(8 c-d x^3\right)\right.\right.$$
$$\left(32 c\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\Bigg)+$$
$$\left(7 x^3\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 339: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(8 c-d x^3\right)\left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$\frac{x \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{8 c^2 \sqrt{c+d x^3}}$$

Result (type 6, 334 leaves) :

$$\frac{1}{27\sqrt{c+d x^3}}2 x\left(\left(176\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1-\frac{7 c d x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]}{\left(8 c-d x^3\right)\left(56 c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)}}{c^2}\right)$$

Problem 340: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^3\left(8 c-d x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\,\text{AppellF1}\left[-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{16 c^2 x^2\sqrt{c+d x^3}}$$

Result (type 6, 351 leaves):

$$\frac{1}{432 c^3 x^2\sqrt{c+d x^3}}\left(-27 c-59 d x^3-\left(7360 c^2 d x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(413 c d^2 x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 341: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^6\left(8 c-d x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{5}{3},1,\frac{3}{2},-\frac{2}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{40 c^2 x^5 \sqrt{c+d x^3}}$$

Result (type 6, 364 leaves):

$$\frac{1}{17280 c^4 x^5 \sqrt{c+d x^3}}\left(-432 c^2+1269 c d x^3+2981 d^2 x^6+\left(382528 c^2 d^2 x^6\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(20867 c d^3 x^9\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 342: Result unnecessarily involves higher level functions.**

$$\int \frac{x\sqrt{a+b x^3}}{2\left(5+3\sqrt{3}\right)a+b x^3}\, dx$$

Optimal (type 4, 737 leaves, 5 steps):

$$\frac{2\sqrt{a+b x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+\frac{3^{3/4} a^{1/6}\ \text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{2\sqrt{2}\ b^{2/3}}+\frac{a^{1/6}\ \text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a+b x^3}}{\sqrt{2}\ 3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\ 3^{1/4}\ b^{2/3}}+\frac{3^{1/4} a^{1/6}\ \text{ArcTanh}\left[\frac{3^{1/4} a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}-2 b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{\sqrt{2}\ b^{2/3}}+\frac{3^{1/4} a^{1/6}\ \text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{2\sqrt{2}\ b^{2/3}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\ a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)+\frac{2\sqrt{2}\ a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\ b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}}$$

Result (type 6, 250 leaves):

$$\left(10\left(26+15\sqrt{3}\right)a\,x^2\sqrt{a+b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\Big/\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a+b\,x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 343: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{a-b\,x^3}}{2\left(5+3\sqrt{3}\right)a-b\,x^3}\,dx$$

Optimal (type 4, 757 leaves, 5 steps):

$$\frac{2\sqrt{a-b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)}+\frac{3^{3/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{a^{1/6}\,\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a-b\,x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}+$$

$$\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2\,b^{1/3}\,x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\sqrt{a-b\,x^3}\right)+\frac{2\sqrt{2}\,a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\sqrt{a-b\,x^3}}$$

Result (type 6, 244 leaves):

$$\left(10\left(26+15\sqrt{3}\right)a\,x^2\sqrt{a-b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a-b\,x^3\right)\left(10\left(5+3\sqrt{3}\right)a\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]+\right.\right.$$
$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 344: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{-a+b\,x^3}}{-2\left(5+3\sqrt{3}\right)a+b\,x^3}\,dx$$

Optimal (type 4, 774 leaves, 5 steps):

$$-\frac{2\sqrt{-a+b\,x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)}+\frac{3^{1/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{-a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+$$
$$\frac{3^{1/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2\,b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{-a+b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}+\frac{3^{3/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{-a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}-\frac{a^{1/6}\,\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-a+b\,x^3}}{\sqrt{2}\,3^{3/4}\,\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}+$$
$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],-7+4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{-a+b\,x^3}\right)-\frac{2\sqrt{2}\,a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{-a+b\,x^3}}$$

Result (type 6, 245 leaves):

$$-\left(10\left(26+15\sqrt{3}\right) a x^2 \sqrt{-a+b x^3}\ \text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10 a+6\sqrt{3}\, a}\right]\right)\Big/$$
$$\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right) a-b x^3\right)\left(10\left(5+3\sqrt{3}\right) a\ \text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10 a+6\sqrt{3}\, a}\right]+\right.\right.$$
$$\left.\left.3 b x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10 a+6\sqrt{3}\, a}\right]-\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10 a+6\sqrt{3}\, a}\right]\right)\right)\right)$$

- **Problem 345: Result unnecessarily involves higher level functions.**

$$\int \frac{x\sqrt{-a-b x^3}}{-2\left(5+3\sqrt{3}\right) a-b x^3}\, dx$$

Optimal (type 4, 768 leaves, 5 steps) :

$$-\frac{2\sqrt{-a-b x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{3^{1/4} a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4} a^{1/6}\left(\left(1+\sqrt{3}\right) a^{1/3}-2 b^{1/3} x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{\sqrt{2}\, b^{2/3}}+$$
$$\frac{3^{1/4} a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right) a^{1/6}\left(a^{1/3}+b^{1/3} x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\, b^{2/3}}+\frac{3^{3/4} a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right) a^{1/6}\left(a^{1/3}+b^{1/3} x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\, b^{2/3}}-\frac{a^{1/6}\,\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-a-b x^3}}{\sqrt{2}\, 3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\, 3^{1/4} b^{2/3}}+$$
$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\, a^{1/3}\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7+4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{-a-b x^3}\right)-\frac{2\sqrt{2}\, a^{1/3}\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7+4\sqrt{3}\right]}{3^{1/4} b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{-a-b x^3}}$$

Result (type 6, 253 leaves) :

$$-\left(10\left(26+15\sqrt{3}\right)a\,x^2\sqrt{-a-b\,x^3}\ \text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a+b\,x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-\right.\right.$$
$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 346: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{a+b\,x^3}}{2\left(5-3\sqrt{3}\right)a+b\,x^3}\,dx$$

Optimal (type 4, 738 leaves, 5 steps):

$$\frac{2\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-\frac{3^{1/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2\,b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a+b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}-$$
$$\frac{3^{1/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{3^{3/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{a^{1/6}\,\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a+b\,x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}-$$
$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\frac{2\sqrt{2}\,a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}$$

Result (type 6, 250 leaves):

$$\left(10\left(-26+15\sqrt{3}\right)a\,x^2\sqrt{a+b\,x^3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a-b\,x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.\right.$$
$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 347: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{a-b\,x^3}}{2\left(5-3\sqrt{3}\right)a-b\,x^3}\,dx$$

Optimal (type 4, 758 leaves, 5 steps) :

$$\frac{2\sqrt{a-b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}-\frac{3^{1/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}-$$
$$\frac{3^{1/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}+2\,b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}+\frac{3^{3/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{a^{1/6}\,\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a-b\,x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}-$$
$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b\,x^3}\right)+\frac{2\sqrt{2}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b\,x^3}}$$

Result (type 6, 242 leaves) :

$$-\left(10\left(26-15\sqrt{3}\right)a\,x^2\sqrt{a-b\,x^3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a+b\,x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]-\right.\right.$$
$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 348: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{-a+b\,x^3}}{2\left(5-3\sqrt{3}\right)a-b\,x^3}\,dx$$

Optimal (type 4, 774 leaves, 5 steps) :

$$\frac{2\sqrt{-a+b\,x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)}-\frac{3^{3/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\sqrt{-a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{a^{1/6}\,\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a+b\,x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}+$$
$$\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\sqrt{-a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}+2\,b^{1/3}\,x\right)}{\sqrt{2}\sqrt{-a+b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}-$$
$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],-7+4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\sqrt{-a+b\,x^3}\right)+\frac{2\sqrt{2}\,a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\sqrt{-a+b\,x^3}}$$

Result (type 6, 243 leaves) :

$$-\left(10\left(26-15\sqrt{3}\right)a\,x^2\sqrt{-a+b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a+b\,x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]-\right.\right.$$
$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 349: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{-a-b\,x^3}}{2\left(5-3\sqrt{3}\right)a+b\,x^3}\,dx$$

Optimal (type 4, 768 leaves, 5 steps) :

$$\frac{2\sqrt{-a-b\,x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-\frac{3^{3/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}\,x\right)}{\sqrt{2}\sqrt{-a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{a^{1/6}\,\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a-b\,x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}+$$
$$\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2\,b^{1/3}\,x\right)}{\sqrt{2}\sqrt{-a-b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}+\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}\,x\right)}{\sqrt{2}\sqrt{-a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}-$$
$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7+4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{-a-b\,x^3}\right)+\frac{2\sqrt{2}\,a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\,b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{-a-b\,x^3}}$$

Result (type 6, 253 leaves) :

$$\left(10\left(-26+15\sqrt{3}\right)a\,x^2\sqrt{-a-b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/$$

$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a-b\,x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.\right.$$

$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 350: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\sqrt{a+b\,x^3}\left(2\left(5+3\sqrt{3}\right)a+b\,x^3\right)}\,dx$$

Optimal (type 3, 318 leaves, 1 step) :

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b\,x^3}}\right]}{2\sqrt{2}\;3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a+b\,x^3}}{\sqrt{2}\;3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\;3^{3/4}a^{5/6}b^{2/3}}-$$

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}-2\,b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b\,x^3}}\right]}{3\sqrt{2}\;3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b\,x^3}}\right]}{6\sqrt{2}\;3^{1/4}a^{5/6}b^{2/3}}$$

Result (type 6, 249 leaves) :

$$\left(10\left(26+15\sqrt{3}\right)a\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\Big/$$

$$\left(\left(5+3\sqrt{3}\right)\sqrt{a+b\,x^3}\left(2\left(5+3\sqrt{3}\right)a+b\,x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-\right.\right.$$

$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 351: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\sqrt{a-b\,x^3}\left(2\left(5+3\sqrt{3}\right)a-b\,x^3\right)}\,dx$$

Optimal (type 3, 324 leaves, 1 step) :

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}$$

Result (type 6, 243 leaves) :

$$\left(10\left(26+15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(5+3\sqrt{3}\right)\sqrt{a-b x^3}\left(2\left(5+3\sqrt{3}\right)a-b x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]+\right.\right.$$
$$\left.\left.3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 352: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{-a+b x^3}\left(-2\left(5+3\sqrt{3}\right)a+b x^3\right)}\,dx$$

Optimal (type 3, 328 leaves, 1 step) :

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Result (type 6, 244 leaves) :

$$-\left(10\left(26+15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a-b x^3\right)\sqrt{-a+b x^3}\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]+\right.\right.$$
$$\left.\left.3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 353: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{-a-b x^3}\left(-2\left(5+3\sqrt{3}\right)a-b x^3\right)}\,dx$$

Optimal (type 3, 330 leaves, 1 step):

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}-2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Result (type 6, 252 leaves):

$$-\left(10\left(26+15\sqrt{3}\right)a\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\Big/\left(\left(5+3\sqrt{3}\right)\sqrt{-a-b x^3}\left(2\left(5+3\sqrt{3}\right)a+b x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]-3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)$$

Problem 354: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{a+b x^3}\left(2\left(5-3\sqrt{3}\right)a+b x^3\right)}\,dx$$

Optimal (type 3, 310 leaves, 1 step):

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Result (type 6, 249 leaves):

$$-\left(10\left(26-15\sqrt{3}\right)a\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{-10\,a+6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a-b\,x^3\right)\sqrt{a+b\,x^3}\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.\right.$$
$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 355: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\sqrt{a-b\,x^3}\left(2\left(5-3\sqrt{3}\right)a-b\,x^3\right)}\,dx$$

Optimal (type 3, 316 leaves, 1 step):

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{6\sqrt{2}\,3^{1/4}\,a^{5/6}\,b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}+2\,b^{1/3}\,x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{3\sqrt{2}\,3^{1/4}\,a^{5/6}\,b^{2/3}}+$$
$$\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\sqrt{a-b\,x^3}}\right]}{2\sqrt{2}\,3^{3/4}\,a^{5/6}\,b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a-b\,x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}\,a^{5/6}\,b^{2/3}}$$

Result (type 6, 242 leaves):

$$-\left(10\left(26-15\sqrt{3}\right)a\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\sqrt{a-b\,x^3}\left(2\left(-5+3\sqrt{3}\right)a+b\,x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]-\right.\right.$$
$$\left.\left.3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

- **Problem 356: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\left(2\left(5-3\sqrt{3}\right)a-b\,x^3\right)\sqrt{-a+b\,x^3}}\,dx$$

Optimal (type 3, 320 leaves, 1 step):

$$\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}$$

Result (type 6, 243 leaves) :

$$-\left(10\left(26-15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]\right)\Big/\left(\left(-5+3\sqrt{3}\right)\sqrt{-a+b x^3}\left(2\left(-5+3\sqrt{3}\right)a+b x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]-3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 357: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{-a-b x^3}\left(2\left(5-3\sqrt{3}\right)a+b x^3\right)}\,dx$$

Optimal (type 3, 322 leaves, 1 step) :

$$\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}$$

Result (type 6, 252 leaves) :

$$-\left(10\left(26-15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{-10a+6\sqrt{3}\,a}\right]\right)\Big/\left(\left(-5+3\sqrt{3}\right)\sqrt{-a-b x^3}\left(2\left(-5+3\sqrt{3}\right)a-b x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a-6\sqrt{3}\,a}\right]+3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a-6\sqrt{3}\,a}\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 361: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x \left(a+b x^3\right)} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{2 \sqrt{c} \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a}+\frac{2 \sqrt{b c-a d} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a \sqrt{b}}$$

Result (type 6, 160 leaves):

$$-\left(2 b d x^3 \sqrt{c+d x^3} \operatorname{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right) \Big/ \left(\left(a+b x^3\right) \left(3 b d x^3 \operatorname{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-2 a d \operatorname{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)$$

Problem 362: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^4 \left(a+b x^3\right)} \, dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{3 a x^3}+\frac{(2 b c-a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 \sqrt{c}}-\frac{2 \sqrt{b} \sqrt{b c-a d} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2}$$

Result (type 6, 407 leaves):

$$\frac{1}{9 x^3 \left(a+b x^3\right) \sqrt{c+d x^3}}\left(\left(6 b c d x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(5 b d x^3 \left(3 a c+b c x^3+4 a d x^3+3 b d x^6\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-3 \left(a+b x^3\right) \left(c+d x^3\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(a \left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)$$

- **Problem 363: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3 \sqrt{c+d x^3}}{a+b x^3}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4 \sqrt{c+d x^3}\, \text{AppellF1}\left[\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 426 leaves):

$$\frac{1}{10 b \left(a+b x^3\right) \sqrt{c+d x^3}} x \left(\left(32 a^2 c^2 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(-7 a c \left(8 a c+11 b c x^3+3 a d x^3+8 b d x^6\right) \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+12 x^3 \left(a+b x^3\right) \left(c+d x^3\right) \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\Big/\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 364: Result more than twice size of optimal antiderivative.

$$\int \frac{x \sqrt{c+d x^3}}{a+b x^3} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2 \sqrt{c+d x^3} \, \text{AppellF1}\left[\frac{2}{3}, 1, -\frac{1}{2}, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 163 leaves):

$$\left(5 a c x^2 \sqrt{c+d x^3} \, \text{AppellF1}\left[\frac{2}{3}, -\frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(\left(a+b x^3\right) \left(10 a c \, \text{AppellF1}\left[\frac{2}{3}, -\frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(-2 b c \, \text{AppellF1}\left[\frac{5}{3}, -\frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 365: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{a+b x^3} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{c+d x^3} \, \text{AppellF1}\left[\frac{1}{3}, 1, -\frac{1}{2}, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 161 leaves):

$$\left(8 a c x \sqrt{c+d x^3} \, \text{AppellF1}\left[\frac{1}{3}, -\frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(\left(a+b x^3\right) \left(8 a c \, \text{AppellF1}\left[\frac{1}{3}, -\frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(-2 b c \, \text{AppellF1}\left[\frac{4}{3}, -\frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 366: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^2 \left(a+b x^3\right)} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a x \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 344 leaves) :

$$\frac{1}{10 x \sqrt{c+d x^3}}\left(-\frac{10\left(c+d x^3\right)}{a}+\left(25 c\ (2 b c-3 a d)\ x^3\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right)\left(-10 a c\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.\right.$$
$$\left.\left.\left.3 x^3\left(2 b c\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\ \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)-$$
$$\left(16 b c d x^6\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right)\left(-16 a c\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$
$$\left.\left.\left.3 x^3\left(2 b c\ \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\ \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

Problem 367: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^3\left(a+b x^3\right)}\, dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a x^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 344 leaves) :

$$\frac{1}{8\,x^2\sqrt{c+d\,x^3}}\left(-\frac{4\,(c+d\,x^3)}{a}+\left(16\,c\,(4\,b\,c-3\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(7\,b\,c\,d\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 371: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d\,x^3)^{3/2}}{x\,(a+b\,x^3)}\,dx$$

Optimal (type 3, 104 leaves, 7 steps):

$$\frac{2\,d\,\sqrt{c+d\,x^3}}{3\,b}-\frac{2\,c^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{3\,a}+\frac{2\,(b\,c-a\,d)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^3}}{\sqrt{b\,c-a\,d}}\right]}{3\,a\,b^{3/2}}$$

Result (type 6, 325 leaves):

$$\frac{1}{9\,b\,\sqrt{c+d\,x^3}}\,2\,d\left(3\,(c+d\,x^3)+\left(6\,a\,c\,(-2\,b\,c+a\,d)\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(5\,b^2\,c^2\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\Big/\left((a+b\,x^3)\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)$$

- **Problem 372: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(c+d\,x^3)^{3/2}}{x^4\,(a+b\,x^3)}\,dx$$

Optimal (type 3, 116 leaves, 7 steps):

$$-\frac{c\sqrt{c+d x^3}}{3 a x^3}+\frac{\sqrt{c}\,(2 b c-3 a d)\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2}-\frac{2 (b c-a d)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2\sqrt{b}}$$

Result (type 6, 414 leaves) :

$$\frac{1}{9 x^3 (a+b x^3)\sqrt{c+d x^3}}c\left(\left(6 d (b c-2 a d) x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\right.$$
$$\left(-4 a c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3\left(2 b c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+$$
$$\left(5 b d x^3 (3 a (c+2 d x^3)+b x^3 (c+3 d x^3))\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-\right.$$
$$3 (a+b x^3)(c+d x^3)\left(2 a d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\Big)\Big/$$
$$\left(a\left(-5 b d x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+\right.\right.$$
$$\left.\left.\left.b c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)$$

Problem 373: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3 (c+d x^3)^{3/2}}{a+b x^3}\,dx$$

Optimal (type 6, 65 leaves, 2 steps) :

$$\frac{c x^4\sqrt{c+d x^3}\,\text{AppellF1}\left[\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a\sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 382 leaves) :

$$\frac{1}{110\,b^2\,\sqrt{c+d\,x^3}}x\left(4\,\left(c+d\,x^3\right)\,\left(14\,b\,c-11\,a\,d+5\,b\,d\,x^3\right)+\left(32\,a^2\,c^2\,\left(14\,b\,c-11\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a+b\,x^3\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)-\left(7\,a\,c\,\left(27\,b^2\,c^2-88\,a\,b\,c\,d+55\,a^2\,d^2\right)\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a+b\,x^3\right)\,\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 374: Result more than twice size of optimal antiderivative.

$$\int\frac{x\,\left(c+d\,x^3\right)^{3/2}}{a+b\,x^3}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{c\,x^2\,\sqrt{c+d\,x^3}\,\text{AppellF1}\left[\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 437 leaves):

$$\frac{1}{35\,b\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}}x^2\left(\left(25\,a\,c^2\,\left(-7\,b\,c+4\,a\,d\right)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(2\,d\,\left(-8\,a\,c\,\left(10\,a\,c+20\,b\,c\,x^3+3\,a\,d\,x^3+10\,b\,d\,x^6\right)\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+15\,x^3\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\Big/\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

Problem 375: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{a+b x^3} \, dx$$

Optimal (type 6, 60 leaves, 2 steps):

$$\frac{c x \sqrt{c+d x^3} \text{ AppellF1}\left[\frac{1}{3}, 1, -\frac{3}{2}, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 434 leaves):

$$\frac{1}{10 b \left(a+b x^3\right) \sqrt{c+d x^3}} x \left(\left(16 a c^2 (-5 b c+2 a d) \text{ AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(-8 a c \text{ AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \text{ AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{ AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) + \left(d \left(-7 a c \left(8 a c + 16 b c x^3 + 3 a d x^3 + 8 b d x^6\right) \text{ AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 12 x^3 \left(a+b x^3\right) \left(c+d x^3\right) \left(2 b c \text{ AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{ AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right) \Big/ \left(-14 a c \text{ AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \text{ AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{ AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 376: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^2 \left(a+b x^3\right)} \, dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$-\frac{c \sqrt{c+d x^3} \text{ AppellF1}\left[-\frac{1}{3}, 1, -\frac{3}{2}, \frac{2}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{a x \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 450 leaves):

$$\frac{1}{10\,x\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}}c\,\left(\left(25\,c\,(2\,b\,c-5\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(16\,a\,\left(b\,c\,x^3\,\left(10\,c+9\,d\,x^3\right)+2\,a\,\left(5\,c^2+5\,c\,d\,x^3-d^2\,x^6\right)\right)\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-30\,x^3\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\Big/\left(a\,\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 377: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(c+d\,x^3\right)^{3/2}}{x^3\,\left(a+b\,x^3\right)}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{c\,\sqrt{c+d\,x^3}\,\text{AppellF1}\left[-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a\,x^2\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 449 leaves):

$$\frac{1}{8 x^2 \left(a+b x^3\right) \sqrt{c+d x^3}} c \left(\left(16 c (4 b c-7 a d) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$

$$\left.3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+$$

$$\left(7 a \left(b c x^3 \left(8 c+9 d x^3\right)+a \left(8 c^2+8 c d x^3-4 d^2 x^6\right)\right) \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-\right.$$

$$\left.12 x^3 \left(a+b x^3\right) \left(c+d x^3\right) \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\Big/$$

$$\left(a \left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$

$$\left.\left.\left.3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 381: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x \left(a+b x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a \sqrt{c}}+\frac{2 \sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a \sqrt{b c-a d}}$$

Result (type 6, 162 leaves):

$$\left(10 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\Big/\left(9 \left(a+b x^3\right) \sqrt{c+d x^3}\right.$$

$$\left.\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)$$

- **Problem 382: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4 \left(a+b x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{3 a c x^3}+\frac{(2 b c+a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 c^{3/2}}-\frac{2 b^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 \sqrt{b c-a d}}$$

Result (type 6, 409 leaves) :

$$\frac{1}{9 x^3 \left(a+b x^3\right) \sqrt{c+d x^3}}\left(\left(6 b d x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(5 b d x^3 \left(3 a c+b c x^3+2 a d x^3+3 b d x^6\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-3 \left(a+b x^3\right) \left(c+d x^3\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(a c \left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)$$

- **Problem 383: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(a+b x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a \sqrt{c+d x^3}}$$

Result (type 6, 165 leaves) :

$$-\left(7 a c x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(2 \left(a+b x^3\right) \sqrt{c+d x^3} \left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

- **Problem 384: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{\left(a+b x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$\frac{x^2 \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a \sqrt{c+d x^3}}$$

Result (type 6, 163 leaves):

$$-\left(5 a c x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right)\sqrt{c+d x^3}\left(-10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 385: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)\sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a \sqrt{c+d x^3}}$$

Result (type 6, 161 leaves):

$$-\left(8 a c x \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right)\sqrt{c+d x^3}\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 386: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\left(a+b x^3\right)\sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a x \sqrt{c+d x^3}}$$

Result (type 6, 345 leaves):

$$\frac{1}{10\,x\,\sqrt{c+d\,x^3}}\left(-\frac{10\,(c+d\,x^3)}{a\,c}+\left(25\,(2\,b\,c-a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)-\left(16\,b\,d\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 387: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\,(a+b\,x^3)\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a\,x^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 344 leaves):

$$\frac{1}{8\,x^2\,\sqrt{c+d\,x^3}}\left(-\frac{4\,(c+d\,x^3)}{a\,c}+\left(16\,(4\,b\,c+a\,d)\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(7\,b\,d\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 391: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x\left(a+b x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 3, 114 leaves, 7 steps):

$$-\frac{2 d}{3 c (b c-a d) \sqrt{c+d x^3}}-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a c^{3/2}}+\frac{2 b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a (b c-a d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(2 d \left(\left(6 a b x^3 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\middle/\right.\right.$$
$$\left(-4 a c \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+x^3 \left(2 b c \operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+a d \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)+$$
$$\left(5 b x^3 \left(2 a d+b \left(c+3 d x^3\right)\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, -\frac{a}{b x^3}\right]-\right.$$
$$\left.3 \left(a+b x^3\right) \left(2 a d \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^3}, -\frac{a}{b x^3}\right]+b c \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^3}, -\frac{a}{b x^3}\right]\right)\right)\Big/$$
$$\left(c \left(-5 b d x^3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, -\frac{a}{b x^3}\right]+2 a d \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^3}, -\frac{a}{b x^3}\right]+\right.\right.$$
$$\left.\left.\left.\left.b c \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^3}, -\frac{a}{b x^3}\right]\right)\right)\right)\right)\Big/\left(9 (b c-a d) \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 392: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4\left(a+b x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 3, 158 leaves, 8 steps):

$$-\frac{d (b c-3 a d)}{3 a c^2 (b c-a d) \sqrt{c+d x^3}}-\frac{1}{3 a c x^3 \sqrt{c+d x^3}}+\frac{(2 b c+3 a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 c^{5/2}}-\frac{2 b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 (b c-a d)^{3/2}}$$

Result (type 6, 501 leaves):

$$\left(\left(6\,b\,c\,d\,(b\,c-3\,a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((b\,c-a\,d)\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)-\left(5\,b\,d\,x^3\left(-3\,a^2\,d\left(c+2\,d\,x^3\right)+b^2\,c\,x^3\left(c+3\,d\,x^3\right)+a\,b\left(3\,c^2-c\,d\,x^3-9\,d^2\,x^6\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+3\left(-b^2\,c\,x^3\left(c+d\,x^3\right)+a^2\,d\left(c+3\,d\,x^3\right)-a\,b\left(c^2-3\,d^2\,x^6\right)\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(a\,(-b\,c+a\,d)\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)\Big/\left(9\,c^2\,x^3\left(a+b\,x^3\right)\sqrt{c+d\,x^3}\right)$$

- **Problem 393: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(a+b\,x^3\right)\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^4\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,a\,c\,\sqrt{c+d\,x^3}}$$

Result (type 6, 332 leaves):

$$\frac{1}{6\,(-b\,c+a\,d)\sqrt{c+d\,x^3}}x\left(-4-\left(32\,a^2\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a+b\,x^3\right)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(7\,a\,b\,c\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a+b\,x^3\right)\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 394: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{\left(a+b\,x^3\right)\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^2 \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a c \sqrt{c+d x^3}}$$

Result (type 6, 366 leaves):

$$\frac{1}{15 \sqrt{c+d x^3}} x^2 \left(\left(25 a (3 b c+a d) \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left((-b c+a d) \left(a+b x^3\right) \left(-10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)+2 d \left(-\frac{5}{b c^2-a c d}+\left(8 a b x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left((-b c+a d) \left(a+b x^3\right) \left(-16 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\right)$$

- **Problem 395: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a c \sqrt{c+d x^3}}$$

Result (type 6, 362 leaves):

$$\frac{1}{6\sqrt{c+d x^3}}x\left(-\frac{4 d}{b c^2-a c d}+\right.$$
$$\left(16 a\,(-3 b c+a d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left((b c-a d)\,\left(a+b x^3\right)\left(-8 a c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$
$$\left.\left.3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)-$$
$$\left(7 a b d x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left((-b c+a d)\,\left(a+b x^3\right)\left(-14 a c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$
$$\left.\left.\left.3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 396: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{x^2\left(a+b x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\;\text{AppellF1}\left[-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a c x\sqrt{c+d x^3}}$$

Result (type 6, 408 leaves):

$$\frac{1}{30 c^2 x\sqrt{c+d x^3}}\left(\left(\left(25 c\left(6 b^2 c^2-3 a b c d+5 a^2 d^2\right) x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\right.\right.$$
$$\left((b c-a d)\,\left(a+b x^3\right)\left(-10 a c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$
$$\left.\left.\left.3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)+$$
$$1/(-b c+a d)\,2\left(\frac{15 b c\left(c+d x^3\right)}{a}-5 d\left(3 c+5 d x^3\right)+\left(8 b c d\,(3 b c-5 a d)\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\right.$$
$$\left(\left(a+b x^3\right)\left(-16 a c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$
$$\left.\left.\left.\left.\left.3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\right)\right)$$

- **Problem 397: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^3 \left(a+b x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[-\frac{2}{3}, 1, \frac{3}{2}, \frac{1}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 a c x^2 \sqrt{c+d x^3}}$$

Result (type 6, 418 leaves):

$$\frac{1}{24 c^2 x^2 \sqrt{c+d x^3}} \left(\frac{12 b c \left(c+d x^3\right)-4 a d \left(3 c+7 d x^3\right)}{a (-b c+a d)}+\left(16 c \left(12 b^2 c^2+3 a b c d-7 a^2 d^2\right) x^3 \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left((b c-a d) \left(a+b x^3\right) \left(-8 a c \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)+\left(7 b c d (3 b c-7 a d) x^6 \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left((b c-a d) \left(a+b x^3\right) \left(-14 a c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 402: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{\sqrt{c+d x^3}}{24 c \left(8 c-d x^3\right)}+\frac{5 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{288 c^{3/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 c^{3/2}}$$

Result (type 6, 316 leaves):

$$\frac{1}{72\sqrt{c+d x^3}}\left(\left(24 d x^3 \,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Bigg/\right.$$
$$\left(\left(8 c-d x^3\right)\left(16 c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left.\frac{-3-\frac{3 d x^3}{c}+\frac{10 d x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]}{5 d x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]}}{-8 c+d x^3}\right)$$

- **Problem 403: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x^4\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 3, 124 leaves, 8 steps):

$$\frac{d\sqrt{c+d x^3}}{96 c^2\left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{24 c x^3\left(8 c-d x^3\right)}+\frac{7 d\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{1152 c^{5/2}}-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{128 c^{5/2}}$$

Result (type 6, 338 leaves):

$$\frac{1}{96 c^2 x^3\sqrt{c+d x^3}}\left(\left(8 c d^2 x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Bigg/\right.$$
$$\left(\left(8 c-d x^3\right)\left(16 c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$1\Big/\left(-8 c+d x^3\right)\left(4 c^2+3 c d x^3-d^2 x^6+\left(10 c d^2 x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Bigg/\right.$$
$$\left.\left.\left(5 d x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 404: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x^7\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 3, 164 leaves, 9 steps):

$$\frac{5\,d^2\sqrt{c+d\,x^3}}{1536\,c^3\,\left(8\,c-d\,x^3\right)}-\frac{\sqrt{c+d\,x^3}}{48\,c\,x^6\,\left(8\,c-d\,x^3\right)}-\frac{7\,d\,\sqrt{c+d\,x^3}}{384\,c^2\,x^3\,\left(8\,c-d\,x^3\right)}+\frac{23\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{18\,432\,c^{7/2}}-\frac{d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{2048\,c^{7/2}}$$

Result (type 6, 349 leaves) :

$$\frac{1}{1536\,c^3\,x^6\,\sqrt{c+d\,x^3}}\left(\left(40\,c\,d^3\,x^9\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.$$
$$\left(\left(8\,c-d\,x^3\right)\,\left(16\,c\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]+d\,x^3\,\left(\text{AppellF1}\left[2,\,\frac{1}{2},\,2,\,3,\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\,\frac{3}{2},\,1,\,3,\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+$$
$$1\Big/\left(-8\,c+d\,x^3\right)\,\left(32\,c^3+60\,c^2\,d\,x^3+23\,c\,d^2\,x^6-5\,d^3\,x^9+\left(10\,c\,d^3\,x^9\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^3},\,\frac{8\,c}{d\,x^3}\right]\right)\Big/\right.$$
$$\left.\left.\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^3},\,\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{2},\,2,\,\frac{7}{2},\,-\frac{c}{d\,x^3},\,\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{3}{2},\,1,\,\frac{7}{2},\,-\frac{c}{d\,x^3},\,\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 405: Result unnecessarily involves higher level functions.**

$$\int\frac{x^7\,\sqrt{c+d\,x^3}}{\left(8\,c-d\,x^3\right)^2}\,dx$$

Optimal (type 4, 663 leaves, 15 steps) :

$$\frac{13\,x^2\sqrt{c+d\,x^3}}{21\,d^2}+\frac{746\,c\sqrt{c+d\,x^3}}{21\,d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}+\frac{x^5\sqrt{c+d\,x^3}}{3\,d\left(8\,c-d\,x^3\right)}+$$

$$\frac{76\,c^{7/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{3\sqrt{3}\,d^{8/3}}-\frac{76\,c^{7/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d\,x^3}}\right]}{9\,d^{8/3}}+\frac{76\,c^{7/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{9\,d^{8/3}}-$$

$$\left(373\sqrt{2-\sqrt{3}}\;c^{4/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{3/4}\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}\right)+$$

$$\frac{746\sqrt{2}\;c^{4/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]}{21\times 3^{1/4}\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}}$$

Result (type 6, 344 leaves) :

$$\left(2\,x^2\left(5\left(c+d\,x^3\right)\left(-52\,c+3\,d\,x^3\right)+\left(10\,400\,c^3\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.\right.$$

$$\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,2,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\,\frac{3}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\right)+$$

$$\left(11\,936\,c^2\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]+\right.$$

$$\left.\left.\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\,\frac{1}{2},\,2,\,\frac{11}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\,\frac{3}{2},\,1,\,\frac{11}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\Big/\left(105\,d^2\left(-8\,c+d\,x^3\right)\sqrt{c+d\,x^3}\right)$$

- **Problem 406: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4\sqrt{c+d\,x^3}}{\left(8\,c-d\,x^3\right)^2}\,dx$$

Optimal (type 4, 641 leaves, 14 steps) :

$$\frac{7\sqrt{c+d x^3}}{3 d^{5/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^2\sqrt{c+d x^3}}{3 d\left(8 c-d x^3\right)}+\frac{5 c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3\sqrt{3}\,d^{5/3}}-\frac{5 c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{9 d^{5/3}}+$$

$$\frac{5 c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{9 d^{5/3}}-\frac{7\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{2\times 3^{3/4} d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}}+$$

$$\frac{7\sqrt{2}\,c^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{3\times 3^{1/4} d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 357 leaves) :

$$\frac{1}{15\sqrt{c+d x^3}}x^2\left(-\frac{5\left(c+d x^3\right)}{d\left(-8 c+d x^3\right)}+\left(200 c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d\left(-8 c+d x^3\right)\right.\right.$$
$$\left.\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$
$$\left(224 c x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$
$$\left.\left.\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 407: Result unnecessarily involves higher level functions.**

$$\int\frac{x\sqrt{c+d x^3}}{\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 644 leaves, 14 steps) :

$$\frac{\sqrt{c+d x^3}}{24 c d^{2/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^2 \sqrt{c+d x^3}}{24 c \left(8 c-d x^3\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{48 \sqrt{3} c^{5/6} d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{144 c^{5/6} d^{2/3}}+$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{144 c^{5/6} d^{2/3}}-\frac{\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]}{16 \times 3^{3/4} c^{2/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}+$$

$$\frac{\left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]}{12 \sqrt{2}\, 3^{1/4} c^{2/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 353 leaves) :

$$\frac{1}{120 \sqrt{c+d x^3}} x^2 \left(\frac{5 \left(c+d x^3\right)}{c \left(8 c-d x^3\right)}+\left(100 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(32 d x^3 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(-8 c+d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 408: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c+d x^3}}{x^2 \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 665 leaves, 15 steps) :

$$-\frac{\sqrt{c+d x^3}}{48 c^2 x}+\frac{d^{1/3}\sqrt{c+d x^3}}{48 c^2\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{\sqrt{c+d x^3}}{24 c x\left(8 c-d x^3\right)}-\frac{d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{48\sqrt{3}\,c^{11/6}}+\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{144 c^{11/6}}-$$

$$\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{144 c^{11/6}}-\frac{\sqrt{2-\sqrt{3}}\,d^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{32\times 3^{3/4}\,c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\sqrt{c+d x^3}}+$$

$$\frac{d^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{24\sqrt{2}\,3^{1/4}\,c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\sqrt{c+d x^3}}$$

Result (type 6, 372 leaves) :

$$\frac{1}{30\sqrt{c+d x^3}}\left(-\frac{5\left(6 c-d x^3\right)\left(c+d x^3\right)}{8 c^2\left(8 c x-d x^4\right)}+\left(125 d x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$

$$\left.\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$

$$\left(4 d^2 x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\right.$$

$$\left.\left.\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 409: Result unnecessarily involves higher level functions.**

$$\int\frac{\sqrt{c+d x^3}}{x^5\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 687 leaves, 16 steps) :

$$-\frac{7\sqrt{c+d x^3}}{768 c^2 x^4}-\frac{d\sqrt{c+d x^3}}{96 c^3 x}+\frac{d^{4/3}\sqrt{c+d x^3}}{96 c^3\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{\sqrt{c+d x^3}}{24 c x^4\left(8 c-d x^3\right)}-\frac{17 d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3072\sqrt{3}\, c^{17/6}}+\frac{17 d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{9216 c^{17/6}}-$$

$$\frac{17 d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{9216 c^{17/6}}-\frac{\sqrt{2-\sqrt{3}}\, d^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{64\times 3^{3/4} c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}}+$$

$$\frac{d^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{48\sqrt{2}\, 3^{1/4} c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 362 leaves) :

$$\left(-5\left(c+d x^3\right)\left(24 c^2+57 c d x^3-8 d^2 x^6\right)+\left(5750 c^2 d^2 x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-$$
$$\left(256 c d^3 x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$
$$\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\Big/\left(3840 c^3 x^4\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 410: Result unnecessarily involves higher level functions.**

$$\int\frac{\sqrt{c+d x^3}}{x^8\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 711 leaves, 17 steps) :

$$-\frac{5\sqrt{c+d x^3}}{672 c^2 x^7}-\frac{53 d\sqrt{c+d x^3}}{21504 c^3 x^4}-\frac{d^2\sqrt{c+d x^3}}{5376 c^4 x}+\frac{d^{7/3}\sqrt{c+d x^3}}{5376 c^4\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+$$

$$\frac{\sqrt{c+d x^3}}{24 c x^7\left(8 c-d x^3\right)}-\frac{13 d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{12288\sqrt{3}\, c^{23/6}}+\frac{13 d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{36864 c^{23/6}}-\frac{13 d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{36864 c^{23/6}}-$$

$$\frac{\sqrt{2-\sqrt{3}}\, d^{7/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{3584\times 3^{3/4} c^{11/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}}+$$

$$\frac{d^{7/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]}{2688\sqrt{2}\, 3^{1/4} c^{11/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}}$$

Result (type 6, 377 leaves) :

$$\Bigg(-5\left(384 c^4+648 c^3 d x^3+243 c^2 d^2 x^6-25 c d^3 x^9-4 d^4 x^{12}\right)+\left(15250 c^2 d^3 x^9 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$
$$\left(40 c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-$$
$$\left(128 c d^4 x^{12} \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64 c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$
$$\left.3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\Bigg)\Big/\left(107520 c^4 x^7\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 415: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x\left(8 c-d x^3\right)^2}\, dx$$

Optimal (type 3, 85 leaves, 7 steps) :

$$\frac{3\sqrt{c+d x^3}}{8\left(8 c-d x^3\right)}-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{32 \sqrt{c}}-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 \sqrt{c}}$$

Result (type 6, 317 leaves):

$$\frac{1}{72\sqrt{c+d x^3}}\left(-\left(168 c d x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left.\frac{-27\left(c+d x^3\right)+\frac{10 c d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]}{5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]}}{-8 c+d x^3}\right)$$

Problem 416: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^4\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 3, 121 leaves, 8 steps):

$$\frac{5 d\sqrt{c+d x^3}}{96 c\left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{24 x^3\left(8 c-d x^3\right)}+\frac{3 d \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{128 c^{3/2}}-\frac{7 d \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{384 c^{3/2}}$$

Result (type 6, 333 leaves):

$$\frac{1}{144\sqrt{c+d x^3}}\left(\left(60 d^2 x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$1\Big/\left(2\left(-8 c+d x^3\right)\right)\left(-3 d+\frac{12 c}{x^3}-\frac{15 d^2 x^3}{c}+\left(70 d^2 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\right.$$
$$\left.\left.\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 417: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^7 \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 3, 161 leaves, 9 steps):

$$\frac{7 d^2 \sqrt{c+d x^3}}{512 c^2 \left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{48 x^6 \left(8 c-d x^3\right)}-\frac{23 d \sqrt{c+d x^3}}{384 c x^3 \left(8 c-d x^3\right)}+\frac{15 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{2048 c^{5/2}}-\frac{17 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{2048 c^{5/2}}$$

Result (type 6, 349 leaves):

$$\frac{1}{1536 c^2 x^6 \sqrt{c+d x^3}}\left(\left(168 c d^3 x^9 \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \right.$$
$$\left(\left(8 c-d x^3\right) \left(16 c \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+d x^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$1 \Big/ \left(-8 c+d x^3\right) \left(32 c^3+124 c^2 d x^3+71 c d^2 x^6-21 d^3 x^9+\left(170 c d^3 x^9 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]\right) \Big/ \right.$$
$$\left.\left.\left(5 d x^3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]+16 c \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]-c \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 418: Result unnecessarily involves higher level functions.**

$$\int \frac{x^7 \left(c+d x^3\right)^{3/2}}{\left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 681 leaves, 16 steps):

$$\frac{103\,c\,x^2\sqrt{c+d\,x^3}}{13\,d^2}+\frac{19\,x^5\sqrt{c+d\,x^3}}{39\,d}+\frac{5906\,c^2\sqrt{c+d\,x^3}}{13\,d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}+\frac{x^5\left(c+d\,x^3\right)^{3/2}}{3\,d\left(8\,c-d\,x^3\right)}+$$

$$\frac{108\sqrt{3}\,c^{13/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{d^{8/3}}-\frac{108\,c^{13/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d\,x^3}}\right]}{d^{8/3}}+\frac{108\,c^{13/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{d^{8/3}}-$$

$$\left(2953\times 3^{1/4}\sqrt{2-\sqrt{3}}\,c^{7/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(13\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}\right)+$$

$$\frac{5906\sqrt{2}\,c^{7/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]}{13\times 3^{1/4}\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}}$$

Result (type 6, 357 leaves) :

$$\left(2\left(5\left(c+d\,x^3\right)\left(-412\,c^2\,x^2+24\,c\,d\,x^5+d^2\,x^8\right)+\left(82\,400\,c^4\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.\right.$$

$$\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)+$$

$$\left(94\,496\,c^3\,d\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+\right.$$

$$\left.\left.\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\Big/\left(65\,d^2\left(-8\,c+d\,x^3\right)\sqrt{c+d\,x^3}\right)$$

- **Problem 419: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4\left(c+d\,x^3\right)^{3/2}}{\left(8\,c-d\,x^3\right)^2}\,dx$$

Optimal (type 4, 657 leaves, 15 steps) :

$$\frac{13 x^2 \sqrt{c+d x^3}}{21 d}+\frac{265 c \sqrt{c+d x^3}}{7 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^2 \left(c+d x^3\right)^{3/2}}{3 d \left(8 c-d x^3\right)}+$$

$$\frac{9 \sqrt{3} c^{7/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{5/3}}-\frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{d^{5/3}}+\frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{d^{5/3}}-$$

$$\left(265\times 3^{1/4} \sqrt{2-\sqrt{3}}\, c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(14 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+$$

$$\frac{265 \sqrt{2}\, c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{7\times 3^{1/4} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 368 leaves):

$$\frac{1}{7 \sqrt{c+d x^3}} x^2 \left(\frac{\left(c+d x^3\right) \left(-37 c+2 d x^3\right)}{d \left(-8 c+d x^3\right)}+\left(1480 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d \left(-8 c+d x^3\right)\right.\right.$$

$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$

$$\left(1696 c^2 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$

$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 420: Result unnecessarily involves higher level functions.**

$$\int \frac{x \left(c+d x^3\right)^{3/2}}{\left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 638 leaves, 14 steps):

$$\frac{19\sqrt{c+d x^3}}{8 d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{3x^2\sqrt{c+d x^3}}{8\left(8c-d x^3\right)}+\frac{9\sqrt{3}\,c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{16 d^{2/3}}-\frac{9c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3c^{1/6}\sqrt{c+d x^3}}\right]}{16 d^{2/3}}+\frac{9c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{16 d^{2/3}}-$$

$$\left(19\times 3^{1/4}\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(16 d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\frac{19c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{4\sqrt{2}\,3^{1/4}d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 330 leaves):

$$\frac{1}{40\left(8c-d x^3\right)\sqrt{c+d x^3}}x^2\left(15\left(c+d x^3\right)-\left(500c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/\right.$$

$$\left(40c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)-$$

$$\left(608c\,d x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/$$

$$\left.\left(64c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)\right)$$

- **Problem 421: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d x^3\right)^{3/2}}{x^2\left(8c-d x^3\right)^2}\,dx$$

Optimal (type 4, 522 leaves, 6 steps):

$$-\frac{\sqrt{c+d x^3}}{16 c x}+\frac{d^{1/3} \sqrt{c+d x^3}}{16 c \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{3 \sqrt{c+d x^3}}{8 x \left(8 c-d x^3\right)}-$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}}\; d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(32 c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \sqrt{c+d x^3}\right)+\frac{d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]}{8 \sqrt{2}\; 3^{1/4} c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \sqrt{c+d x^3}}$$

Result (type 4, 242 leaves) :

$$\frac{\left(2 c-d x^3\right) \sqrt{c+d x^3}}{16 c x \left(-8 c+d x^3\right)}-\frac{1}{16\times 3^{1/4} c^{1/3} \sqrt{c+d x^3}} (-1)^{1/6} (-d)^{1/3} \sqrt{(-1)^{5/6} \left(-1+\frac{(-d)^{1/3} x}{c^{1/3}}\right)} \sqrt{1+\frac{(-d)^{1/3} x}{c^{1/3}}+\frac{(-d)^{2/3} x^2}{c^{2/3}}}$$

$$\left(-i \sqrt{3}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-d)^{1/3} x}{c^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-d)^{1/3} x}{c^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 422: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^5 \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 684 leaves, 16 steps) :

$$-\frac{13\sqrt{c+d x^3}}{256 c x^4}-\frac{d\sqrt{c+d x^3}}{32 c^2 x}+\frac{d^{4/3}\sqrt{c+d x^3}}{32 c^2\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{3\sqrt{c+d x^3}}{8 x^4\left(8 c-d x^3\right)}-$$

$$\frac{9\sqrt{3}\,d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{1024 c^{11/6}}+\frac{9 d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{1024 c^{11/6}}-\frac{9 d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{1024 c^{11/6}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(64 c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\frac{d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{16\sqrt{2}\,3^{1/4}c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 361 leaves):

$$\left(-\frac{5\left(c+d x^3\right)\left(8 c^2+51 c d x^3-8 d^2 x^6\right)}{c^2}+\left(7250 d^2 x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-$$
$$\left(256 d^3 x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(1280 x^4\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 423: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^8\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 708 leaves, 17 steps):

$$-\frac{11\sqrt{c+d x^3}}{224 c x^7}-\frac{83 d\sqrt{c+d x^3}}{7168 c^2 x^4}-\frac{19 d^2\sqrt{c+d x^3}}{1792 c^3 x}+\frac{19 d^{7/3}\sqrt{c+d x^3}}{1792 c^3\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+$$

$$\frac{3\sqrt{c+d x^3}}{8 x^7\left(8 c-d x^3\right)}-\frac{9\sqrt{3}\, d^{7/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{4096 c^{17/6}}+\frac{9 d^{7/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{4096 c^{17/6}}-\frac{9 d^{7/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{4096 c^{17/6}}-$$

$$\left(19\times 3^{1/4}\sqrt{2-\sqrt{3}}\, d^{7/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3584 c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\frac{19 d^{7/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{896\sqrt{2}\, 3^{1/4} c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 373 leaves):

$$\left(-5\left(c+d x^3\right)\left(128 c^3+312 c^2 d x^3+525 c d^2 x^6-76 d^3 x^9\right)+\left(58750 c^2 d^3 x^9\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$

$$\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-$$

$$\left(2432 c d^4 x^{12}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$

$$\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\Big/\left(35840 c^3 x^7\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 428: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{1}{x\left(8 c-d x^3\right)^2\sqrt{c+d x^3}}\,dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{\sqrt{c+d x^3}}{216 c^2\left(8 c-d x^3\right)}+\frac{13\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{2592 c^{5/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 c^{5/2}}$$

Result (type 6, 329 leaves):

$$\frac{1}{216 c^2 \sqrt{c+d x^3}}\left(\frac{c+d x^3}{8 c-d x^3}+\left(8 c d x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3 \left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(30 c d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 429: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4 \left(8 c-d x^3\right)^2 \sqrt{c+d x^3}}\, dx$$

Optimal (type 3, 124 leaves, 8 steps):

$$\frac{5 d \sqrt{c+d x^3}}{864 c^3 \left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{24 c^2 x^3 \left(8 c-d x^3\right)}+\frac{11 d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{10368 c^{7/2}}+\frac{d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{384 c^{7/2}}$$

Result (type 6, 347 leaves):

$$\frac{1}{864 c^3 x^3 \sqrt{c+d x^3}}\left(-\frac{\left(c+d x^3\right)\left(-36 c+5 d x^3\right)}{-8 c+d x^3}+\left(40 c d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3 \left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(30 c d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 430: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^7 \left(8 c-d x^3\right)^2 \sqrt{c+d x^3}}\, dx$$

Optimal (type 3, 164 leaves, 9 steps):

$$-\frac{35 d^2 \sqrt{c+d x^3}}{13824 c^4 \left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{48 c^2 x^6 \left(8 c-d x^3\right)}+\frac{3 d \sqrt{c+d x^3}}{128 c^3 x^3 \left(8 c-d x^3\right)}+\frac{31 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{165888 c^{9/2}}-\frac{19 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{6144 c^{9/2}}$$

Result (type 6, 349 leaves) :

$$\frac{1}{13824\, c^4 x^6 \sqrt{c+d x^3}}\left(-\left(280\, c\, d^3 x^9\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16\, c\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+1\Big/\left(-8 c+d x^3\right)\left(288 c^3-36 c^2 d x^3-289 c d^2 x^6+35 d^3 x^9+\left(570\, c\, d^3 x^9\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(5 d x^3\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16\, c\, \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\, \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

- **Problem 431: Result unnecessarily involves higher level functions.**

$$\int \frac{x^7}{\left(8 c-d x^3\right)^2 \sqrt{c+d x^3}}\, dx$$

Optimal (type 4, 641 leaves, 14 steps) :

$$\frac{62\sqrt{c+d x^3}}{27\, d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{8 x^2\sqrt{c+d x^3}}{27\, d^2\left(8 c-d x^3\right)}+\frac{44\, c^{1/6}\, \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{27\sqrt{3}\, d^{8/3}}-\frac{44\, c^{1/6}\, \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{81\, d^{8/3}}+\frac{44\, c^{1/6}\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{81\, d^{8/3}}-\left(31\sqrt{2-\sqrt{3}}\, c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(9\times 3^{3/4}\, d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\frac{62\sqrt{2}\, c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{27\times 3^{1/4}\, d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 333 leaves) :

$$\left(8 x^2 \left(5 \left(c+d x^3\right)-\left(200 c^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.\right.$$
$$\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-$$
$$\left(248 c d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$
$$\left.\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(135 d^2 \left(8 c-d x^3\right) \sqrt{c+d x^3}\right)$$

- **Problem 432: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4}{\left(8 c-d x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 4, 647 leaves, 14 steps):

$$\frac{\sqrt{c+d x^3}}{27 c d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^2 \sqrt{c+d x^3}}{27 c d \left(8 c-d x^3\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{27 \sqrt{3} c^{5/6} d^{5/3}}-\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{81 c^{5/6} d^{5/3}}+$$
$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{81 c^{5/6} d^{5/3}}-\frac{\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{18\times 3^{3/4} c^{2/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}+$$
$$\frac{\sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{27\times 3^{1/4} c^{2/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 360 leaves):

$$\frac{1}{135\sqrt{c+d\,x^3}}x^2\left(\frac{5\,c+5\,d\,x^3}{8\,c^2\,d-c\,d^2\,x^3}+\left(200\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(d\left(-8\,c+d\,x^3\right)\right.\right.$$

$$\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\Big)-$$

$$\left(32\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+\right.\right.$$

$$\left.\left.\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

- **Problem 433: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\left(8\,c-d\,x^3\right)^2\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 4, 644 leaves, 14 steps):

$$\frac{\sqrt{c+d\,x^3}}{216\,c^2\,d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}+\frac{x^2\sqrt{c+d\,x^3}}{216\,c^2\left(8\,c-d\,x^3\right)}-\frac{7\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{432\sqrt{3}\,c^{11/6}\,d^{2/3}}+\frac{7\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d\,x^3}}\right]}{1296\,c^{11/6}\,d^{2/3}}-$$

$$\frac{7\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{1296\,c^{11/6}\,d^{2/3}}-\frac{\sqrt{2-\sqrt{3}}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]}{144\times3^{3/4}\,c^{5/3}\,d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}}+$$

$$\frac{\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]}{108\sqrt{2}\,3^{1/4}\,c^{5/3}\,d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}}$$

Result (type 6, 332 leaves):

$$\left(x^2\left(\left(2500\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\middle/\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\right.$$
$$1/c^2\left(5\left(c+d x^3\right)-\left(32\,c\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\middle/\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\middle/\left(1080\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 434: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2\left(8 c-d x^3\right)^2\sqrt{c+d x^3}}\,dx$$

Optimal (type 4, 665 leaves, 15 steps):

$$-\frac{7\sqrt{c+d x^3}}{432\,c^3\,x}+\frac{7\,d^{1/3}\sqrt{c+d x^3}}{432\,c^3\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}+\frac{\sqrt{c+d x^3}}{216\,c^2\,x\left(8 c-d x^3\right)}-\frac{d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d x^3}}\right]}{216\sqrt{3}\,c^{17/6}}+\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d x^3}}\right]}{648\,c^{17/6}}-$$
$$\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{648\,c^{17/6}}-\frac{7\sqrt{2-\sqrt{3}}\,d^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]}{288\times 3^{3/4}\,c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d x^3}}+$$
$$\frac{7\,d^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]}{216\sqrt{2}\,3^{1/4}\,c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 375 leaves):

$$\frac{1}{135\sqrt{c+d x^3}}\left(-\frac{5\left(54 c-7 d x^3\right)\left(c+d x^3\right)}{16 c^3\left(8 c x-d x^4\right)}+\left(250 d x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$
$$\left(14 d^2 x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c^2\left(8 c-d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 435: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^5\left(8 c-d x^3\right)^2\sqrt{c+d x^3}}\,dx$$

Optimal (type 4, 687 leaves, 16 steps):

$$-\frac{31\sqrt{c+d x^3}}{6912 c^3 x^4}+\frac{5 d\sqrt{c+d x^3}}{864 c^4 x}-\frac{5 d^{4/3}\sqrt{c+d x^3}}{864 c^4\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{\sqrt{c+d x^3}}{216 c^2 x^4\left(8 c-d x^3\right)}-$$
$$\frac{25 d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{27648\sqrt{3}\,c^{23/6}}+\frac{25 d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{82944 c^{23/6}}-\frac{25 d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{82944 c^{23/6}}+$$
$$\frac{5\sqrt{2-\sqrt{3}}\,d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{576\times 3^{3/4}c^{11/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}-$$
$$\frac{5 d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{432\sqrt{2}\,3^{1/4}c^{11/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}}$$

Result (type 6, 384 leaves):

$$\frac{1}{6912\, c^4 x^4 \sqrt{c+d x^3}}\left(\frac{(c+d x^3)(216 c^2-351 c d x^3+40 d^2 x^6)}{-8 c+d x^3}-\left(2450 c^2 d^2 x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left((8 c-d x^3)\left(40 c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(256 c d^3 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left((8 c-d x^3)\left(64 c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 436: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^8 \left(8 c-d x^3\right)^2 \sqrt{c+d x^3}}\, dx$$

Optimal (type 4, 711 leaves, 17 steps):

$$-\frac{17 \sqrt{c+d x^3}}{6048 c^3 x^7}+\frac{391 d \sqrt{c+d x^3}}{193536 c^4 x^4}-\frac{289 d^2 \sqrt{c+d x^3}}{48384 c^5 x}+\frac{289 d^{7/3} \sqrt{c+d x^3}}{48384 c^5 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{\sqrt{c+d x^3}}{216 c^2 x^7 \left(8 c-d x^3\right)}-\frac{17 d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{110592 \sqrt{3}\, c^{29/6}}+\frac{17 d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{331776 c^{29/6}}-\frac{17 d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{331776 c^{29/6}}-\left(289 \sqrt{2-\sqrt{3}}\, d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(32256\times 3^{3/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\frac{289 d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{24192 \sqrt{2}\, 3^{1/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 377 leaves):

$$\left(-5\left(3456c^4-216c^3dx^3+5967c^2d^2x^6+8483cd^3x^9-1156d^4x^{12}\right)+\left(480250c^2d^3x^9\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(40c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)-\left(36992cd^4x^{12}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(64c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)\Big/\left(967680c^5x^7\left(8c-dx^3\right)\sqrt{c+dx^3}\right)$$

- **Problem 437: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^6}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^7\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left[\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right]}{448c^2\sqrt{c+dx^3}}$$

Result (type 6, 331 leaves):

$$\left(2x\left(4\left(c+dx^3\right)-\left(128c^2\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(32c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)-\left(161cdx^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\Big/\left(56c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]+3dx^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right]\right)\right)\right)\right)\Big/\left(27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}\right)$$

- **Problem 438: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left[\frac{4}{3},2,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right]}{256c^2\sqrt{c+dx^3}}$$

Result (type 6, 355 leaves):

$$\frac{1}{27\sqrt{c+d x^3}} x \left(\frac{c+d x^3}{8 c^2 d-c d^2 x^3}+\left(32 c \,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d \left(-8 c+d x^3\right) \left(32 c \,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(7 x^3 \,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(56 c \,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 439: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(8 c-d x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}} \,\text{AppellF1}\left[\frac{1}{3},2,\frac{1}{2},\frac{4}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{64 c^2 \sqrt{c+d x^3}}$$

Result (type 6, 327 leaves):

$$\frac{1}{216 \left(8 c-d x^3\right) \sqrt{c+d x^3}} x \left(\left(832 \,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(32 c \,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)+\frac{c+d x^3+\frac{7 c d x^3 \,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]}{56 c \,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)}}{c^2}\right)$$

Problem 440: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(8 c - d x^3\right)^2 \sqrt{c + d x^3}} \, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1 + \frac{d x^3}{c}} \text{ AppellF1}\left[-\frac{2}{3}, 2, \frac{1}{2}, \frac{1}{3}, \frac{d x^3}{8 c}, -\frac{d x^3}{c}\right]}{128 c^2 x^2 \sqrt{c + d x^3}}$$

Result (type 6, 372 leaves):

$$\frac{1}{3456 c^3 x^2 \sqrt{c + d x^3}} \left(-\frac{\left(c + d x^3\right) \left(-216 c + 29 d x^3\right)}{-8 c + d x^3} - \left(64 c^2 d x^3 \text{ AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right)\right.\right.$$
$$\left(32 c \text{ AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \text{ AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\Big) +$$
$$\left(203 c d^2 x^6 \text{ AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right)\right.$$
$$\left.\left.\left(56 c \text{ AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \text{ AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 441: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6 \left(8 c - d x^3\right)^2 \sqrt{c + d x^3}} \, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1 + \frac{d x^3}{c}} \text{ AppellF1}\left[-\frac{5}{3}, 2, \frac{1}{2}, -\frac{2}{3}, \frac{d x^3}{8 c}, -\frac{d x^3}{c}\right]}{320 c^2 x^5 \sqrt{c + d x^3}}$$

Result (type 6, 384 leaves):

$$\frac{1}{34560\, c^4\, x^5 \sqrt{c+d\, x^3}} \left(\frac{(c+d\, x^3)\, (864\, c^2 - 1080\, c\, d\, x^3 + 119\, d^2\, x^6)}{-8\, c + d\, x^3} + \left(21952\, c^2\, d^2\, x^6\, \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right]\right) \Big/ \left((8\, c - d\, x^3) \left(32\, c\, \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right] + 3\, d\, x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right] - 4\, \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right]\right)\right)\right) - \left(833\, c\, d^3\, x^9\, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right]\right) \Big/ \left((8\, c - d\, x^3) \left(56\, c\, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right] + 3\, d\, x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right] - 4\, \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right]\right)\right)\right)\right)$$

- **Problem 446: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\, (8\, c - d\, x^3)^2\, (c+d\, x^3)^{3/2}}\, dx$$

Optimal (type 3, 106 leaves, 8 steps) :

$$\frac{5}{648\, c^3 \sqrt{c+d\, x^3}} + \frac{1}{216\, c^2\, (8\, c - d\, x^3) \sqrt{c+d\, x^3}} + \frac{7\, \text{ArcTanh}\left[\frac{\sqrt{c+d\, x^3}}{3\sqrt{c}}\right]}{7776\, c^{7/2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\, x^3}}{\sqrt{c}}\right]}{96\, c^{7/2}}$$

Result (type 6, 338 leaves) :

$$\frac{1}{324 \sqrt{c+d\, x^3}} \left(\frac{43\, c - 5\, d\, x^3}{16\, c^4 - 2\, c^3\, d\, x^3} - \left(20\, d\, x^3\, \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right]\right) \Big/ \left(c^2\, (8\, c - d\, x^3) \left(16\, c\, \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right] + d\, x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right] - 4\, \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{d\, x^3}{c}, \frac{d\, x^3}{8\, c}\right]\right)\right)\right) + \left(45\, d\, x^3\, \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d\, x^3}, \frac{8\, c}{d\, x^3}\right]\right) \Big/ \left(c^2\, (-8\, c + d\, x^3) \left(5\, d\, x^3\, \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d\, x^3}, \frac{8\, c}{d\, x^3}\right] + 16\, c\, \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d\, x^3}, \frac{8\, c}{d\, x^3}\right] - c\, \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d\, x^3}, \frac{8\, c}{d\, x^3}\right]\right)\right)\right)$$

- **Problem 447: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4\, (8\, c - d\, x^3)^2\, (c+d\, x^3)^{3/2}}\, dx$$

Optimal (type 3, 143 leaves, 9 steps) :

$$-\frac{35\, d}{2592\, c^4 \sqrt{c+d\, x^3}} + \frac{5\, d}{864\, c^3\, (8\, c - d\, x^3) \sqrt{c+d\, x^3}} - \frac{1}{24\, c^2\, x^3\, (8\, c - d\, x^3) \sqrt{c+d\, x^3}} + \frac{5\, d\, \text{ArcTanh}\left[\frac{\sqrt{c+d\, x^3}}{3\sqrt{c}}\right]}{31104\, c^{9/2}} + \frac{5\, d\, \text{ArcTanh}\left[\frac{\sqrt{c+d\, x^3}}{\sqrt{c}}\right]}{384\, c^{9/2}}$$

Result (type 6, 350 leaves):

$$\frac{1}{2592\,c^4\,x^3\,\sqrt{c+d\,x^3}}\left(\frac{108\,c^2+265\,c\,d\,x^3-35\,d^2\,x^6}{-8\,c+d\,x^3}+\left(280\,c\,d^2\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\left(450\,c\,d^2\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 448: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^7\left(8\,c-d\,x^3\right)^2\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 3, 185 leaves, 10 steps):

$$\frac{665\,d^2}{41\,472\,c^5\,\sqrt{c+d\,x^3}}-\frac{71\,d^2}{13\,824\,c^4\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}-\frac{1}{48\,c^2\,x^6\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}+\frac{17\,d}{384\,c^3\,x^3\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}+\frac{13\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{497\,664\,c^{11/2}}-\frac{33\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{2048\,c^{11/2}}$$

Result (type 6, 349 leaves):

$$\frac{1}{41\,472\,c^5\,x^6\,\sqrt{c+d\,x^3}}\left(-\left(5320\,c\,d^3\,x^9\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+1\Big/\left(-8\,c+d\,x^3\right)\left(864\,c^3-1836\,c^2\,d\,x^3-5107\,c\,d^2\,x^6+665\,d^3\,x^9+\left(8910\,c\,d^3\,x^9\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 449: Result unnecessarily involves higher level functions.**

$$\int\frac{x^7}{\left(8\,c-d\,x^3\right)^2\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 668 leaves, 15 steps):

$$-\frac{2 x^2}{81 c d^2 \sqrt{c+d x^3}}+\frac{8 x^2}{27 d^2 \left(8 c-d x^3\right) \sqrt{c+d x^3}}+\frac{2 \sqrt{c+d x^3}}{81 c d^{8/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{4 \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{81 \sqrt{3} c^{5/6} d^{8/3}}-\frac{4 \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{243 c^{5/6} d^{8/3}}+$$

$$\frac{4 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{243 c^{5/6} d^{8/3}}-\frac{\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{27\times 3^{3/4} c^{2/3} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}+$$

$$\frac{2 \sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{81\times 3^{1/4} c^{2/3} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 357 leaves):

$$\frac{1}{405 d^2 \sqrt{c+d x^3}} 2 x^2 \left(\frac{20 c+5 d x^3}{8 c^2-c d x^3}+\left(800 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(32 d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 450: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4}{\left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 671 leaves, 15 steps):

$$-\frac{x^2}{81 c^2 d \sqrt{c+d x^3}}+\frac{x^2}{27 c d \left(8 c-d x^3\right) \sqrt{c+d x^3}}+\frac{\sqrt{c+d x^3}}{81 c^2 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{81 \sqrt{3} c^{11/6} d^{5/3}}+\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{243 c^{11/6} d^{5/3}}-$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{243 c^{11/6} d^{5/3}}-\frac{\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{54\times 3^{3/4} c^{5/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}+$$

$$\frac{\sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{81\times 3^{1/4} c^{5/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 337 leaves) :

$$\frac{1}{405 \left(8 c-d x^3\right) \sqrt{c+d x^3}} x^2 \left(\left(1000 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \right.$$

$$\left(d \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$

$$1 \Big/ c^2 \left(5 \left(-\frac{5 c}{d}+x^3\right)-\left(32 c x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \right.$$

$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 451: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{\left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 665 leaves, 15 steps) :

$$\frac{5 x^2}{648 c^3 \sqrt{c+d x^3}}+\frac{x^2}{216 c^2 \left(8 c-d x^3\right) \sqrt{c+d x^3}}-\frac{5 \sqrt{c+d x^3}}{648 c^3 d^{2/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{5 \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{1296 \sqrt{3} c^{17/6} d^{2/3}}+\frac{5 \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{3888 c^{17/6} d^{2/3}}-$$

$$\frac{5 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{3888 c^{17/6} d^{2/3}}+\frac{5 \sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{432\times 3^{3/4} c^{8/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}-$$

$$\frac{5 \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{324 \sqrt{2}\, 3^{1/4} c^{8/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 366 leaves):

$$\frac{1}{162 \sqrt{c+d x^3}}\left(\frac{43 c x^2-5 d x^5}{32 c^4-4 c^3 d x^3}-\left(25 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c \left(8 c-d x^3\right)\right.\right.$$
$$\left.\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(8 d x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c^2 \left(8 c-d x^3\right)\right.$$
$$\left.\left.\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 452: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 \left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 4, 686 leaves, 16 steps):

$$\frac{5}{648\,c^3\,x\,\sqrt{c+d\,x^3}}+\frac{1}{216\,c^2\,x\,\left(8\,c-d\,x^3\right)\,\sqrt{c+d\,x^3}}-\frac{31\,\sqrt{c+d\,x^3}}{1296\,c^4\,x}+\frac{31\,d^{1/3}\,\sqrt{c+d\,x^3}}{1296\,c^4\,\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)}-$$

$$\frac{d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{1296\,\sqrt{3}\,c^{23/6}}+\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\,\sqrt{c+d\,x^3}}\right]}{3888\,c^{23/6}}-\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{3888\,c^{23/6}}-$$

$$\left(31\,\sqrt{2-\sqrt{3}}\,d^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(864\times 3^{3/4}\,c^{11/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)+$$

$$\frac{31\,d^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{648\,\sqrt{2}\,3^{1/4}\,c^{11/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}}$$

Result (type 6, 374 leaves):

$$\frac{1}{6480\,c^4\,\sqrt{c+d\,x^3}}\left(\frac{5\,\left(162\,c^2+227\,c\,d\,x^3-31\,d^2\,x^6\right)}{-8\,c\,x+d\,x^4}+\left(13\,000\,c^2\,d\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\right.\right.$$

$$\left.\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)-$$

$$\left(992\,c\,d^2\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\right.$$

$$\left.\left.\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

- **Problem 453: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^5\,\left(8\,c-d\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 708 leaves, 17 steps):

$$\frac{5}{648\,c^3\,x^4\,\sqrt{c+d\,x^3}}+\frac{1}{216\,c^2\,x^4\,\left(8\,c-d\,x^3\right)\,\sqrt{c+d\,x^3}}-\frac{253\,\sqrt{c+d\,x^3}}{20\,736\,c^4\,x^4}+\frac{77\,d\,\sqrt{c+d\,x^3}}{2592\,c^5\,x}-$$

$$\frac{77\,d^{4/3}\,\sqrt{c+d\,x^3}}{2592\,c^5\,\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)}-\frac{11\,d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{82\,944\,\sqrt{3}\,c^{29/6}}+\frac{11\,d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\,\sqrt{c+d\,x^3}}\right]}{248\,832\,c^{29/6}}-\frac{11\,d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{248\,832\,c^{29/6}}+$$

$$\left(77\,\sqrt{2-\sqrt{3}}\,d^{4/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(1728\times 3^{3/4}\,c^{14/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}\right)-$$

$$\frac{77\,d^{4/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{1296\,\sqrt{2}\,3^{1/4}\,c^{14/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}}$$

Result (type 6, 389 leaves) :

$$\frac{1}{103\,680\,c^5\,x^4\,\sqrt{c+d\,x^3}}\left(\frac{5\,\left(648\,c^3-2997\,c^2\,d\,x^3-4565\,c\,d^2\,x^6+616\,d^3\,x^9\right)}{-8\,c+d\,x^3}-\left(244\,750\,c^2\,d^2\,x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\right.\right.$$

$$\left.\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+$$

$$\left(19\,712\,c\,d^3\,x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\right.$$

$$\left.\left.\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

- **Problem 454: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^8\,\left(8\,c-d\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 732 leaves, 18 steps) :

$$\frac{5}{648\,c^3\,x^7\,\sqrt{c+d\,x^3}}+\frac{1}{216\,c^2\,x^7\,\left(8\,c-d\,x^3\right)\,\sqrt{c+d\,x^3}}-\frac{191\,\sqrt{c+d\,x^3}}{18\,144\,c^4\,x^7}+\frac{8257\,d\,\sqrt{c+d\,x^3}}{580\,608\,c^5\,x^4}-\frac{5179\,d^2\,\sqrt{c+d\,x^3}}{145\,152\,c^6\,x}+$$

$$\frac{5179\,d^{7/3}\,\sqrt{c+d\,x^3}}{145\,152\,c^6\,\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)}-\frac{7\,d^{7/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{331\,776\,\sqrt{3}\,c^{35/6}}+\frac{7\,d^{7/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\,\sqrt{c+d\,x^3}}\right]}{995\,328\,c^{35/6}}-\frac{7\,d^{7/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{995\,328\,c^{35/6}}-$$

$$\left(5179\,\sqrt{2-\sqrt{3}}\,d^{7/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(96\,768\times 3^{3/4}\,c^{17/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)+$$

$$\frac{5179\,d^{7/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{72\,576\,\sqrt{2}\,3^{1/4}\,c^{17/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}}$$

Result (type 6, 374 leaves) :

$$\left(-51\,840\,c^4+93\,960\,c^3\,d\,x^3-509\,085\,c^2\,d^2\,x^6-766\,345\,c\,d^3\,x^9+103\,580\,d^4\,x^{12}+\left(8\,293\,750\,c^2\,d^3\,x^9\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.$$
$$\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,2,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\,\frac{3}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\right)-$$
$$\left(662\,912\,c\,d^4\,x^{12}\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]+\right.$$
$$\left.\left.3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{8}{3},\,\frac{1}{2},\,2,\,\frac{11}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\,\frac{3}{2},\,1,\,\frac{11}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\Big/\left(2\,903\,040\,c^6\,x^7\,\left(8\,c-d\,x^3\right)\,\sqrt{c+d\,x^3}\right)$$

- **Problem 455: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^6}{\left(8\,c-d\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2x\left(4c+dx^3\right)}{81cd^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{81\times 3^{1/4}cd^{7/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}}$$

Result (type 4, 189 leaves) :

$$-\left(6(-d)^{1/3}x\left(4c+dx^3\right)+2\,i\,3^{3/4}c^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-c^{1/3}+(-d)^{1/3}x\right)}{c^{1/3}}}\sqrt{1+\frac{(-d)^{1/3}x}{c^{1/3}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}\right.$$
$$\left.\left(-8c+dx^3\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-d)^{1/3}x}{c^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(243c(-d)^{7/3}\left(-8c+dx^3\right)\sqrt{c+dx^3}\right)$$

Problem 456: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps) :

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left[\frac{4}{3},2,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right]}{256c^3\sqrt{c+dx^3}}$$

Result (type 6, 333 leaves) :

$$\frac{1}{81\,(8\,c-d\,x^3)\,\sqrt{c+d\,x^3}}\left(\left(160\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(d\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\frac{x\left(-\frac{5\,c}{d}+x^3+\frac{7\,c\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]}{56\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)}\right)}{c^2}\right)$$

- **Problem 457: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(8\,c-d\,x^3)^2\,(c+d\,x^3)^{3/2}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{d\,x^3}{8\,c},-\frac{d\,x^3}{c}\right]}{64\,c^3\,\sqrt{c+d\,x^3}}$$

Result (type 6, 331 leaves):

$$\left(x\left(43\,c-5\,d\,x^3+\left(1216\,c^2\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)-\left(35\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(56\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\Big/\left(648\,c^3\,(8\,c-d\,x^3)\,\sqrt{c+d\,x^3}\right)$$

- **Problem 458: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^3\,(8\,c-d\,x^3)^2\,(c+d\,x^3)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{128 c^3 x^2 \sqrt{c+d x^3}}$$

Result (type 6, 375 leaves):

$$\frac{1}{10368 c^4 x^2 \sqrt{c+d x^3}}\left(\frac{648 c^2+1249 c d x^3-167 d^2 x^6}{-8 c+d x^3}-\left(19648 c^2 d x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(1169 c d^2 x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 459: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^6 \left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{5}{3},2,\frac{3}{2},-\frac{2}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{320 c^3 x^5 \sqrt{c+d x^3}}$$

Result (type 6, 388 leaves):

$$\frac{1}{103680 c^5 x^5 \sqrt{c+d x^3}}\left(\frac{2592 c^3-7128 c^2 d x^3-15373 c d^2 x^6+2027 d^3 x^9}{-8 c+d x^3}+\left(262336 c^2 d^2 x^6 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(14189 c d^3 x^9 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

- **Problem 463: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x \left(a+b x^3\right)^2} \, dx$$

Optimal (type 3, 121 leaves, 7 steps):

$$\frac{\sqrt{c+d x^3}}{3 a \left(a+b x^3\right)}-\frac{2 \sqrt{c} \,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2}+\frac{(2 b c-a d) \,\text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 \sqrt{b} \sqrt{b c-a d}}$$

Result (type 6, 306 leaves):

$$\frac{1}{9 \left(a+b x^3\right) \sqrt{c+d x^3}}\left(-\left(6 c d x^3 \,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\right.$$
$$\left(-4 a c \,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3 \left(2 b c \,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+$$
$$1/a\left(3 \left(c+d x^3\right)+\left(10 b c d x^3 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\Big/\left(-5 b d x^3 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+\right.\right.$$
$$\left.\left.\left.2 a d \,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)$$

- **Problem 464: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+d x^3}}{x^4 \left(a+b x^3\right)^2} \, dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$-\frac{2 b \sqrt{c+d x^3}}{3 a^2 \left(a+b x^3\right)}-\frac{\sqrt{c+d x^3}}{3 a x^3 \left(a+b x^3\right)}+\frac{(4 b c-a d) \,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^3 \sqrt{c}}-\frac{\sqrt{b} (4 b c-3 a d) \,\text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^3 \sqrt{b c-a d}}$$

Result (type 6, 410 leaves):

$$\left(\left(12\,a\,b\,c\,d\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\right.$$
$$\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+$$
$$\left(5\,b\,d\,x^3\left(3\,a\,c+2\,b\,c\,x^3+4\,a\,d\,x^3+6\,b\,d\,x^6\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]-\right.$$
$$\left.3\left(a+2\,b\,x^3\right)\left(c+d\,x^3\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/$$
$$\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+\right.$$
$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(9\,a^2\,x^3\left(a+b\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 465: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3\sqrt{c+d\,x^3}}{\left(a+b\,x^3\right)^2}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4\sqrt{c+d\,x^3}\;\text{AppellF1}\left[\frac{4}{3},2,-\frac{1}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,a^2\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 324 leaves):

$$\frac{1}{12\,b\left(a+b\,x^3\right)\sqrt{c+d\,x^3}}x\left(-4\left(c+d\,x^3\right)+\left(32\,a\,c^2\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-\right.\right.$$
$$\left.3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)-$$
$$\left(35\,a\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+\right.$$
$$\left.\left.3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

Problem 466: Result more than twice size of optimal antiderivative.

$$\int\frac{x\sqrt{c+d\,x^3}}{\left(a+b\,x^3\right)^2}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2\sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{2}{3},2,-\frac{1}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2\sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 324 leaves):

$$\frac{1}{15\left(a+b x^3\right)\sqrt{c+d x^3}}x^2\left(\frac{5\left(c+d x^3\right)}{a}+\left(25 c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(10 a c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(8 c d x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-16 a c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

- **Problem 467: Result more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{c+d x^3}}{\left(a+b x^3\right)^2}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{1}{3},2,-\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2\sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 322 leaves):

$$\frac{1}{12\left(a+b x^3\right)\sqrt{c+d x^3}}x\left(\frac{4\left(c+d x^3\right)}{a}+\left(64 c^2\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(8 a c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)-\left(7 c d x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-14 a c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 468: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^2 \left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2 x \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 347 leaves):

$$\left(-10 \left(3 a+4 b x^3\right) \left(c+d x^3\right)+\left(25 a c \left(-8 b c+9 a d\right) x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3 \left(2 b c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(64 a b c d x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(16 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3 \left(2 b c \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\Big/\left(30 a^2 x \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 469: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^3 \left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 x^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 347 leaves):

$$\left(-4\left(3a+5bx^3\right)\left(c+dx^3\right)+\left(16ac\left(-20bc+9ad\right)x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\Big/\left(8ac\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]-3x^3\left(2bc\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+ad\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\right)+\left(35abcdx^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\Big/\left(-14ac\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+3x^3\left(2bc\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+ad\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\right)\right)\Big/\left(24a^2x^2\left(a+bx^3\right)\sqrt{c+dx^3}\right)$$

- **Problem 473: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$\frac{\left(bc-ad\right)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)}-\frac{2c^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right]}{3a^2}+\frac{\sqrt{bc-ad}\,\left(2bc+ad\right)\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right]}{3a^2b^{3/2}}$$

Result (type 6, 328 leaves):

$$\frac{1}{9b\left(a+bx^3\right)\sqrt{c+dx^3}}\left(-\left(6cd\left(bc+ad\right)x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\Big/\left(-4ac\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+x^3\left(2bc\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+ad\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\right)+1/a\left(3\left(bc-ad\right)\left(c+dx^3\right)+\left(10b^2c^2dx^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]\right)\Big/\left(-5bdx^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]+2ad\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]+bc\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]\right)\right)\right)$$

- **Problem 474: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx$$

Optimal (type 3, 170 leaves, 8 steps):

$$-\frac{\left(2bc-ad\right)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}+\frac{\sqrt{c}\,\left(4bc-3ad\right)\text{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right]}{3a^3}-\frac{\sqrt{bc-ad}\,\left(4bc-ad\right)\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right]}{3a^3\sqrt{b}}$$

Result (type 6, 439 leaves):

$$\left(\left(6\,a\,c\,d\,(-2\,b\,c+a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\right.$$
$$\left(4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+$$
$$\left(5\,b\,d\,x^3\left(2\,b\,c\,x^3\left(c+3\,d\,x^3\right)+3\,a\left(c^2+c\,d\,x^3-d^2\,x^6\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]-\right.$$
$$3\left(c+d\,x^3\right)\left(2\,b\,c\,x^3+a\left(c-d\,x^3\right)\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/$$
$$\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+\right.$$
$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(9\,a^2\,x^3\left(a+b\,x^3\right)\sqrt{c+d\,x^3}\right)$$

- **Problem 475: Result more than twice size of optimal antiderivative.**

$$\int\frac{x^3\left(c+d\,x^3\right)^{3/2}}{\left(a+b\,x^3\right)^2}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{c\,x^4\sqrt{c+d\,x^3}\,\text{AppellF1}\left[\frac{4}{3},2,-\frac{3}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,a^2\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 358 leaves):

$$\frac{1}{60\,b^2\left(a+b\,x^3\right)\sqrt{c+d\,x^3}}x\left(-4\left(c+d\,x^3\right)\left(5\,b\,c-11\,a\,d-6\,b\,d\,x^3\right)-\right.$$
$$\left(32\,a\,c^2\,(-5\,b\,c+11\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-\right.$$
$$\left.3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+$$
$$\left(7\,a\,c\,d\,(-43\,b\,c+55\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+\right.$$
$$\left.\left.3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

Problem 476: Result more than twice size of optimal antiderivative.

$$\int \frac{x \left(c+d x^3\right)^{3/2}}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{c x^2 \sqrt{c+d x^3} \text{ AppellF1}\left[\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 439 leaves):

$$\frac{1}{15 b \left(a+b x^3\right) \sqrt{c+d x^3}} x^2 \left(-\left(25 c^2 (b c+2 a d) \text{ AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(-10 a c \text{ AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.$$
$$\left. 3 x^3 \left(2 b c \text{ AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{ AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+$$
$$\left(-8 a c \left(a d \left(10 c+3 d x^3\right)-b c \left(10 c+9 d x^3\right)\right) \text{ AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-\right.$$
$$\left. 15 (b c-a d) x^3 \left(c+d x^3\right) \left(2 b c \text{ AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{ AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right) \Big/$$
$$\left(a \left(16 a c \text{ AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-\right.\right.$$
$$\left.\left.\left. 3 x^3 \left(2 b c \text{ AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{ AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

Problem 477: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 60 leaves, 2 steps):

$$\frac{c x \sqrt{c+d x^3} \text{ AppellF1}\left[\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 437 leaves):

$$\frac{1}{12\,b\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}}x\left(-\left(32\,c^2\,(2\,b\,c+a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(-7\,a\,c\,\left(a\,d\,\left(8\,c+3\,d\,x^3\right)-b\,c\,\left(8\,c+9\,d\,x^3\right)\right)\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-12\,(b\,c-a\,d)\,x^3\,\left(c+d\,x^3\right)\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\Big/\left(a\,\left(14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 478: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(c+d\,x^3\right)^{3/2}}{x^2\,\left(a+b\,x^3\right)^2}\,dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$-\frac{c\,\sqrt{c+d\,x^3}\,\text{AppellF1}\left[-\frac{1}{3},2,-\frac{3}{2},\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a^2\,x\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 365 leaves):

$$\left(-10\,\left(c+d\,x^3\right)\,\left(3\,a\,c+4\,b\,c\,x^3-a\,d\,x^3\right)+\left(25\,a\,c^2\,(-8\,b\,c+11\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(16\,a\,c\,d\,(-4\,b\,c+a\,d)\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\Big/\left(30\,a^2\,x\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}\right)$$

Problem 479: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^3 \left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{c \sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{2}{3}, 2, -\frac{3}{2}, \frac{1}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 a^2 x^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 366 leaves):

$$\Bigg(-4 \left(c+d x^3\right) \left(3 a c+5 b c x^3-2 a d x^3\right)+\left(16 a c^2 (-20 b c+17 a d) x^3 \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(8 a c \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] - 3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) - \left(7 a c d (-5 b c+2 a d) x^6 \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(-14 a c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\Bigg) \Big/ \left(24 a^2 x^2 \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 483: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b \sqrt{c+d x^3}}{3 a (b c-a d) \left(a+b x^3\right)} - \frac{2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 \sqrt{c}} + \frac{\sqrt{b} (2 b c-3 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 (b c-a d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(b\left(\left(6 c d x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3\left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(5 d x^3\left(2 a d+b\left(c+3 d x^3\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-3\left(c+d x^3\right)\left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(a\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)\right)\Big/\left(9(-b c+a d)\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 484: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4\left(a+b x^3\right)^2\sqrt{c+d x^3}}\,dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b(2 b c-a d)\sqrt{c+d x^3}}{3 a^2 c(b c-a d)\left(a+b x^3\right)}-\frac{\sqrt{c+d x^3}}{3 a c x^3\left(a+b x^3\right)}+\frac{(4 b c+a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^3 c^{3/2}}-\frac{b^{3/2}(4 b c-5 a d) \text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^3(b c-a d)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6 a b d(-2 b c+a d) x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left((-b c+a d)\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3\left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)+\left(5 b d x^3\left(-a^2 d\left(3 c+2 d x^3\right)+2 b^2 c x^3\left(c+3 d x^3\right)+3 a b\left(c^2+c d x^3-d^2 x^6\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+3\left(c+d x^3\right)\left(a^2 d-2 b^2 c x^3+a b\left(-c+d x^3\right)\right)\left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(c(b c-a d)\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)\Big/\left(9 a^2 x^3\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 485: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a^2 \sqrt{c+d x^3}}$$

Result (type 6, 331 leaves) :

$$\left(x \left(4 \left(c+d x^3\right)+\left(32 a c^2 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.\right.\right.$$
$$\left.3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)-$$
$$\left(7 a c d x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\right.$$
$$\left.\left.\left.\left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right) \Big/ \left(12 (-b c+a d) \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 486: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$\frac{x^2 \sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 \sqrt{c+d x^3}}$$

Result (type 6, 342 leaves) :

$$\left(x^2\left(-\frac{5\,b\,(c+d\,x^3)}{a}+\left(25\,c\,(b\,c-3\,a\,d)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)-\left(8\,b\,c\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\Big/\left(15\,(-b\,c+a\,d)\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 487: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+b\,x^3)^2\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 341 leaves):

$$\left(x\left(-\frac{4\,b\,(c+d\,x^3)}{a}+\left(32\,c\,(2\,b\,c-3\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(7\,b\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\Big/\left(12\,(-b\,c+a\,d)\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 488: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^2\,(a+b\,x^3)^2\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2 x \sqrt{c+d x^3}}$$

Result (type 6, 399 leaves) :

$$\left(\frac{10\left(c+d x^3\right)\left(-3 a^2 d+4 b^2 c x^3+3 a b\left(c-d x^3\right)\right)}{c}-\left(25 a\left(8 b^2 c^2-15 a b c d+3 a^2 d^2\right) x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-10 a c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(16 a b d(4 b c-3 a d) x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-16 a c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\Big/\left(30 a^2(-b c+a d) x\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 489: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^3\left(a+b x^3\right)^2\sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 x^2 \sqrt{c+d x^3}}$$

Result (type 6, 399 leaves) :

$$\left(\frac{4\left(c+d x^3\right)\left(-3 a^2 d+5 b^2 c x^3+3 a b\left(c-d x^3\right)\right)}{c}+\left(16 a\left(-20 b^2 c^2+21 a b c d+3 a^2 d^2\right) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(7 a b d(-5 b c+3 a d) x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\Big/\left(24 a^2(-b c+a d) x^2\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 493: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\left(a+b x^3\right)^2\left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 3, 172 leaves, 8 steps):

$$\frac{d(b c+2 a d)}{3 a c(b c-a d)^2\sqrt{c+d x^3}}+\frac{b}{3 a(b c-a d)\left(a+b x^3\right)\sqrt{c+d x^3}}-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 c^{3/2}}+\frac{b^{3/2}(2 b c-5 a d) \text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2(b c-a d)^{5/2}}$$

Result (type 6, 453 leaves):

$$\left(-\left(6 b d(b c+2 a d) x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3\left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(-5 b d x^3\left(4 a^2 d^2+b^2 c\left(c+3 d x^3\right)+2 a b d\left(2 c+3 d x^3\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+3\left(2 a^2 d^2+2 a b d^2 x^3+b^2 c\left(c+d x^3\right)\right)\left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(a c\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)\Big/\left(9(b c-a d)^2\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 494: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4 \left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 241 leaves, 9 steps):

$$-\frac{d \left(2 b^2 c^2-2 a b c d+3 a^2 d^2\right)}{3 a^2 c^2 (b c-a d)^2 \sqrt{c+d x^3}}-\frac{b (2 b c-a d)}{3 a^2 c (b c-a d) \left(a+b x^3\right) \sqrt{c+d x^3}}-\frac{1}{3 a c x^3 \left(a+b x^3\right) \sqrt{c+d x^3}}+\frac{(4 b c+3 a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^3 c^{5/2}}-\frac{b^{5/2} (4 b c-7 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^3 (b c-a d)^{5/2}}$$

Result (type 6, 582 leaves):

$$\frac{1}{9 a^2 c^2 (b c-a d)^2 x^3 \left(a+b x^3\right) \sqrt{c+d x^3}}\left(\left(6 a b c d \left(2 b^2 c^2-2 a b c d+3 a^2 d^2\right) x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\right.$$
$$\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)-$$
$$\left(-5 b d x^3 \left(3 a^3 d^2 \left(c+2 d x^3\right)+2 b^3 c^2 x^3 \left(c+3 d x^3\right)+a b^2 c \left(3 c^2+2 c d x^3-6 d^2 x^6\right)+a^2 b d \left(-6 c^2-c d x^3+9 d^2 x^6\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},\right.\right.$$
$$\left.1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+3 \left(2 b^3 c^2 x^3 \left(c+d x^3\right)+a^3 d^2 \left(c+3 d x^3\right)+a b^2 c \left(c^2-c d x^3-2 d^2 x^6\right)+a^2 b d \left(-2 c^2-c d x^3+3 d^2 x^6\right)\right)$$
$$\left.\left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/$$
$$\left.\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)$$

- **Problem 495: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a^2 c \sqrt{c+d x^3}}$$

Result (type 6, 346 leaves):

$$\left(x\left(-4\left(b c+2 a d+3 b d x^3\right)+\left(32 a c\,(b c+2 a d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(8 a c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(21 a b c d x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-14 a c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\Big/\left(12\,(b c-a d)^2\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 496: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{\left(a+b x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^2\sqrt{1+\frac{d x^3}{c}}\;\text{AppellF1}\left[\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 c\sqrt{c+d x^3}}$$

Result (type 6, 482 leaves):

$$\left(x^2\left(-\left(25\left(b^2 c^2-6 a b c d-a^2 d^2\right)\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-10 a c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(8 a c\left(20 a^2 d^2+18 a b d^2 x^3+b^2 c\left(10 c+9 d x^3\right)\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-15 x^3\left(2 a^2 d^2+2 a b d^2 x^3+b^2 c\left(c+d x^3\right)\right)\left(2 b c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\Big/\left(a c\left(16 a c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\right)\Big/\left(15\,(b c-a d)^2\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

- **Problem 497: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}}\, \text{AppellF1}\left[\frac{1}{3}, 2, \frac{3}{2}, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{a^2 c \sqrt{c+d x^3}}$$

Result (type 6, 480 leaves) :

$$\left(x \left(-\left(32 \left(2 b^2 c^2 - 6 a b c d + a^2 d^2\right) \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(-8 a c\, \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c\, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d\, \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) + \left(7 a c \left(16 a^2 d^2 + 18 a b d^2 x^3 + b^2 c \left(8 c + 9 d x^3\right)\right) \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] - 12 x^3 \left(2 a^2 d^2 + 2 a b d^2 x^3 + b^2 c \left(c + d x^3\right)\right) \left(2 b c\, \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d\, \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) \Big/ \left(a c \left(14 a c\, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] - 3 x^3 \left(2 b c\, \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d\, \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)\right) \Big/ \left(12 \left(b c - a d\right)^2 \left(a + b x^3\right) \sqrt{c + d x^3}\right)$$

- **Problem 498: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^2 \left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 65 leaves, 2 steps) :

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\, \text{AppellF1}\left[-\frac{1}{3}, 2, \frac{3}{2}, \frac{2}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{a^2 c x \sqrt{c+d x^3}}$$

Result (type 6, 483 leaves) :

$$\frac{1}{30\,a^2\,c^2\,(b\,c-a\,d)^2\,x\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}}\left(-10\,\left(4\,b^3\,c^2\,x^3\,\left(c+d\,x^3\right)+a^3\,d^2\,\left(3\,c+5\,d\,x^3\right)+3\,a\,b^2\,c\,\left(c^2-c\,d\,x^3-2\,d^2\,x^6\right)+a^2\,b\,d\,\left(-6\,c^2-3\,c\,d\,x^3+5\,d^2\,x^6\right)\right)+\left(25\,a\,c\,\left(-8\,b^3\,c^3+21\,a\,b^2\,c^2\,d-6\,a^2\,b\,c\,d^2+5\,a^3\,d^3\right)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)-\left(16\,a\,b\,c\,d\,\left(4\,b^2\,c^2-6\,a\,b\,c\,d+5\,a^2\,d^2\right)\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

Problem 499: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\,\left(a+b\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a^2\,c\,x^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 483 leaves):

$$\frac{1}{24\,a^2\,c^2\,(b\,c-a\,d)^2\,x^2\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}}\left(-4\,\left(5\,b^3\,c^2\,x^3\,\left(c+d\,x^3\right)+a^3\,d^2\,\left(3\,c+7\,d\,x^3\right)+3\,a\,b^2\,c\,\left(c^2-c\,d\,x^3-2\,d^2\,x^6\right)+a^2\,b\,d\,\left(-6\,c^2-3\,c\,d\,x^3+7\,d^2\,x^6\right)\right)+\left(16\,a\,c\,\left(20\,b^3\,c^3-33\,a\,b^2\,c^2\,d-6\,a^2\,b\,c\,d^2+7\,a^3\,d^3\right)\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(7\,a\,b\,c\,d\,\left(5\,b^2\,c^2-6\,a\,b\,c\,d+7\,a^2\,d^2\right)\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

- **Problem 508: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\sqrt{a+b x^3}\,\sqrt{c+d x^3}}\,dx$$

Optimal (type 3, 48 leaves, 3 steps):

$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{a+b x^3}}{\sqrt{a}\,\sqrt{c+d x^3}}\right]}{3\sqrt{a}\,\sqrt{c}}$$

Result (type 6, 155 leaves):

$$\left(4\,b\,d\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\Big/\left(3\sqrt{a+b x^3}\,\sqrt{c+d x^3}\right.$$
$$\left.\left(-4\,b\,d\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+b\,c\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)$$

- **Problem 509: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4\sqrt{a+b x^3}\,\sqrt{c+d x^3}}\,dx$$

Optimal (type 3, 91 leaves, 4 steps):

$$-\frac{\sqrt{a+b x^3}\,\sqrt{c+d x^3}}{3\,a\,c\,x^3}+\frac{(b\,c+a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{a+b x^3}}{\sqrt{a}\,\sqrt{c+d x^3}}\right]}{3\,a^{3/2}\,c^{3/2}}$$

Result (type 6, 192 leaves):

$$\left(-\left(a+b x^3\right)\left(c+d x^3\right)+\left(2\,b\,d\,(b\,c+a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\Big/\left(4\,b\,d\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]-\right.\right.$$
$$\left.\left.b\,c\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]-a\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)\Big/\left(3\,a\,c\,x^3\sqrt{a+b x^3}\,\sqrt{c+d x^3}\right)$$

- **Problem 513: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a+b x^3}\,\sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{x\sqrt{1+\frac{b x^3}{a}}\,\sqrt{1+\frac{d x^3}{c}}\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{\sqrt{a+b x^3}\,\sqrt{c+d x^3}}$$

Result (type 6, 170 leaves):

$$-\left(8\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\sqrt{a+b\,x^3}\,\sqrt{c+d\,x^3}\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,x^3\left(a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},\frac{1}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

Problem 514: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\,\sqrt{a+b\,x^3}\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 86 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{b\,x^3}{a}}\,\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{x\,\sqrt{a+b\,x^3}\,\sqrt{c+d\,x^3}}$$

Result (type 6, 357 leaves):

$$\frac{1}{10\,x\,\sqrt{a+b\,x^3}\,\sqrt{c+d\,x^3}}\left(-\frac{10\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)}{a\,c}-\left(25\,(b\,c+a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,x^3\left(a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},\frac{3}{2},\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},\frac{1}{2},\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)-\left(64\,b\,d\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+3\,x^3\left(a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},\frac{3}{2},\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},\frac{1}{2},\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

Problem 515: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\,\sqrt{a+b\,x^3}\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{b\,x^3}{a}}\,\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,x^2\,\sqrt{a+b\,x^3}\,\sqrt{c+d\,x^3}}$$

Result (type 6, 357 leaves) :

$$\frac{1}{2 x^2 \sqrt{a+b x^3} \sqrt{c+d x^3}} \left(-\frac{\left(a+b x^3\right) \left(c+d x^3\right)}{a c}+\left(4 (b c+a d) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+3 x^3 \left(a d \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+\left(7 b d x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(28 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-6 x^3 \left(a d \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},\frac{3}{2},\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},\frac{1}{2},\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

- **Problem 517: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e x)^{5/2} \sqrt{a+b x^3} \left(A+B x^3\right) \, dx$$

Optimal (type 4, 324 leaves, 5 steps) :

$$\frac{3 a (16 A b-7 a B) e^2 \sqrt{e x} \sqrt{a+b x^3}}{320 b^2}+\frac{(16 A b-7 a B) (e x)^{7/2} \sqrt{a+b x^3}}{80 b e}+\frac{B (e x)^{7/2} \left(a+b x^3\right)^{3/2}}{8 b e}-\left(3^{3/4} a^{5/3} (16 A b-7 a B) e^2 \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/\left(640 b^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 234 leaves) :

$$\frac{1}{320\,(-a)^{1/3}\,b^2\,\sqrt{a+b\,x^3}}\,e^2\,\sqrt{e\,x}\,\left(-(-a)^{1/3}\,\left(a+b\,x^3\right)\,\left(21\,a^2\,B-12\,a\,b\,\left(4\,A+B\,x^3\right)-8\,b^2\,x^3\,\left(8\,A+5\,B\,x^3\right)\right)+\right.$$

$$\left.i\,3^{3/4}\,a^2\,b^{1/3}\,(16\,A\,b-7\,a\,B)\,x\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 518: Result unnecessarily involves imaginary or complex numbers.

$$\int (e\,x)^{3/2}\,\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)\,dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$\frac{(14\,A\,b-5\,a\,B)\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}{56\,b\,e}+\frac{3\,\left(1+\sqrt{3}\right)\,a\,(14\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{112\,b^{5/3}\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}+\frac{B\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{3/2}}{7\,b\,e}-$$

$$\left(3\times 3^{1/4}\,a^{4/3}\,(14\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(112\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(3^{3/4}\,\left(1-\sqrt{3}\right)\,a^{4/3}\,(14\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(224\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 279 leaves):

$$\frac{1}{112\,b^2\sqrt{a+b\,x^3}}x\,(e\,x)^{3/2}\left(2\,b\,\left(a+b\,x^3\right)\,\left(14\,A\,b+3\,a\,B+8\,b\,B\,x^3\right)-\right.$$

$$a\,(14\,A\,b-5\,a\,B)\left(-3\,\left(b+\frac{a}{x^3}\right)+1\Big/\left(\left((-a)^{2/3}\,x\right)\,(-1)^{1/6}\,3^{3/4}\,a\,b^{2/3}\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.\right.$$

$$\left.\left.\left.\left(-i\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)\right)$$

- **Problem 520: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{\sqrt{e\,x}}\,dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$\frac{(10\,A\,b-a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{20\,b\,e}+\frac{B\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{3/2}}{5\,b\,e}+$$

$$\left(3^{3/4}\,a^{2/3}\,(10\,A\,b-a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(40\,b\,e\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 209 leaves):

$$\left((-a)^{1/3}\,x\,\left(a+b\,x^3\right)\,\left(10\,A\,b+3\,a\,B+4\,b\,B\,x^3\right)-i\,3^{3/4}\,a\,b^{1/3}\,(10\,A\,b-a\,B)\,x^2\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\right.$$

$$\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Bigg/\left(20\,(-a)^{1/3}\,b\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}\right)$$

Problem 521: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{(e\,x)^{3/2}}\,dx$$

Optimal (type 4, 580 leaves, 6 steps):

$$\frac{(8\,A\,b+a\,B)\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}{4\,a\,e^4}+\frac{3\,\left(1+\sqrt{3}\right)\,(8\,A\,b+a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{8\,b^{2/3}\,e^2\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}-\frac{2\,A\,\left(a+b\,x^3\right)^{3/2}}{a\,e\,\sqrt{e\,x}}-$$

$$\left(3\times 3^{1/4}\,a^{1/3}\,(8\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(8\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(3^{3/4}\,\left(1-\sqrt{3}\right)\,a^{1/3}\,(8\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(16\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 283 leaves):

$$\frac{1}{8\,(e\,x)^{3/2}\sqrt{a+b\,x^3}}x^{3/2}\left(\frac{2\,(a+b\,x^3)\,(-8\,A+B\,x^3)}{\sqrt{x}}-\right.$$

$$1/b\,(8\,A\,b+a\,B)\,x^{5/2}\left(-3\left(b+\frac{a}{x^3}\right)+1\Big/\left((-a)^{2/3}\,x\right)(-1)^{1/6}\,3^{3/4}\,a\,b^{2/3}\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.$$

$$\left.\left.\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\right)$$

- **Problem 523: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{(e\,x)^{7/2}}\,dx$$

Optimal (type 4, 283 leaves, 4 steps):

$$\frac{(4\,A\,b+5\,a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{10\,a\,e^4}-\frac{2\,A\,\left(a+b\,x^3\right)^{3/2}}{5\,a\,e\,(e\,x)^{5/2}}+$$

$$\left(3^{3/4}\,(4\,A\,b+5\,a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(20\,a^{1/3}\,e^4\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 199 leaves):

$$\left(x \left((-a)^{1/3} \left(a+b x^3\right) \left(-4 A+5 B x^3\right)-i\, 3^{3/4} b^{1/3} (4 A b+5 a B) x^4 \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}}\right.\right.$$

$$\left.\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Big/ \left(10 (-a)^{1/3} (e x)^{7/2} \sqrt{a+b x^3}\right)$$

Problem 524: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3} \left(A+B x^3\right)}{x^{9/2}} \, dx$$

Optimal (type 4, 564 leaves, 6 steps):

$$-\frac{2 (2 A b+7 a B) \sqrt{a+b x^3}}{7 a \sqrt{x}}+\frac{3 \left(1+\sqrt{3}\right) b^{1/3} (2 A b+7 a B) \sqrt{x} \sqrt{a+b x^3}}{7 a \left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}-\frac{2 A \left(a+b x^3\right)^{3/2}}{7 a x^{7/2}}-$$

$$\left(3\times 3^{1/4} b^{1/3} (2 A b+7 a B) \sqrt{x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$

$$\left(7 a^{2/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(3^{3/4} \left(1-\sqrt{3}\right) b^{1/3} (2 A b+7 a B) \sqrt{x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$

$$\left(14 a^{2/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 285 leaves):

$$-\frac{1}{7\,(-a)^{5/3}\,x^{7/2}\,\sqrt{a+b\,x^3}}\left(-2\,(-a)^{2/3}\,\left(a+b\,x^3\right)\,\left(a\,A+(3\,A\,b+7\,a\,B)\,x^3\right)+\right.$$

$$(2\,A\,b+7\,a\,B)\,x^3\left(3\,(-a)^{2/3}\,\left(a+b\,x^3\right)+(-1)^{2/3}\,3^{3/4}\,a\,b^{2/3}\,x^2\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.$$

$$\left.\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)$$

- **Problem 526: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{x^{13/2}}\,dx$$

Optimal (type 4, 269 leaves, 4 steps):

$$\frac{2\,(2\,A\,b-11\,a\,B)\,\sqrt{a+b\,x^3}}{55\,a\,x^{5/2}}-\frac{2\,A\,\left(a+b\,x^3\right)^{3/2}}{11\,a\,x^{11/2}}-$$

$$\left(3^{3/4}\,b\,(2\,A\,b-11\,a\,B)\,\sqrt{x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(55\,a^{4/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 206 leaves):

$$\left(-\frac{2\,A}{11\,x^{11/2}}-\frac{2\,(3\,A\,b+11\,a\,B)}{55\,a\,x^{5/2}}\right)\sqrt{a+b\,x^3}-\frac{1}{55\,(-a)^{1/3}\,a\,\sqrt{a+b\,x^3}}$$

$$2\,i\,3^{3/4}\,b^{4/3}\,(-2\,A\,b+11\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-a)^{1/3}}{b^{1/3}\,x}\right)}\,\sqrt{1+\frac{(-a)^{2/3}}{b^{2/3}\,x^2}+\frac{(-a)^{1/3}}{b^{1/3}\,x}}\,x^{3/2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]$$

Problem 528: Result unnecessarily involves imaginary or complex numbers.

$$\int (e\,x)^{5/2}\,\left(a+b\,x^3\right)^{3/2}\,\left(A+B\,x^3\right)\,dx$$

Optimal (type 4, 364 leaves, 6 steps):

$$\frac{27\,a^2\,(22\,A\,b-7\,a\,B)\,e^2\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{7040\,b^2}+\frac{9\,a\,(22\,A\,b-7\,a\,B)\,(e\,x)^{7/2}\,\sqrt{a+b\,x^3}}{1760\,b\,e}+\frac{(22\,A\,b-7\,a\,B)\,(e\,x)^{7/2}\,\left(a+b\,x^3\right)^{3/2}}{176\,b\,e}+\frac{B\,(e\,x)^{7/2}\,\left(a+b\,x^3\right)^{5/2}}{11\,b\,e}-$$
$$\left(9\times 3^{3/4}\,a^{8/3}\,(22\,A\,b-7\,a\,B)\,e^2\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$
$$\left(14\,080\,b^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 256 leaves):

$$\frac{1}{7040\,(-a)^{1/3}\,b^2\,\sqrt{a+b\,x^3}}e^2\,\sqrt{e\,x}\,\left(-(-a)^{1/3}\,\left(a+b\,x^3\right)\,\left(189\,a^3\,B-54\,a^2\,b\,\left(11\,A+2\,B\,x^3\right)-80\,b^3\,x^6\,\left(11\,A+8\,B\,x^3\right)-8\,a\,b^2\,x^3\,\left(209\,A+125\,B\,x^3\right)\right)+\right.$$
$$\left.9\,i\,3^{3/4}\,a^3\,b^{1/3}\,(22\,A\,b-7\,a\,B)\,x\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 529: Result unnecessarily involves imaginary or complex numbers.

$$\int (e\,x)^{3/2}\,\left(a+b\,x^3\right)^{3/2}\,\left(A+B\,x^3\right)\,dx$$

Optimal (type 4, 621 leaves, 7 steps):

$$\frac{9 a (4 A b-a B) (e x)^{5/2} \sqrt{a+b x^3}}{224 b e}+\frac{27 \left(1+\sqrt{3}\right) a^2 (4 A b-a B) e \sqrt{e x} \sqrt{a+b x^3}}{448 b^{5/3} \left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}+\frac{(4 A b-a B) (e x)^{5/2} \left(a+b x^3\right)^{3/2}}{28 b e}+\frac{B (e x)^{5/2} \left(a+b x^3\right)^{5/2}}{10 b e}-$$

$$\left(27\times 3^{1/4} a^{7/3} (4 A b-a B) e \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(448 b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(9\times 3^{3/4} \left(1-\sqrt{3}\right) a^{7/3} (4 A b-a B) e \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(896 b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 303 leaves) :

$$\frac{1}{2240 b^2 \sqrt{a+b x^3}} x (e x)^{3/2} \left(2 b \left(a+b x^3\right) \left(27 a^2 B+16 b^2 x^3 \left(10 A+7 B x^3\right)+4 a b \left(85 A+46 B x^3\right)\right)+\right.$$

$$45 a^2 (-4 A b+a B) \left(-3 \left(b+\frac{a}{x^3}\right)+1\Big/\left(\left((-a)^{2/3} x\right) (-1)^{1/6} 3^{3/4} a b^{2/3} \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}\right.\right.$$

$$\left.\left.\left.\left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)\right)\right)$$

Problem 531: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{\sqrt{e x}} \, dx$$

Optimal (type 4, 324 leaves, 5 steps):

$$\frac{9 a (16 A b-a B) \sqrt{e x} \sqrt{a+b x^3}}{320 b e}+\frac{(16 A b-a B) \sqrt{e x} \left(a+b x^3\right)^{3/2}}{80 b e}+\frac{B \sqrt{e x} \left(a+b x^3\right)^{5/2}}{8 b e}+$$
$$\left(9\times 3^{3/4} a^{5/3} (16 A b-a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$
$$\left(640 b e \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 234 leaves):

$$\left((-a)^{1/3} x \left(a+b x^3\right) \left(27 a^2 B+8 b^2 x^3 \left(8 A+5 B x^3\right)+4 a b \left(52 A+19 B x^3\right)\right)-9 i 3^{3/4} a^2 b^{1/3} (16 A b-a B) x^2 \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}}\right.$$
$$\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right) \Big/ \left(320 (-a)^{1/3} b \sqrt{e x} \sqrt{a+b x^3}\right)$$

Problem 532: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{(e x)^{3/2}} \, dx$$

Optimal (type 4, 614 leaves, 7 steps):

$$\frac{9\,(14\,A\,b+a\,B)\,(e\,x)^{5/2}\sqrt{a+b\,x^3}}{56\,e^4}+\frac{27\left(1+\sqrt{3}\right)a\,(14\,A\,b+a\,B)\sqrt{e\,x}\sqrt{a+b\,x^3}}{112\,b^{2/3}\,e^2\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)}+\frac{(14\,A\,b+a\,B)\,(e\,x)^{5/2}\left(a+b\,x^3\right)^{3/2}}{7\,a\,e^4}-\frac{2\,A\left(a+b\,x^3\right)^{5/2}}{a\,e\sqrt{e\,x}}-$$

$$\left(27\times 3^{1/4}\,a^{4/3}\,(14\,A\,b+a\,B)\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(112\,b^{2/3}\,e^2\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)-$$

$$\left(9\times 3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}\,(14\,A\,b+a\,B)\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(224\,b^{2/3}\,e^2\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 301 leaves) :

$$\frac{1}{112\,(e\,x)^{3/2}\sqrt{a+b\,x^3}}x^{3/2}\left(\frac{2\left(a+b\,x^3\right)\left(-112\,a\,A+14\,A\,b\,x^3+17\,a\,B\,x^3+8\,b\,B\,x^6\right)}{\sqrt{x}}-\right.$$

$$1\,/\,b9\,a\,(14\,A\,b+a\,B)\,x^{5/2}\left(-3\left(b+\frac{a}{x^3}\right)+1\Big/\left(\left(-a\right)^{2/3}x\right)(-1)^{1/6}\,3^{3/4}\,a\,b^{2/3}\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.$$

$$\left.\left.\left(-\mathrm{i}\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\right)$$

Problem 534: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{(e x)^{7/2}} \, dx$$

Optimal (type 4, 314 leaves, 5 steps):

$$\frac{9 (2 A b+a B) \sqrt{e x} \sqrt{a+b x^3}}{20 e^4}+\frac{(2 A b+a B) \sqrt{e x} \left(a+b x^3\right)^{3/2}}{5 a e^4}-\frac{2 A \left(a+b x^3\right)^{5/2}}{5 a e (e x)^{5/2}}+$$
$$\left(9\times 3^{3/4} a^{2/3} (2 A b+a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$
$$\left(40 e^4 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 215 leaves):

$$\left(x \left((-a)^{1/3} \left(a+b x^3\right) \left(-8 a A+10 A b x^3+13 a B x^3+4 b B x^6\right)-9 i\, 3^{3/4} a b^{1/3} (2 A b+a B) x^4 \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}}\right.\right.$$
$$\left.\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}} \,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(20 (-a)^{1/3} (e x)^{7/2} \sqrt{a+b x^3}\right)$$

Problem 536: Result unnecessarily involves imaginary or complex numbers.

$$\int (e x)^{5/2} \left(a+b x^3\right)^{5/2} \left(A+B x^3\right) \, dx$$

Optimal (type 4, 404 leaves, 7 steps):

$$\frac{81 a^3 (4 A b - a B) e^2 \sqrt{e x} \sqrt{a + b x^3}}{5632 b^2} + \frac{27 a^2 (4 A b - a B) (e x)^{7/2} \sqrt{a + b x^3}}{1408 b e} +$$
$$\frac{15 a (4 A b - a B) (e x)^{7/2} \left(a + b x^3\right)^{3/2}}{704 b e} + \frac{(4 A b - a B) (e x)^{7/2} \left(a + b x^3\right)^{5/2}}{44 b e} + \frac{B (e x)^{7/2} \left(a + b x^3\right)^{7/2}}{14 b e} -$$
$$\left(27 \times 3^{3/4} a^{11/3} (4 A b - a B) e^2 \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1 - \sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2 + \sqrt{3}\right)\right]\right) \Big/$$
$$\left(11264 b^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 276 leaves) :

$$\left(e^2 \sqrt{e x} \left(-(-a)^{1/3} \left(a + b x^3\right) \left(567 a^4 B - 324 a^3 b \left(7 A + B x^3\right) - 256 b^4 x^9 \left(14 A + 11 B x^3\right) - 32 a b^3 x^6 \left(329 A + 236 B x^3\right) - 8 a^2 b^2 x^3 \left(1246 A + 727 B x^3\right)\right) + \right.\right.$$
$$189 i 3^{3/4} a^4 b^{1/3} (4 A b - a B) x \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3} - b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}}$$
$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(39424 (-a)^{1/3} b^2 \sqrt{a + b x^3}\right)$$

- **Problem 537: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e x)^{3/2} \left(a + b x^3\right)^{5/2} \left(A + B x^3\right) dx$$

Optimal (type 4, 661 leaves, 8 steps) :

$$\frac{27 a^2 (26 A b-5 a B) (e x)^{5/2} \sqrt{a+b x^3}}{5824 b e}+\frac{81 \left(1+\sqrt{3}\right) a^3 (26 A b-5 a B) e \sqrt{e x} \sqrt{a+b x^3}}{11648 b^{5/3} \left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}+$$

$$\frac{3 a (26 A b-5 a B) (e x)^{5/2} \left(a+b x^3\right)^{3/2}}{728 b e}+\frac{(26 A b-5 a B) (e x)^{5/2} \left(a+b x^3\right)^{5/2}}{260 b e}+\frac{B (e x)^{5/2} \left(a+b x^3\right)^{7/2}}{13 b e}-$$

$$\left(81\times 3^{1/4} a^{10/3} (26 A b-5 a B) e \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$

$$\left(11648 b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\left(27\times 3^{3/4} \left(1-\sqrt{3}\right) a^{10/3} (26 A b-5 a B) e \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/ \left(23296 b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 337 leaves) :

$$\frac{1}{58240 (-a)^{2/3} b^2 \sqrt{e x} \sqrt{a+b x^3}}$$

$$e^2 \left(2 (-a)^{2/3} b x^3 \left(a+b x^3\right) \left(a^2 (9542 A b+405 a B)+8 a b (1118 A b+625 a B) x^3+112 b^2 (26 A b+55 a B) x^6+2240 b^3 B x^9\right)+\right.$$

$$135 a^3 (26 A b-5 a B) \left(3 (-a)^{2/3} \left(a+b x^3\right)+(-1)^{2/3} 3^{3/4} a b^{2/3} x^2 \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}\right.$$

$$\left.\left.\left(\sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)\right)$$

Problem 539: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{5/2} \left(A+B x^3\right)}{\sqrt{e x}} \, dx$$

Optimal (type 4, 364 leaves, 6 steps):

$$\frac{27 a^2 (22 A b-a B) \sqrt{e x} \sqrt{a+b x^3}}{1408 b e}+\frac{3 a (22 A b-a B) \sqrt{e x} \left(a+b x^3\right)^{3/2}}{352 b e}+\frac{(22 A b-a B) \sqrt{e x} \left(a+b x^3\right)^{5/2}}{176 b e}+\frac{B \sqrt{e x} \left(a+b x^3\right)^{7/2}}{11 b e}+$$
$$\left(27\times 3^{3/4} a^{8/3} (22 A b-a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Bigg/$$
$$\left(2816 b e \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 255 leaves):

$$\Bigg((-a)^{1/3} x \left(a+b x^3\right) \left(81 a^3 B+16 b^3 x^6 \left(11 A+8 B x^3\right)+8 a b^2 x^3 \left(77 A+47 B x^3\right)+2 a^2 b \left(517 A+178 B x^3\right)\right)-27 i\, 3^{3/4} a^3 b^{1/3} (22 A b-a B) x^2$$
$$\sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\Bigg) \Bigg/ \left(1408 (-a)^{1/3} b \sqrt{e x} \sqrt{a+b x^3}\right)$$

Problem 540: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{5/2} \left(A+B x^3\right)}{(e x)^{3/2}} \, dx$$

Optimal (type 4, 650 leaves, 8 steps):

$$\frac{27\,a\,(20\,A\,b+a\,B)\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}{224\,e^4}+\frac{81\,\left(1+\sqrt{3}\right)\,a^2\,(20\,A\,b+a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{448\,b^{2/3}\,e^2\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}+$$

$$\frac{3\,(20\,A\,b+a\,B)\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{3/2}}{28\,e^4}+\frac{(20\,A\,b+a\,B)\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{5/2}}{10\,a\,e^4}-\frac{2\,A\,\left(a+b\,x^3\right)^{7/2}}{a\,e\,\sqrt{e\,x}}-$$

$$\left(81\times3^{1/4}\,a^{7/3}\,(20\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(448\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(27\times3^{3/4}\,\left(1-\sqrt{3}\right)\,a^{7/3}\,(20\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(896\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 329 leaves) :

$$\frac{1}{448\,(e\,x)^{3/2}\,\sqrt{a+b\,x^3}}x^{3/2}\left(\frac{2\,\left(a+b\,x^3\right)\,\left(16\,b^2\,x^6\,\left(10\,A+7\,B\,x^3\right)+4\,a\,b\,x^3\,\left(155\,A+86\,B\,x^3\right)+a^2\,\left(-2240\,A+367\,B\,x^3\right)\right)}{5\,\sqrt{x}}-\right.$$

$$1\,/\,b27\,a^2\,(20\,A\,b+a\,B)\,x^{5/2}\left(-3\,\left(b+\frac{a}{x^3}\right)+1\Big/\left(\left(-a\right)^{2/3}\,x\right)\,(-1)^{1/6}\,3^{3/4}\,a\,b^{2/3}\,\sqrt{\frac{(-1)^{5/6}\,\left(\left(-a\right)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.$$

$$\left.\left.\left(-i\,\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)$$

Problem 542: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{5/2} \left(A+B x^3\right)}{(e x)^{7/2}} \, dx$$

Optimal (type 4, 352 leaves, 6 steps):

$$\frac{27 a (16 A b+5 a B) \sqrt{e x} \sqrt{a+b x^3}}{320 e^4}+\frac{3 (16 A b+5 a B) \sqrt{e x} \left(a+b x^3\right)^{3/2}}{80 e^4}+\frac{(16 A b+5 a B) \sqrt{e x} \left(a+b x^3\right)^{5/2}}{40 a e^4}-\frac{2 A \left(a+b x^3\right)^{7/2}}{5 a e (e x)^{5/2}}+$$
$$\left(27\times 3^{3/4} a^{5/3} (16 A b+5 a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Bigg/$$
$$\left(640 e^4 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 242 leaves):

$$\Bigg(x \Bigg((-a)^{1/3} \left(a+b x^3\right) \left(8 b^2 x^6 \left(8 A+5 B x^3\right)+4 a b x^3 \left(92 A+35 B x^3\right)+a^2 \left(-128 A+235 B x^3\right)\right)-$$
$$27 i\, 3^{3/4} a^2 b^{1/3} (16 A b+5 a B) x^4 \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}$$
$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\Bigg)\Bigg) \Bigg/ \left(320 (-a)^{1/3} (e x)^{7/2} \sqrt{a+b x^3}\right)$$

Problem 544: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e x)^{5/2} \left(A+B x^3\right)}{\sqrt{a+b x^3}} \, dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$\frac{(10\,A\,b-7\,a\,B)\,e^2\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{20\,b^2}+\frac{B\,(e\,x)^{7/2}\,\sqrt{a+b\,x^3}}{5\,b\,e}-$$

$$\left(a^{2/3}\,(10\,A\,b-7\,a\,B)\,e^2\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\;\mathrm{EllipticF}\left[\mathrm{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(40\times 3^{1/4}\,b^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 210 leaves) :

$$\frac{1}{60\,(-a)^{1/3}\,b^2\,\sqrt{a+b\,x^3}}e^2\,\sqrt{e\,x}\,\left(-3\,(-a)^{1/3}\,\left(a+b\,x^3\right)\,\left(-10\,A\,b+7\,a\,B-4\,b\,B\,x^3\right)+\right.$$

$$\left.\mathrm{i}\,3^{3/4}\,a\,b^{1/3}\,(10\,A\,b-7\,a\,B)\,x\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\;\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 545: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{3/2}\,\left(A+B\,x^3\right)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 543 leaves, 5 steps) :

$$\frac{B\,(e\,x)^{5/2}\sqrt{a+b\,x^3}}{4\,b\,e}+\frac{\left(1+\sqrt{3}\right)(8\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{8\,b^{5/3}\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\,a^{1/3}\,(8\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(8\,b^{5/3}\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(\left(1-\sqrt{3}\right)a^{1/3}\,(8\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(16\times 3^{1/4}\,b^{5/3}\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 263 leaves) :

$$\frac{1}{24\,b^2\sqrt{a+b\,x^3}}$$

$$x\,(e\,x)^{3/2}\left(6\,b\,B\left(a+b\,x^3\right)-(8\,A\,b-5\,a\,B)\left(-3\left(b+\frac{a}{x^3}\right)+1\Big/\left(\left((-a)^{2/3}\,x\right)(-1)^{1/6}\,3^{3/4}\,a\,b^{2/3}\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.\right.\right.$$

$$\left.\left.\left.\left(-i\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\right)\right)$$

- **Problem 547: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{\sqrt{e\,x}\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 249 leaves, 3 steps) :

$$\frac{B\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{2\,b\,e}+\left((4\,A\,b-a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(4\times 3^{1/4}\,a^{1/3}\,b\,e\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 184 leaves) :

$$\frac{1}{6\,b\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}\times\left(3\,B\,\left(a+b\,x^3\right)+\right.$$

$$\left.1\Big/(-a)^{1/3}\,\mathrm{i}\,3^{3/4}\,b^{1/3}\,(-4\,A\,b+a\,B)\,x\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 548: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{(e\,x)^{3/2}\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 542 leaves, 5 steps) :

$$-\frac{2 A \sqrt{a+b x^3}}{a e \sqrt{e x}}+\frac{\left(1+\sqrt{3}\right) (2 A b+a B) \sqrt{e x} \sqrt{a+b x^3}}{a b^{2/3} e^2 \left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}-$$

$$\left(3^{1/4} (2 A b+a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(a^{2/3} b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(\left(1-\sqrt{3}\right) (2 A b+a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(2\times 3^{1/4} a^{2/3} b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 355 leaves) :

$$\frac{1}{a (e x)^{3/2} \sqrt{a+b x^3}}$$

$$x \left(-2 A \left(a+b x^3\right)+1\Big/\left(\left(-1+(-1)^{2/3}\right) a^{1/3} b\right) (2 A b+a B) \left(-\left(-1+(-1)^{2/3}\right) a^{1/3} b^{1/3} x \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \left((-1)^{2/3} a^{1/3}+b^{1/3} x\right)-\right.\right.$$

$$(-1)^{2/3} a^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2 \sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(a^{1/3}+b^{1/3} x\right)^2}} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{a^{1/3}+b^{1/3} x}} \left(\left(1+(-1)^{1/3}\right) \text{EllipticE}\left[\right.\right.$$

$$\left.\left.\left.\left.\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x}{a^{1/3}+b^{1/3} x}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\left(1+(-1)^{2/3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x}{a^{1/3}+b^{1/3} x}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\right)$$

- **Problem 550: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x^3}{(e x)^{7/2} \sqrt{a+b x^3}} \, dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$-\frac{2 A \sqrt{a+b x^3}}{5 a e (e x)^{5/2}}-\left((2 A b-5 a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$

$$\left(5\times 3^{1/4} a^{4/3} e^4 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 187 leaves):

$$\frac{1}{15 a (e x)^{7/2} \sqrt{a+b x^3}} 2 x \left(-3 A \left(a+b x^3\right)+\right.$$

$$\left.1 \Big/ (-a)^{1/3} \mathbb{i}\, 3^{3/4} b^{1/3} (2 A b-5 a B) x^4 \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathbb{i} (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 552: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e x)^{5/2} \left(A+B x^3\right)}{\left(a+b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$-\frac{(4 A b-7 a B) e^2 \sqrt{e x}}{6 b^2 \sqrt{a+b x^3}}+\frac{B (e x)^{7/2}}{2 b e \sqrt{a+b x^3}}+$$

$$\left((4 A b-7 a B) e^2 \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$

$$\left(12\times 3^{1/4} a^{1/3} b^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 202 leaves):

$$\frac{1}{18\,(-a)^{1/3}\,b^2\,\sqrt{a+b\,x^3}}e^2\,\sqrt{e\,x}\,\left(3\,(-a)^{1/3}\,\left(-4\,A\,b+7\,a\,B+3\,b\,B\,x^3\right)-\right.$$

$$\left.i\,3^{3/4}\,b^{1/3}\,(4\,A\,b-7\,a\,B)\,x\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 553: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(e\,x)^{3/2}\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 553 leaves, 5 steps):

$$\frac{2\,(A\,b-a\,B)\,(e\,x)^{5/2}}{3\,a\,b\,e\,\sqrt{a+b\,x^3}}-\frac{\left(1+\sqrt{3}\right)\,(2\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{3\,a\,b^{5/3}\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}+$$

$$\left((2\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(3^{3/4}\,a^{2/3}\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(\left(1-\sqrt{3}\right)\,(2\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(6\times 3^{1/4}\,a^{2/3}\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 266 leaves):

$$\frac{1}{9\,a\,b^2\sqrt{a+b\,x^3}}\,x\,(e\,x)^{3/2}\left(6\,b\,(A\,b-a\,B)-(-2\,A\,b+5\,a\,B)\left(-3\left(b+\frac{a}{x^3}\right)+1\Big/\left(\left((-a)^{2/3}\,x\right)(-1)^{1/6}\,3^{3/4}\,a\,b^{2/3}\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.\right.\right.$$

$$\left.\left.\left.\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)\right)$$

- **Problem 555: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{\sqrt{e\,x}\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 258 leaves, 3 steps):

$$\frac{2\,(A\,b-a\,B)\,\sqrt{e\,x}}{3\,a\,b\,e\sqrt{a+b\,x^3}}+\left((2\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(3\times3^{1/4}\,a^{4/3}\,b\,e\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 193 leaves):

$$-\left(6\,(-a)^{1/3}\,(A\,b-a\,B)\,x-2\,i\,3^{3/4}\,b^{1/3}\,(2\,A\,b+a\,B)\,x^2\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\right.$$

$$\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/\left(9\,(-a)^{4/3}\,b\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}\right)$$

■ Problem 556: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^3}{(e x)^{3/2} \left(a + b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 585 leaves, 6 steps):

$$-\frac{2 A}{a e \sqrt{e x} \sqrt{a + b x^3}} - \frac{2 (4 A b - a B) (e x)^{5/2}}{3 a^2 e^4 \sqrt{a + b x^3}} + \frac{2 \left(1 + \sqrt{3}\right) (4 A b - a B) \sqrt{e x} \sqrt{a + b x^3}}{3 a^2 b^{2/3} e^2 \left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)} -$$

$$\left(2 (4 A b - a B) \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \, \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1 - \sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2 + \sqrt{3}\right)\right]\right) \Big/$$

$$\left(3^{3/4} a^{5/3} b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) -$$

$$\left(\left(1 - \sqrt{3}\right) (4 A b - a B) \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \, \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1 - \sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2 + \sqrt{3}\right)\right]\right) \Big/$$

$$\left(3 \times 3^{1/4} a^{5/3} b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 372 leaves):

$$\frac{1}{3 a^2 (e x)^{3/2} \sqrt{a+b x^3}} 2 x \left(-(A b-a B) x^3-3 A \left(a+b x^3\right)+1\Big/\left(\left(-1+(-1)^{2/3}\right) a^{1/3} b\right) (4 A b-a B) \left(-\left(-1+(-1)^{2/3}\right) a^{1/3} b^{1/3} x \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \left((-1)^{2/3} a^{1/3}+b^{1/3} x\right)-\right.\right.$$
$$(-1)^{2/3} a^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2 \sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(a^{1/3}+b^{1/3} x\right)^2}} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{a^{1/3}+b^{1/3} x}} \left(\left(1+(-1)^{1/3}\right) \text{EllipticE}\left[\right.\right.$$
$$\left.\left.\left.\left.\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x}{a^{1/3}+b^{1/3} x}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\left(1+(-1)^{2/3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x}{a^{1/3}+b^{1/3} x}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\right)$$

- **Problem 558: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+B x^3}{(e x)^{7/2} \left(a+b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 283 leaves, 4 steps) :

$$-\frac{2 A}{5 a e (e x)^{5/2} \sqrt{a+b x^3}}-\frac{2 (8 A b-5 a B) \sqrt{e x}}{15 a^2 e^4 \sqrt{a+b x^3}}-$$
$$\left(2 (8 A b-5 a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$
$$\left(15 \times 3^{1/4} a^{7/3} e^4 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 202 leaves) :

$$\left(x\left(-6(-a)^{1/3}\left(3aA+8Abx^3-5aBx^3\right)+4i\,3^{3/4}b^{1/3}(8Ab-5aB)x^4\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}x\right)}{b^{1/3}x}}\right.\right.$$

$$\left.\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}x}{b^{1/3}}+x^2}{x^2}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(45(-a)^{7/3}(ex)^{7/2}\sqrt{a+bx^3}\right)$$

- **Problem 560: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$$

Optimal (type 4, 299 leaves, 4 steps):

$$\frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}+$$

$$\left((2Ab+7aB)e^2\sqrt{ex}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(27\times3^{1/4}a^{4/3}b^2\sqrt{\frac{b^{1/3}x\left(a^{1/3}+b^{1/3}x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 216 leaves):

$$\frac{1}{81(-a)^{4/3}b^2\left(a+bx^3\right)^{3/2}}2ie^2\sqrt{ex}\left(-3i(-a)^{1/3}\left(7a^2B-Ab^2x^3+2ab\left(A+5Bx^3\right)\right)+\right.$$

$$\left.3^{3/4}b^{1/3}(2Ab+7aB)\sqrt{(-1)^{5/6}\left(-1+\frac{(-a)^{1/3}}{b^{1/3}x}\right)}\,x\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}x}{b^{1/3}}+x^2}{x^2}}\left(a+bx^3\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 561: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e\,x)^{3/2}\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 596 leaves, 6 steps) :

$$\frac{2\,(A\,b-a\,B)\,(e\,x)^{5/2}}{9\,a\,b\,e\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,(4\,A\,b+5\,a\,B)\,(e\,x)^{5/2}}{27\,a^2\,b\,e\,\sqrt{a+b\,x^3}}-\frac{2\left(1+\sqrt{3}\right)(4\,A\,b+5\,a\,B)\,e\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{27\,a^2\,b^{5/3}\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)}+$$

$$\left(2\,(4\,A\,b+5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\mathrm{EllipticE}\left[\mathrm{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(9\times3^{3/4}\,a^{5/3}\,b^{5/3}\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(\left(1-\sqrt{3}\right)(4\,A\,b+5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\mathrm{EllipticF}\left[\mathrm{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(27\times3^{1/4}\,a^{5/3}\,b^{5/3}\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 307 leaves) :

$$\frac{1}{81\,(-a)^{8/3}\,b^2\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{3/2}}\,2\,e^2\left(3\,(-a)^{2/3}\,b\,x^3\,\left(2\,a^2\,B+4\,A\,b^2\,x^3+a\,b\,\left(7\,A+5\,B\,x^3\right)\right)-\right.$$

$$(4\,A\,b+5\,a\,B)\,\left(a+b\,x^3\right)\left(3\,(-a)^{2/3}\,\left(a+b\,x^3\right)+(-1)^{2/3}\,3^{3/4}\,a\,b^{2/3}\,x^2\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\right.$$

$$\left.\left.\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)$$

- **Problem 563: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{\sqrt{e\,x}\,\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\frac{2\,(A\,b-a\,B)\,\sqrt{e\,x}}{9\,a\,b\,e\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,(8\,A\,b+a\,B)\,\sqrt{e\,x}}{27\,a^2\,b\,e\,\sqrt{a+b\,x^3}}+$$

$$\left(2\,(8\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(27\times3^{1/4}\,a^{7/3}\,b\,e\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 214 leaves):

$$\left(2\left(3(-a)^{1/3}x\left(3a(Ab-aB)+(8Ab+aB)\left(a+bx^3\right)\right)-2\,i\,3^{3/4}b^{1/3}(8Ab+aB)\sqrt{(-1)^{5/6}\left(-1+\frac{(-a)^{1/3}}{b^{1/3}x}\right)}\,x^2\right.\right.$$

$$\left.\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}x}{b^{1/3}}+x^2}{x^2}}\left(a+bx^3\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(81(-a)^{7/3}b\sqrt{ex}\left(a+bx^3\right)^{3/2}\right)$$

Problem 564: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx$$

Optimal (type 4, 624 leaves, 7 steps):

$$-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}-\frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}}+\frac{8\left(1+\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{27a^3b^{2/3}e^2\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)}-$$

$$\left(8(10Ab-aB)\sqrt{ex}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(9\times3^{3/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{b^{1/3}x\left(a^{1/3}+b^{1/3}x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)-$$

$$\left(4\left(1-\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(27\times3^{1/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{b^{1/3}x\left(a^{1/3}+b^{1/3}x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 401 leaves):

$$\frac{1}{27\,a^3\,(e\,x)^{3/2}\,\sqrt{a+b\,x^3}}\,2\,x\left(\frac{-40\,A\,b^2\,x^6+a^2\,\left(-27\,A+7\,B\,x^3\right)+a\,\left(-70\,A\,b\,x^3+4\,b\,B\,x^6\right)}{a+b\,x^3}+\right.$$

$$\frac{1}{\left(-1+(-1)^{2/3}\right)\,a^{1/3}\,b}\,4\,(10\,A\,b-a\,B)\left(-\left(-1+(-1)^{2/3}\right)\,a^{1/3}\,b^{1/3}\,x\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\left((-1)^{2/3}\,a^{1/3}+b^{1/3}\,x\right)-\right.$$

$$(-1)^{2/3}\,a^{2/3}\,\left(a^{1/3}+b^{1/3}\,x\right)^2\,\sqrt{\frac{\left(1+(-1)^{1/3}\right)\,b^{1/3}\,x\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{a^{1/3}+b^{1/3}\,x}}\,\left(\left(1+(-1)^{1/3}\right)\,\text{EllipticE}\left[\right.\right.$$

$$\left.\left.\left.\left.\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right)\,b^{1/3}\,x}{a^{1/3}+b^{1/3}\,x}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\left(1+(-1)^{2/3}\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right)\,b^{1/3}\,x}{a^{1/3}+b^{1/3}\,x}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\right)$$

- **Problem 566: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{A+B\,x^3}{(e\,x)^{7/2}\,\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 320 leaves, 5 steps) :

$$-\frac{2\,A}{5\,a\,e\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{3/2}}-\frac{2\,(14\,A\,b-5\,a\,B)\,\sqrt{e\,x}}{45\,a^2\,e^4\,\left(a+b\,x^3\right)^{3/2}}-\frac{16\,(14\,A\,b-5\,a\,B)\,\sqrt{e\,x}}{135\,a^3\,e^4\,\sqrt{a+b\,x^3}}-$$

$$\left(16\,(14\,A\,b-5\,a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(135\times3^{1/4}\,a^{10/3}\,e^4\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 232 leaves) :

$$-\left(2\,i\,\sqrt{e\,x}\,\left(3\,i\,(-a)^{1/3}\,\left(112\,A\,b^2\,x^6+a^2\,\left(27\,A-55\,B\,x^3\right)+2\,a\,b\,x^3\,\left(77\,A-20\,B\,x^3\right)\right)+16\times 3^{3/4}\,b^{1/3}\,(14\,A\,b-5\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-a)^{1/3}}{b^{1/3}\,x}\right)}\,x^4\right.\right.$$

$$\left.\left.\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\left(a+b\,x^3\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(405\,(-a)^{10/3}\,e^4\,x^3\,\left(a+b\,x^3\right)^{3/2}\right)$$

Problem 567: Result unnecessarily involves higher level functions.

$$\int\frac{x^{11}\,\left(a+b\,x^3\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 3, 220 leaves, 8 steps):

$$-\frac{a^3\,\left(a+b\,x^3\right)^{1/3}}{b^4\,d}-\frac{a^2\,\left(a+b\,x^3\right)^{4/3}}{4\,b^4\,d}+\frac{a\,\left(a+b\,x^3\right)^{7/3}}{7\,b^4\,d}-\frac{\left(a+b\,x^3\right)^{10/3}}{10\,b^4\,d}+$$

$$\frac{2^{1/3}\,a^{10/3}\,\text{ArcTan}\left[\frac{a^{1/3}+2^{2/3}\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,b^4\,d}+\frac{a^{10/3}\,\text{Log}\left[a-b\,x^3\right]}{3\times 2^{2/3}\,b^4\,d}-\frac{a^{10/3}\,\text{Log}\left[2^{1/3}\,a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,b^4\,d}$$

Result (type 5, 80 leaves):

$$-\frac{\left(a+b\,x^3\right)^{1/3}\,\left(169\,a^3+37\,a^2\,b\,x^3+22\,a\,b^2\,x^6+14\,b^3\,x^9-140\,a^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,1,\,\frac{4}{3},\,\frac{a+b\,x^3}{2\,a}\right]\right)}{140\,b^4\,d}$$

Problem 568: Result unnecessarily involves higher level functions.

$$\int\frac{x^8\,\left(a+b\,x^3\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$-\frac{a^2\,\left(a+b\,x^3\right)^{1/3}}{b^3\,d}-\frac{\left(a+b\,x^3\right)^{7/3}}{7\,b^3\,d}+\frac{2^{1/3}\,a^{7/3}\,\text{ArcTan}\left[\frac{a^{1/3}+2^{2/3}\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,b^3\,d}+\frac{a^{7/3}\,\text{Log}\left[a-b\,x^3\right]}{3\times 2^{2/3}\,b^3\,d}-\frac{a^{7/3}\,\text{Log}\left[2^{1/3}\,a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,b^3\,d}$$

Result (type 5, 68 leaves):

$$-\frac{\left(a+b\,x^3\right)^{1/3}\,\left(8\,a^2+2\,a\,b\,x^3+b^2\,x^6-7\,a^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,1,\,\frac{4}{3},\,\frac{a+b\,x^3}{2\,a}\right]\right)}{7\,b^3\,d}$$

Problem 569: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 172 leaves, 7 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{b^2 d}-\frac{\left(a+b x^3\right)^{4/3}}{4 b^2 d}+\frac{2^{1/3} a^{4/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} b^2 d}+\frac{a^{4/3} \text{Log}\left[a-b x^3\right]}{3\times 2^{2/3} b^2 d}-\frac{a^{4/3} \text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^2 d}$$

Result (type 5, 55 leaves):

$$-\frac{\left(a+b x^3\right)^{1/3} \left(5 a+b x^3-4 a \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{a+b x^3}{2 a}\right]\right)}{4 b^2 d}$$

Problem 570: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 150 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{b d}+\frac{2^{1/3} a^{1/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} b d}+\frac{a^{1/3} \text{Log}\left[a-b x^3\right]}{3\times 2^{2/3} b d}-\frac{a^{1/3} \text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b d}$$

Result (type 5, 42 leaves):

$$\frac{\left(a+b x^3\right)^{1/3} \left(-1+\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{a+b x^3}{2 a}\right]\right)}{b d}$$

Problem 571: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 214 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} d}+\frac{2^{1/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} d}-\frac{\text{Log}[x]}{2 a^{2/3} d}+\frac{\text{Log}\left[a-b x^3\right]}{3\times 2^{2/3} a^{2/3} d}+\frac{\text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2 a^{2/3} d}-\frac{\text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a^{2/3} d}$$

Result (type 6, 158 leaves):

$$-\left(5\,b\,x^3\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]\right)\Big/\left(2\,d\,\left(a-b\,x^3\right)\right.$$
$$\left.\left(5\,b\,x^3\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]+a\,\left(3\,\text{AppellF1}\left[\frac{5}{3},-\frac{1}{3},2,\frac{8}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]+\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]\right)\right)\right)$$

Problem 572: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{1/3}}{x^4\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$\frac{b\,\left(a+b\,x^3\right)^{1/3}}{3\,a^2\,d}-\frac{\left(a+b\,x^3\right)^{4/3}}{3\,a^2\,d\,x^3}-\frac{4\,b\,\text{ArcTan}\left[\frac{a^{1/3}+2\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{3\,\sqrt{3}\,a^{5/3}\,d}+\frac{2^{1/3}\,b\,\text{ArcTan}\left[\frac{a^{1/3}+2^{2/3}\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{5/3}\,d}-$$
$$\frac{2\,b\,\text{Log}[x]}{3\,a^{5/3}\,d}+\frac{b\,\text{Log}\left[a-b\,x^3\right]}{3\times 2^{2/3}\,a^{5/3}\,d}+\frac{2\,b\,\text{Log}\left[a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{3\,a^{5/3}\,d}-\frac{b\,\text{Log}\left[2^{1/3}\,a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,a^{5/3}\,d}$$

Result (type 6, 308 leaves):

$$\frac{1}{15\,d\,x^3\,\left(a+b\,x^3\right)^{2/3}}\left(-5-\frac{5\,b\,x^3}{a}+\left(20\,b^2\,x^6\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\right.$$
$$\left(\left(a-b\,x^3\right)\,\left(6\,a\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[2,\frac{2}{3},2,3,-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[2,\frac{5}{3},1,3,-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)+$$
$$\left(32\,b^2\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]\right)\Big/\left(\left(-a+b\,x^3\right)\right.$$
$$\left.\left.\left(8\,b\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]+3\,a\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]-2\,a\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},\frac{a}{b\,x^3}\right]\right)\right)\right)$$

Problem 573: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{1/3}}{x^7\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 283 leaves, 12 steps):

$$-\frac{2\,b\,\left(a+b\,x^3\right)^{1/3}}{9\,a^2\,d\,x^3}-\frac{\left(a+b\,x^3\right)^{4/3}}{6\,a^2\,d\,x^6}-\frac{11\,b^2\,\text{ArcTan}\left[\frac{a^{1/3}+2\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{9\,\sqrt{3}\,a^{8/3}\,d}+\frac{2^{1/3}\,b^2\,\text{ArcTan}\left[\frac{a^{1/3}+2^{2/3}\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{8/3}\,d}-$$
$$\frac{11\,b^2\,\text{Log}[x]}{18\,a^{8/3}\,d}+\frac{b^2\,\text{Log}\left[a-b\,x^3\right]}{3\times 2^{2/3}\,a^{8/3}\,d}+\frac{11\,b^2\,\text{Log}\left[a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{18\,a^{8/3}\,d}-\frac{b^2\,\text{Log}\left[2^{1/3}\,a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,a^{8/3}\,d}$$

Result (type 6, 325 leaves):

$$\frac{1}{90\,a^2\,d\,x^6\,\left(a+b\,x^3\right)^{2/3}}\left(-5\,\left(3\,a^2+10\,a\,b\,x^3+7\,b^2\,x^6\right)+\left(140\,a\,b^3\,x^9\,\text{AppellF1}\left[1,\,\frac{2}{3},\,1,\,2,\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\Big/\right.$$
$$\left(\left(a-b\,x^3\right)\,\left(6\,a\,\text{AppellF1}\left[1,\,\frac{2}{3},\,1,\,2,\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[2,\,\frac{2}{3},\,2,\,3,\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[2,\,\frac{5}{3},\,1,\,3,\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\right)\right)-$$
$$\left(176\,a\,b^3\,x^9\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{a}{b\,x^3},\,\frac{a}{b\,x^3}\right]\right)\Big/\left(\left(a-b\,x^3\right)\right.$$
$$\left.\left.\left(8\,b\,x^3\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{a}{b\,x^3},\,\frac{a}{b\,x^3}\right]+3\,a\,\text{AppellF1}\left[\frac{8}{3},\,\frac{2}{3},\,2,\,\frac{11}{3},\,-\frac{a}{b\,x^3},\,\frac{a}{b\,x^3}\right]-2\,a\,\text{AppellF1}\left[\frac{8}{3},\,\frac{5}{3},\,1,\,\frac{11}{3},\,-\frac{a}{b\,x^3},\,\frac{a}{b\,x^3}\right]\right)\right)\right)$$

- **Problem 574: Result unnecessarily involves higher level functions.**

$$\int \frac{x^7\,\left(a+b\,x^3\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 3, 268 leaves, 6 steps) :

$$-\frac{7\,a\,x^2\,\left(a+b\,x^3\right)^{1/3}}{18\,b^2\,d}-\frac{x^5\,\left(a+b\,x^3\right)^{1/3}}{6\,b\,d}+\frac{11\,a^2\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,b^{8/3}\,d}-\frac{2^{1/3}\,a^2\,\text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{8/3}\,d}+$$
$$\frac{a^2\,\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times 2^{2/3}\,b^{8/3}\,d}+\frac{11\,a^2\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{18\,b^{8/3}\,d}-\frac{a^2\,\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,b^{8/3}\,d}$$

Result (type 6, 247 leaves) :

$$\frac{1}{90\,b^2\,d\,\left(a+b\,x^3\right)^{2/3}}\left(-5\,\left(a+b\,x^3\right)\,\left(7\,a\,x^2+3\,b\,x^5\right)+\left(176\,a^3\,b\,x^5\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\right.\right.$$
$$\left.\left(8\,a\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[\frac{8}{3},\,\frac{2}{3},\,2,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{8}{3},\,\frac{5}{3},\,1,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\right)\right)+$$
$$\left.35\,a^2\,x^2\,\left(\frac{a+b\,x^3}{a-b\,x^3}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,-\frac{2\,b\,x^3}{a-b\,x^3}\right]\right)$$

- **Problem 575: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4\,\left(a+b\,x^3\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 3, 233 leaves, 5 steps) :

$$-\frac{x^2\left(a+b x^3\right)^{1/3}}{3 b d}+\frac{4 a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{5/3} d}-\frac{2^{1/3} a \operatorname{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{5/3} d}+$$
$$\frac{a \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} b^{5/3} d}+\frac{2 a \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{3 b^{5/3} d}-\frac{a \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^{5/3} d}$$

Result (type 6, 231 leaves) :

$$\frac{1}{15 d\left(a+b x^3\right)^{2/3}} x^2\left(\left(32 a^2 x^3 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]\right) \Big/ \left(\left(a-b x^3\right)\right.\right.$$
$$\left.\left(8 a \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]+b x^3\left(3 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]-2 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]\right)\right)\right)-$$
$$\left.\frac{5\left(a+b x^3-a\left(\frac{a+b x^3}{a-b x^3}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{2 b x^3}{a-b x^3}\right]\right)}{b}\right)$$

Problem 576: Result unnecessarily involves higher level functions.

$$\int \frac{x\left(a+b x^3\right)^{1/3}}{a d-b d x^3}\, dx$$

Optimal (type 3, 201 leaves, 3 steps) :

$$\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}-\frac{2^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{\operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} b^{2/3} d}+\frac{\operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3} d}-\frac{\operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^{2/3} d}$$

Result (type 6, 158 leaves) :

$$\left(5 a x^2\left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]\right) \Big/ \left(2 d\left(a-b x^3\right)\right.$$
$$\left.\left(5 a \operatorname{AppellF1}\left[\frac{2}{3}, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]+b x^3\left(3 \operatorname{AppellF1}\left[\frac{5}{3}, -\frac{1}{3}, 2, \frac{8}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]+\operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 577: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^2\left(a d-b d x^3\right)}\, dx$$

Optimal (type 3, 156 leaves, 3 steps) :

$$-\frac{\left(a+b x^3\right)^{1/3}}{a d x}-\frac{2^{1/3} b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a d}+\frac{b^{1/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} a d}-\frac{b^{1/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a d}$$

Result (type 5, 84 leaves):

$$\frac{-a-b x^3+b x^3 \left(\frac{a+b x^3}{a-b x^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{2 b x^3}{a-b x^3}\right]}{a d x \left(a+b x^3\right)^{2/3}}$$

Problem 578: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^5 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 183 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{4 a d x^4}-\frac{5 b \left(a+b x^3\right)^{1/3}}{4 a^2 d x}-\frac{2^{1/3} b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^2 d}+\frac{b^{4/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} a^2 d}-\frac{b^{4/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a^2 d}$$

Result (type 5, 101 leaves):

$$\frac{-a^2-6 a b x^3-5 b^2 x^6+4 b^2 x^6 \left(\frac{a+b x^3}{a-b x^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{2 b x^3}{a-b x^3}\right]}{4 a^2 d x^4 \left(a+b x^3\right)^{2/3}}$$

Problem 579: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^8 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{7 a d x^7}-\frac{2 b \left(a+b x^3\right)^{1/3}}{7 a^2 d x^4}-\frac{8 b^2 \left(a+b x^3\right)^{1/3}}{7 a^3 d x}-\frac{2^{1/3} b^{7/3} \text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^3 d}+\frac{b^{7/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} a^3 d}-\frac{b^{7/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a^3 d}$$

Result (type 5, 112 leaves):

$$\frac{-a^3-3 a^2 b x^3-10 a b^2 x^6-8 b^3 x^9+7 b^3 x^9 \left(\frac{a+b x^3}{a-b x^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{2 b x^3}{a-b x^3}\right]}{7 a^3 d x^7 \left(a+b x^3\right)^{2/3}}$$

Problem 580: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^{11} \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 237 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{10 a d x^{10}}-\frac{11 b \left(a+b x^3\right)^{1/3}}{70 a^2 d x^7}-\frac{37 b^2 \left(a+b x^3\right)^{1/3}}{140 a^3 d x^4}-\frac{169 b^3 \left(a+b x^3\right)^{1/3}}{140 a^4 d x}-\frac{2^{1/3} b^{10/3} \text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^4 d}+\frac{b^{10/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} a^4 d}-\frac{b^{10/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a^4 d}$$

Result (type 5, 123 leaves):

$$\frac{1}{140 a^4 d x^{10} \left(a+b x^3\right)^{2/3}}\left(-14 a^4-36 a^3 b x^3-59 a^2 b^2 x^6-206 a b^3 x^9-169 b^4 x^{12}+140 b^4 x^{12} \left(\frac{a+b x^3}{a-b x^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{2 b x^3}{a-b x^3}\right]\right)$$

Problem 581: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 521 leaves, 22 steps):

$$-\frac{3 a x \left(a+b x^3\right)^{1/3}}{5 b^2 d}-\frac{x^4 \left(a+b x^3\right)^{1/3}}{5 b d}-\frac{2^{1/3} a^{5/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{7/3} d}-\frac{a^{5/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3} b^{7/3} d}-\frac{2 a^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{5 b^2 d \left(a+b x^3\right)^{2/3}}-\frac{a^{5/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{7/3} d}+\frac{a^{5/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{7/3} d}-\frac{2^{1/3} a^{5/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{7/3} d}+\frac{a^{5/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} b^{7/3} d}$$

Result (type 6, 332 leaves):

$$\frac{1}{20\,b^2\,d\,\left(a+b\,x^3\right)^{2/3}}\left(-4\,\left(a+b\,x^3\right)\,\left(3\,a\,x+b\,x^4\right)+\left(48\,a^4\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(49\,a^3\,b\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\left(7\,a\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 582: Result unnecessarily involves higher level functions.**

$$\int\frac{x^3\,\left(a+b\,x^3\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 5, 494 leaves, 21 steps):

$$-\frac{x\,\left(a+b\,x^3\right)^{1/3}}{2\,b\,d}-\frac{2^{1/3}\,a^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{4/3}\,d}-\frac{a^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}\,b^{4/3}\,d}-\frac{a\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,b\,d\,\left(a+b\,x^3\right)^{2/3}}-\frac{a^{2/3}\,\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times2^{2/3}\,b^{4/3}\,d}+\frac{a^{2/3}\,\text{Log}\left[1+\frac{2^{2/3}\,\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}-\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times2^{2/3}\,b^{4/3}\,d}-\frac{2^{1/3}\,a^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{4/3}\,d}+\frac{a^{2/3}\,\text{Log}\left[2\times2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{2^{2/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\times2^{2/3}\,b^{4/3}\,d}$$

Result (type 6, 324 leaves):

$$\frac{1}{8\,d\,\left(a+b\,x^3\right)^{2/3}}x\left(-\frac{4\,\left(a+b\,x^3\right)}{b}+\left(16\,a^3\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left(b\,\left(a-b\,x^3\right)\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(21\,a^2\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\left(7\,a\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 583: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{a d-b d x^3}\, dx$$

Optimal (type 3, 416 leaves, 14 steps):

$$-\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{1/3}\, b^{1/3}\, d}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}\, a^{1/3}\, b^{1/3}\, d}-\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\, a^{1/3}\, b^{1/3}\, d}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\, a^{1/3}\, b^{1/3}\, d}-\frac{2^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\, a^{1/3}\, b^{1/3}\, d}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3}\, a^{1/3}\, b^{1/3}\, d}$$

Result (type 6, 154 leaves):

$$\left(4\, a\, x\left(a+b x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(d\left(a-b x^3\right)\right.$$
$$\left.\left(4\, a\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 584: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^3\left(a d-b d x^3\right)}\, dx$$

Optimal (type 5, 496 leaves, 21 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{2\, a\, d\, x^2}-\frac{2^{1/3}\, b^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{4/3}\, d}-\frac{b^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}\, a^{4/3}\, d}+$$
$$\frac{b\, x\left(1+\frac{b x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2\, a\, d\left(a+b x^3\right)^{2/3}}-\frac{b^{2/3}\,\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\, a^{4/3}\, d}+$$
$$\frac{b^{2/3}\,\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\, a^{4/3}\, d}-\frac{2^{1/3}\, b^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\, a^{4/3}\, d}+\frac{b^{2/3}\,\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3}\, a^{4/3}\, d}$$

Result (type 6, 323 leaves):

$$\frac{1}{8\,d\,x^2\,\left(a+b\,x^3\right)^{2/3}}\left(-4-\frac{4\,b\,x^3}{a}+\left(48\,a\,b\,x^3\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\right.\right.$$
$$\left.\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,2,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\,\frac{5}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\right)\right)+$$
$$\left(7\,b^2\,x^6\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\right.$$
$$\left.\left.\left(7\,a\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\,\frac{2}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{7}{3},\,\frac{5}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 585: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^6\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 5, 523 leaves, 22 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{5\,a\,d\,x^5}-\frac{3\,b\,\left(a+b\,x^3\right)^{1/3}}{5\,a^2\,d\,x^2}-\frac{2^{1/3}\,b^{5/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,a^{7/3}\,d}-$$
$$\frac{b^{5/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}\,a^{7/3}\,d}+\frac{2\,b^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{4}{3},\,-\frac{b\,x^3}{a}\right]}{5\,a^2\,d\,\left(a+b\,x^3\right)^{2/3}}-\frac{b^{5/3}\,\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times2^{2/3}\,a^{7/3}\,d}+$$
$$\frac{b^{5/3}\,\text{Log}\left[1+\frac{2^{2/3}\,\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}-\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times2^{2/3}\,a^{7/3}\,d}-\frac{2^{1/3}\,b^{5/3}\,\text{Log}\left[1+\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,a^{7/3}\,d}+\frac{b^{5/3}\,\text{Log}\left[2\times2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{2^{2/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\times2^{2/3}\,a^{7/3}\,d}$$

Result (type 6, 339 leaves):

$$\frac{1}{20\,d\,\left(a+b\,x^3\right)^{2/3}}\left(\left(112\,b^2\,x\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\right.\right.$$
$$\left.\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,2,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\,\frac{5}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\right)\right)+$$
$$1\Big/\left(a^2\,x^5\right)\left(-4\,\left(a^2+4\,a\,b\,x^3+3\,b^2\,x^6\right)+\left(21\,a\,b^3\,x^9\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\right.\right.$$
$$\left.\left.\left.\left(7\,a\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\,\frac{2}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]-2\,\text{AppellF1}\left[\frac{7}{3},\,\frac{5}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\right)$$

Problem 586: Result unnecessarily involves higher level functions.

$$\int \frac{x^{11} \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 223 leaves, 8 steps):

$$-\frac{a^3 \left(a+b x^3\right)^{2/3}}{2 b^4 d}-\frac{a^2 \left(a+b x^3\right)^{5/3}}{5 b^4 d}+\frac{a \left(a+b x^3\right)^{8/3}}{8 b^4 d}-\frac{\left(a+b x^3\right)^{11/3}}{11 b^4 d}-\frac{2^{2/3} a^{11/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} b^4 d}+\frac{a^{11/3} \text{Log}\left[a-b x^3\right]}{3\times 2^{1/3} b^4 d}-\frac{a^{11/3} \text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^4 d}$$

Result (type 5, 80 leaves):

$$-\frac{\left(a+b x^3\right)^{2/3} \left(293 a^3+98 a^2 b x^3+65 a b^2 x^6+40 b^3 x^9-220 a^3 \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{a+b x^3}{2 a}\right]\right)}{440 b^4 d}$$

Problem 587: Result unnecessarily involves higher level functions.

$$\int \frac{x^8 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 177 leaves, 8 steps):

$$-\frac{a^2 \left(a+b x^3\right)^{2/3}}{2 b^3 d}-\frac{\left(a+b x^3\right)^{8/3}}{8 b^3 d}-\frac{2^{2/3} a^{8/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} b^3 d}+\frac{a^{8/3} \text{Log}\left[a-b x^3\right]}{3\times 2^{1/3} b^3 d}-\frac{a^{8/3} \text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^3 d}$$

Result (type 5, 68 leaves):

$$-\frac{\left(a+b x^3\right)^{2/3} \left(5 a^2+2 a b x^3+b^2 x^6-4 a^2 \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{a+b x^3}{2 a}\right]\right)}{8 b^3 d}$$

Problem 588: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$-\frac{a \left(a+b x^3\right)^{2/3}}{2 b^2 d}-\frac{\left(a+b x^3\right)^{5/3}}{5 b^2 d}-\frac{2^{2/3} a^{5/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} b^2 d}+\frac{a^{5/3} \text{Log}\left[a-b x^3\right]}{3\times 2^{1/3} b^2 d}-\frac{a^{5/3} \text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^2 d}$$

Result (type 5, 56 leaves):

$$-\frac{\left(a+b x^3\right)^{2/3} \left(7 a+2 b x^3-5 a \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{a+b x^3}{2 a}\right]\right)}{10 b^2 d}$$

- **Problem 589: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 b d}-\frac{2^{2/3} a^{2/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} b d}+\frac{a^{2/3} \text{Log}\left[a-b x^3\right]}{3\times 2^{1/3} b d}-\frac{a^{2/3} \text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b d}$$

Result (type 5, 45 leaves):

$$\frac{\left(a+b x^3\right)^{2/3} \left(-1+\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{a+b x^3}{2 a}\right]\right)}{2 b d}$$

- **Problem 590: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 214 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3} d}-\frac{2^{2/3} \text{ArcTan}\left[\frac{a^{1/3}+2^{2/3} \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3} d}-\frac{\text{Log}[x]}{2 a^{1/3} d}+\frac{\text{Log}\left[a-b x^3\right]}{3\times 2^{1/3} a^{1/3} d}+\frac{\text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2 a^{1/3} d}-\frac{\text{Log}\left[2^{1/3} a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a^{1/3} d}$$

Result (type 6, 158 leaves):

$$\left(4 b x^3 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]\right) \Big/ \left(d \left(-a+b x^3\right)\right.$$
$$\left.\left(4 b x^3 \text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]+3 a \text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]+2 a \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]\right)\right)$$

- **Problem 591: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^4 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 269 leaves, 13 steps):

$$\frac{b\,(a+b x^3)^{2/3}}{3 a^2 d}-\frac{(a+b x^3)^{5/3}}{3 a^2 d x^3}+\frac{5 b \,\text{ArcTan}\left[\frac{a^{1/3}+2 (a+b x^3)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{3\sqrt{3}\, a^{4/3} d}-\frac{2^{2/3} b\,\text{ArcTan}\left[\frac{a^{1/3}+2^{2/3}(a+b x^3)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{\sqrt{3}\, a^{4/3} d}-\frac{5 b \,\text{Log}[x]}{6 a^{4/3} d}+\frac{b\,\text{Log}[a-b x^3]}{3\times 2^{1/3} a^{4/3} d}+\frac{5 b\,\text{Log}\left[a^{1/3}-(a+b x^3)^{1/3}\right]}{6 a^{4/3} d}-\frac{b\,\text{Log}\left[2^{1/3} a^{1/3}-(a+b x^3)^{1/3}\right]}{2^{1/3} a^{4/3} d}$$

Result (type 6, 308 leaves):

$$\frac{1}{12 d x^3 (a+b x^3)^{1/3}}\left(-4-\frac{4 b x^3}{a}+\left(8 b^2 x^6 \,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left((a-b x^3)\left(6 a\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3\,\text{AppellF1}\left[2,\frac{1}{3},2,3,-\frac{b x^3}{a},\frac{b x^3}{a}\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)+\left(35 b^2 x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]\right)\Big/\left((-a+b x^3)\left(7 b x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]+3 a\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]-a\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]\right)\right)\right)$$

- **Problem 592: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x^3)^{2/3}}{x^7 (a d-b d x^3)}\,dx$$

Optimal (type 3, 284 leaves, 12 steps):

$$-\frac{5 b\,(a+b x^3)^{2/3}}{18 a^2 d x^3}-\frac{(a+b x^3)^{5/3}}{6 a^2 d x^6}+\frac{14 b^2\,\text{ArcTan}\left[\frac{a^{1/3}+2 (a+b x^3)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{9\sqrt{3}\, a^{7/3} d}-\frac{2^{2/3} b^2\,\text{ArcTan}\left[\frac{a^{1/3}+2^{2/3}(a+b x^3)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{\sqrt{3}\, a^{7/3} d}-\frac{7 b^2\,\text{Log}[x]}{9 a^{7/3} d}+\frac{b^2\,\text{Log}[a-b x^3]}{3\times 2^{1/3} a^{7/3} d}+\frac{7 b^2\,\text{Log}\left[a^{1/3}-(a+b x^3)^{1/3}\right]}{9 a^{7/3} d}-\frac{b^2\,\text{Log}\left[2^{1/3} a^{1/3}-(a+b x^3)^{1/3}\right]}{2^{1/3} a^{7/3} d}$$

Result (type 6, 322 leaves):

$$\frac{1}{18 d (a+b x^3)^{1/3}}\left(-\frac{8 b^2}{a^2}-\frac{3}{x^6}-\frac{11 b}{a x^3}+\left(16 b^3 x^3\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(a (a-b x^3)\left(6 a\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3\,\text{AppellF1}\left[2,\frac{1}{3},2,3,-\frac{b x^3}{a},\frac{b x^3}{a}\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)-\left(49 b^3 x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]\right)\Big/\left(a (a-b x^3)\left(7 b x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]+3 a\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]-a\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b x^3},\frac{a}{b x^3}\right]\right)\right)\right)$$

Problem 593: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 264 leaves, 5 steps):

$$-\frac{4 a x \left(a+b x^3\right)^{2/3}}{9 b^2 d}-\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 b d}-\frac{14 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{7/3} d}+\frac{2^{2/3} a^2 \text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{7/3} d}+$$
$$\frac{a^2 \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{7/3} d}-\frac{a^2 \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{7/3} d}+\frac{7 a^2 \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{9 b^{7/3} d}$$

Result (type 6, 335 leaves):

$$\frac{1}{54 b^{7/3} d}\left(-3 b^{1/3} \left(a+b x^3\right)^{2/3} \left(8 a x+3 b x^4\right)+\left(147 a^3 b^{4/3} x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right) \left(a+b x^3\right)^{1/3}\right.\right.$$
$$\left.\left.\left(7 a \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3 \left(3 \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)+$$
$$\left.2\times 2^{2/3} a^2 \left(2 \sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(b+a x^3\right)^{1/3}}}{\sqrt{3}}\right]-2 \text{Log}\left[1-\frac{2^{1/3} b^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]+\text{Log}\left[1+\frac{2^{2/3} b^{2/3} x^2}{\left(b+a x^3\right)^{2/3}}+\frac{2^{1/3} b^{1/3} x}{\left(b+a x^3\right)^{1/3}}\right]\right)\right)$$

Problem 594: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 229 leaves, 4 steps):

$$-\frac{x \left(a+b x^3\right)^{2/3}}{3 b d}-\frac{5 a \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{4/3} d}+\frac{2^{2/3} a \text{ArcTan}\left[\frac{1+\frac{2\times 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{4/3} d}+$$
$$\frac{a \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{4/3} d}-\frac{a \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{4/3} d}+\frac{5 a \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 b^{4/3} d}$$

Result (type 6, 315 leaves):

$$\frac{1}{36\,d}\left(-\frac{12\,x\,\left(a+b\,x^3\right)^{2/3}}{b}+\left(105\,a^2\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a-b\,x^3\right)\,\left(a+b\,x^3\right)^{1/3}\right.\right.$$

$$\left.\left(7\,a\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)+$$

$$\left.\frac{2^{2/3}\,a\,\left(2\,\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\times2^{1/3}\,b^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}}{\sqrt{3}}\right]-2\,\text{Log}\left[1-\frac{2^{1/3}\,b^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]+\text{Log}\left[1+\frac{2^{2/3}\,b^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{2^{1/3}\,b^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)}{b^{4/3}}\right)$$

- **Problem 595: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 3, 200 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{1/3}\,d}+\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\times2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{1/3}\,d}+\frac{\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times2^{1/3}\,b^{1/3}\,d}-\frac{\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{1/3}\,b^{1/3}\,d}+\frac{\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{1/3}\,d}$$

Result (type 6, 156 leaves):

$$\left(4\,a\,x\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left(d\,\left(a-b\,x^3\right)\right.$$

$$\left.\left(4\,a\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\,\left(3\,\text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)$$

- **Problem 600: Result unnecessarily involves higher level functions.**

$$\int\frac{x^7\,\left(a+b\,x^3\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 5, 512 leaves, 14 steps):

$$-\frac{9 a x^2 \left(a+b x^3\right)^{2/3}}{28 b^2 d}-\frac{x^5 \left(a+b x^3\right)^{2/3}}{7 b d}+\frac{2^{2/3} a^{7/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{8/3} d}+\frac{a^{7/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, b^{8/3} d}-$$

$$\frac{19 a^2 x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{28 b^2 d \left(a+b x^3\right)^{1/3}}+\frac{a^{7/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{8/3} d}+$$

$$\frac{a^{7/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{8/3} d}-\frac{2^{2/3} a^{7/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{8/3} d}-\frac{a^{7/3} \text{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{8/3} d}$$

Result (type 6, 338 leaves) :

$$\frac{1}{140 b^2 d \left(a+b x^3\right)^{1/3}} x^2 \left(-5 \left(9 a^2+13 a b x^3+4 b^2 x^6\right)+\left(225 a^4 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right) \Big/ \left(\left(a-b x^3\right)\right.\right.$$
$$\left.\left(5 a \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3 \left(3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)+$$
$$\left(304 a^3 b x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right) \Big/ \left(\left(a-b x^3\right)\right.$$
$$\left.\left.\left(8 a \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3 \left(3 \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 601: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 485 leaves, 13 steps) :

$$-\frac{x^2 \left(a+b x^3\right)^{2/3}}{4 b d}+\frac{2^{2/3} a^{4/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{5/3} d}+\frac{a^{4/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, b^{5/3} d}-$$

$$\frac{3 a x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{4 b d \left(a+b x^3\right)^{1/3}}+\frac{a^{4/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{5/3} d}+$$

$$\frac{a^{4/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{5/3} d}-\frac{2^{2/3} a^{4/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{5/3} d}-\frac{a^{4/3} \text{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{5/3} d}$$

Result (type 6, 326 leaves) :

$$\frac{1}{20\,d\,(a+b x^3)^{1/3}} x^2 \left(-\frac{5\,(a+b x^3)}{b} + \left(25\,a^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right) \Big/ \left(b\,(a-b x^3)\left(5\,a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right] + b x^3 \left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right] - \text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right) + \left(48\,a^2 x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right) \Big/ \left((a-b x^3)\left(8\,a\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right] + b x^3 \left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right] - \text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

- **Problem 602: Result unnecessarily involves higher level functions.**

$$\int \frac{x\,(a+b x^3)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 5, 457 leaves, 11 steps):

$$\frac{2^{2/3} a^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(a^{1/3}+b^{1/3}x)}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}\,d} + \frac{a^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(a^{1/3}+b^{1/3}x)}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}\,b^{2/3}\,d} - \frac{x^2\left(1+\frac{b x^3}{a}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{2\,d\,(a+b x^3)^{1/3}} + \frac{a^{1/3}\,\text{Log}\left[\frac{(a^{1/3}-b^{1/3}x)^2\,(a^{1/3}+b^{1/3}x)}{a}\right]}{6\times 2^{1/3}\,b^{2/3}\,d} + \frac{a^{1/3}\,\text{Log}\left[1+\frac{2^{2/3}(a^{1/3}+b^{1/3}x)^2}{(a+b x^3)^{2/3}}-\frac{2^{1/3}(a^{1/3}+b^{1/3}x)}{(a+b x^3)^{1/3}}\right]}{3\times 2^{1/3}\,b^{2/3}\,d} - \frac{2^{2/3} a^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}(a^{1/3}+b^{1/3}x)}{(a+b x^3)^{1/3}}\right]}{3\,b^{2/3}\,d} - \frac{a^{1/3}\,\text{Log}\left[\frac{b^{1/3}(a^{1/3}+b^{1/3}x)}{a^{1/3}}-\frac{2^{2/3}b^{1/3}(a+b x^3)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3}\,b^{2/3}\,d}$$

Result (type 6, 160 leaves):

$$\left(5\,a\,x^2\,(a+b x^3)^{2/3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right) \Big/ \left(2\,d\,(a-b x^3)\left(5\,a\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right] + b x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},2,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right] + 2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)$$

- **Problem 603: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x^3)^{2/3}}{x^2\,(a\,d-b\,d\,x^3)}\,dx$$

Optimal (type 5, 483 leaves, 13 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{a d x}+\frac{2^{2/3} b^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{2/3} d}+\frac{b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}\, a^{2/3} d}+$$

$$\frac{b x^2\left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{2 a d\left(a+b x^3\right)^{1/3}}+\frac{b^{1/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2\left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} a^{2/3} d}+$$

$$\frac{b^{1/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} a^{2/3} d}-\frac{2^{2/3} b^{1/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{2/3} d}-\frac{b^{1/3} \text{Log}\left[\frac{b^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}\left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} a^{2/3} d}$$

Result (type 6, 326 leaves) :

$$\frac{1}{10 d x\left(a+b x^3\right)^{1/3}}\left(\left(75 a b x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\right.\right.$$
$$\left.\left.\left(5 a \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)+$$
$$2\left(-5-\frac{5 b x^3}{a}-\left(8 b^2 x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\Big/\left(\left(a-b x^3\right)\right.\right.$$
$$\left.\left.\left.\left(8 a \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]+b x^3\left(3 \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]\right)\right)\right)\right)\right)$$

- **Problem 604: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^5\left(a d-b d x^3\right)}\, dx$$

Optimal (type 5, 512 leaves, 14 steps) :

$$-\frac{\left(a+b x^3\right)^{2/3}}{4 a d x^4}-\frac{3 b\left(a+b x^3\right)^{2/3}}{2 a^2 d x}+\frac{2^{2/3} b^{4/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{5/3} d}+\frac{b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}\, a^{5/3} d}+$$

$$\frac{3 b^2 x^2\left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{4 a^2 d\left(a+b x^3\right)^{1/3}}+\frac{b^{4/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2\left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} a^{5/3} d}+$$

$$\frac{b^{4/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} a^{5/3} d}-\frac{2^{2/3} b^{4/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{5/3} d}-\frac{b^{4/3} \text{Log}\left[\frac{b^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}\left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} a^{5/3} d}$$

Result (type 6, 341 leaves) :

$$\frac{1}{20\,d\,(a+b\,x^3)^{1/3}}\left(\left(175\,b^2\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left((a-b\,x^3)\left(5\,a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)+1\Big/(a^2\,x^4)\left(-5\,(a^2+7\,a\,b\,x^3+6\,b^2\,x^6)-\left(48\,a\,b^3\,x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\Big/\left((a-b\,x^3)\left(8\,a\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]+b\,x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\right)$$

- **Problem 605: Result unnecessarily involves higher level functions.**

$$\int\frac{x^{14}}{(1-x^3)^{1/3}\,(1+x^3)}\,dx$$

Optimal (type 3, 127 leaves, 7 steps) :

$$\frac{2}{5}(1-x^3)^{5/3}-\frac{1}{4}(1-x^3)^{8/3}+\frac{1}{11}(1-x^3)^{11/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}[1+x^3]}{6\times2^{1/3}}+\frac{\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2\times2^{1/3}}$$

Result (type 5, 74 leaves) :

$$\frac{(-1+x^3)^2\,(53+15\,x^3+20\,x^6)-220\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{220\,(1-x^3)^{1/3}}$$

- **Problem 606: Result unnecessarily involves higher level functions.**

$$\int\frac{x^{11}}{(1-x^3)^{1/3}\,(1+x^3)}\,dx$$

Optimal (type 3, 128 leaves, 7 steps) :

$$-\frac{1}{2}(1-x^3)^{2/3}+\frac{1}{5}(1-x^3)^{5/3}-\frac{1}{8}(1-x^3)^{8/3}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}[1+x^3]}{6\times2^{1/3}}-\frac{\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2\times2^{1/3}}$$

Result (type 5, 70 leaves) :

$$\frac{-17+19\,x^3-7\,x^6+5\,x^9+40\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{40\,(1-x^3)^{1/3}}$$

Problem 607: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 5, 61 leaves):

$$\frac{\left(-1+x^3\right)^2-5\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{5\left(1-x^3\right)^{1/3}}$$

Problem 608: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 5, 58 leaves):

$$\frac{-1+x^3+2\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{2\left(1-x^3\right)^{1/3}}$$

Problem 610: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 137 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 6, 111 leaves):

$$-\left(7 x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/\left(4\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(7 x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)$$

- **Problem 611: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 157 leaves, 11 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{3 x^3}-\frac{2\,\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}[x]}{3}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{1}{3}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 6, 209 leaves):

$$\frac{1}{6 x^3\left(1-x^3\right)^{1/3}}\left(-2+2 x^3-\left(4 x^6 \text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[2,\frac{1}{3},2,3,x^3,-x^3\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,x^3,-x^3\right]\right)\right)\right)+\left(7 x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/\left(\left(1+x^3\right)\left(7 x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)\right)$$

- **Problem 612: Result unnecessarily involves higher level functions.**

$$\int \frac{x^6}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 154 leaves, 4 steps):

$$-\frac{1}{3}x\left(1-x^3\right)^{2/3}+\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}-\frac{1}{3}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 233 leaves):

$$\frac{1}{36}\left(-12\,x\,\left(1-x^3\right)^{2/3}+\left(42\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\Big/\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-7\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]\right)\right)\right)+2^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\frac{2\times 2^{1/3}x}{\left(-1+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(-1+x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]+2\,\text{Log}\left[1+\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]\right)\right)$$

- **Problem 613: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 135 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}+\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 115 leaves):

$$-\left(7\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\Big/\left(4\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-7\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]\right)\right)\right)$$

- **Problem 618: Result unnecessarily involves higher level functions.**

$$\int \frac{x^7}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 271 leaves, 12 steps):

$$-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 119 leaves):

$$\frac{1}{4} x^2 \left(1-x^3\right)^{2/3} \left(-1-\left(5 \text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left.\left(\left(1+x^3\right)\left(-5 \text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3 \left(3 \text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},2,\frac{8}{3},x^3,-x^3\right]+2 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 619: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 254 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{1}{2} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x) (1+x)^2\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$-\left(8 x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/$$
$$\left(5 \left(1-x^3\right)^{1/3} \left(1+x^3\right) \left(-8 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3 \left(3 \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)$$

Problem 620: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[(1-x) (1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$-\left(5 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/$$
$$\left(2 \left(1-x^3\right)^{1/3} \left(1+x^3\right) \left(-5 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3 \left(3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)$$

Problem 621: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 270 leaves, 12 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}-\frac{1}{2} x^2 \,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x) (1+x)^2\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 229 leaves):

$$\frac{1}{5 x \left(1-x^3\right)^{1/3}}\left(-5+5 x^3+\left(25 x^3 \,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-5 \,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3 \left(3 \,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+$$
$$\left(8 x^6 \,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(-8 \,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3 \left(3 \,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 622: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 289 leaves, 14 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{4 x^4}+\frac{\left(1-x^3\right)^{2/3}}{2 x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{1}{4} x^2 \,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$
$$\frac{\text{Log}\left[(1-x) (1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 234 leaves):

$$-\frac{1}{20 x^4 \left(1-x^3\right)^{1/3}}\left(5-15 x^3+10 x^6+\left(75 x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-5 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+$$
$$\left(16 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(-8 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

- **Problem 623: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 125 leaves, 7 steps) :

$$-\left(1-x^3\right)^{1/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 70 leaves) :

$$\frac{-25+26 x^3-5 x^6+4 x^9+14\left(\frac{-1+x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2}{1+x^3}\right]}{28\left(1-x^3\right)^{2/3}}$$

- **Problem 624: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{8}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 98 leaves, 7 steps) :

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 61 leaves) :

$$\frac{\left(-1+x^3\right)^2-2\left(\frac{-1+x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2}{1+x^3}\right]}{4\left(1-x^3\right)^{2/3}}$$

Problem 625: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 95 leaves, 6 steps):

$$-\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 59 leaves):

$$\frac{-2+2x^3+\left(\frac{-1+x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2}{1+x^3}\right]}{2\left(1-x^3\right)^{2/3}}$$

Problem 627: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 137 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 113 leaves):

$$-\left(8x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/$$
$$\left(5\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(8x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+2\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)$$

Problem 628: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 158 leaves, 11 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{3x^3}+\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}[x]}{6}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{1}{6}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 110 leaves):

$$-\left(11\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},1,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/\left(8\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(11\,x^3\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},1,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{11}{3},\frac{2}{3},2,\frac{14}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+2\,\text{AppellF1}\left[\frac{11}{3},\frac{5}{3},1,\frac{14}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)$$

- **Problem 629: Result unnecessarily involves higher level functions.**

$$\int\frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$-\frac{1}{3}x^2\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times2^{2/3}}+\frac{1}{6}\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times2^{2/3}}$$

Result (type 6, 170 leaves):

$$\frac{1}{15\left(1-x^3\right)^{2/3}}x^2\left(\left(8\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},x^3,-x^3\right]-2\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)+5\left(-1+x^3+\left(\frac{1-x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2x^3}{1+x^3}\right]\right)\right)$$

- **Problem 630: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 139 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times2^{2/3}}-\frac{1}{2}\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times2^{2/3}}$$

Result (type 6, 115 leaves):

$$-\left(8\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/\left(5\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},x^3,-x^3\right]-2\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)$$

Problem 631: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 59 leaves):

$$\frac{x^2 \left(\frac{1-x^3}{1+x^3}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2 x^3}{1+x^3}\right]}{2 \left(1-x^3\right)^{2/3}}$$

Problem 632: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 103 leaves, 2 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 154 leaves):

$$\left(5 \left(2+x^3-3 x^6\right) \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2 x^3}{-1+x^3}\right]-12 x^3 \left(1+x^3\right) \text{Hypergeometric2F1}\left[\frac{5}{3},2,\frac{8}{3},\frac{2 x^3}{-1+x^3}\right]\right) \Big/$$
$$\left(2 x \left(1-x^3\right)^{2/3} \left(5 \left(2-5 x^3+3 x^6\right)+15 \left(-1+x^6\right) \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2 x^3}{-1+x^3}\right]+18 \left(x^3+x^6\right) \text{Hypergeometric2F1}\left[\frac{5}{3},2,\frac{8}{3},\frac{2 x^3}{-1+x^3}\right]\right)\right)$$

Problem 633: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{4 x^4}+\frac{\left(1-x^3\right)^{1/3}}{4 x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 680 leaves):

$$-\left(\left(\left(1-x^3\right)^{4/3}\left(5\left(-1-9x^3+x^6+9x^9+\left(4-13x^3-20x^6+9x^9\right)\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]\right)+216\left(x^6+x^9\right)\right.\right.\right.$$
$$\left.\left.\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+81x^3\left(1+x^3\right)^2\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]\right)\right)\Big/$$
$$\left(3x^4\left(-20+70x^3+60x^6-200x^9+40x^{12}+50x^{15}+40\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]-125x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]+\right.\right.$$
$$90x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]+180x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]-$$
$$130x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]-55x^{15}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]+$$
$$144x^6\left(-1-4x^3+x^6+4x^9\right)\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+$$
$$432x^9\left(1+x^3\right)^2\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3\right\},\left\{2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+27x^3\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]-$$
$$270x^6\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]-324x^9\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+$$
$$270x^{12}\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+297x^{15}\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+$$
$$324x^6\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+972x^9\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+$$
$$\left.\left.972x^{12}\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+324x^{15}\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]\right)\right)\right)$$

- **Problem 634: Result unnecessarily involves higher level functions.**

$$\int\frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 291 leaves, 15 steps):

$$-\frac{1}{2}x\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{2/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times2^{2/3}}-\frac{\text{Log}\left[2\times2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times2^{2/3}}$$

Result (type 6, 115 leaves):

$$\frac{1}{2} x \left(1-x^3\right)^{1/3} \left(-1-\left(4 \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-4 \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right]+\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

- **Problem 635: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx$$

Optimal (type 5, 294 leaves, 18 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{1}{2} x \,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]-\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 115 leaves):

$$-\left(7 x^4 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\Big/\left(4\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(-7 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},x^3,-x^3\right]-2 \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},x^3,-x^3\right]\right)\right)\right)$$

- **Problem 636: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx$$

Optimal (type 5, 293 leaves, 16 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{1}{2} x \,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 111 leaves):

$$-\left(4\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]\right)\Big/$$
$$\left(\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(-4\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right]-2\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\right)\right)$$

Problem 637: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 294 leaves, 16 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{2\,x^2}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{2/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times2^{2/3}}+\frac{\text{Log}\left[2\times2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times2^{2/3}}$$

Result (type 6, 120 leaves):

$$\frac{\left(1-x^3\right)^{1/3}\left(-1+\frac{4\,x^3\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]}{\left(1+x^3\right)\left(-4\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right]+\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\right)}\right)}{2\,x^2}$$

Problem 638: Result unnecessarily involves higher level functions.

$$\int\frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 141 leaves, 11 steps):

$$\frac{1}{2\left(1-x^3\right)^{1/3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times2^{1/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{4\times2^{1/3}}$$

Result (type 5, 70 leaves):

$$\frac{49-23\,x^3-x^6-5\,x^9-20\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{40\left(1-x^3\right)^{1/3}}$$

Problem 639: Result unnecessarily involves higher level functions.

$$\int\frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 130 leaves, 11 steps):

$$\frac{1}{2\left(1-x^3\right)^{1/3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 65 leaves):

$$\frac{8-x^3-2x^6+5\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{10\left(1-x^3\right)^{1/3}}$$

- **Problem 640: Result unnecessarily involves higher level functions.**

$$\int\frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 115 leaves, 9 steps):

$$\frac{1}{2\left(1-x^3\right)^{1/3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 60 leaves):

$$\frac{2-x^3-\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{2\left(1-x^3\right)^{1/3}}$$

- **Problem 641: Result unnecessarily involves higher level functions.**

$$\int\frac{x^5}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$\frac{1}{2\left(1-x^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 54 leaves):

$$\frac{1+\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{2\left(1-x^3\right)^{1/3}}$$

Problem 643: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 154 leaves, 11 steps):

$$\frac{1}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 201 leaves):

$$\frac{1}{4\left(1-x^3\right)^{1/3}}\left(2-\left(4x^3\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[2,\frac{1}{3},2,3,x^3,-x^3\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,x^3,-x^3\right]\right)\right)\right)-$$
$$\left(7x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(7x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)\right)$$

Problem 644: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 175 leaves, 13 steps):

$$\frac{5}{6\left(1-x^3\right)^{1/3}}-\frac{1}{3x^3\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}[x]}{6}-\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{1}{6}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 206 leaves):

$$\frac{1}{12\left(1-x^3\right)^{1/3}}\left(10-\frac{4}{x^3}-\left(20x^3\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[2,\frac{1}{3},2,3,x^3,-x^3\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,x^3,-x^3\right]\right)\right)\right)-$$
$$\left(7x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(7x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)\right)$$

Problem 645: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 174 leaves, 5 steps):

$$\frac{x^4}{2\left(1-x^3\right)^{1/3}}+\frac{5}{6}x\left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}-\frac{1}{6}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 241 leaves):

$$\frac{1}{72}\left(-\frac{12x\left(-5+2x^3\right)}{\left(1-x^3\right)^{1/3}}+\left(42x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-7\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]\right)\right)\right)-$$
$$\left.5\times 2^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\frac{2\times 2^{1/3}x}{\left(-1+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(-1+x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]+2\,\text{Log}\left[1+\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]\right)\right)$$

Problem 646: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 231 leaves):

$$\frac{1}{24}\left(\frac{12\,x}{\left(1-x^3\right)^{1/3}}+\left(42\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\Big/\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-7\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]\right)\right)\right)+2^{2/3}\left(-2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\frac{2\times 2^{1/3}\,x}{\left(-1+x^3\right)^{1/3}}}{\sqrt{3}}\right]+\text{Log}\left[1+\frac{2^{2/3}\,x^2}{\left(-1+x^3\right)^{2/3}}-\frac{2^{1/3}\,x}{\left(-1+x^3\right)^{1/3}}\right]-2\,\text{Log}\left[1+\frac{2^{1/3}\,x}{\left(-1+x^3\right)^{1/3}}\right]\right)\right)$$

Problem 652: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\frac{x^5}{2\left(1-x^3\right)^{1/3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 226 leaves):

$$\frac{1}{20\left(1-x^3\right)^{1/3}}x^2\left(15-5\,x^3+\left(75\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+\left(32\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 653: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 274 leaves, 12 steps):

$$\frac{x^2}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$
$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 221 leaves) :

$$\frac{1}{10\left(1-x^3\right)^{1/3}}x^2\left(5+\left(25\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+$$
$$\left(24\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

- **Problem 654: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 274 leaves, 12 steps) :

$$\frac{x^2}{2\left(1-x^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 221 leaves) :

$$\frac{1}{10\left(1-x^3\right)^{1/3}}x^2\left(5+\left(25\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+$$
$$\left(8\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

- **Problem 655: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 274 leaves, 11 steps):

$$\frac{x^2}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times2^{1/3}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$
$$\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times2^{1/3}}$$

Result (type 6, 122 leaves):

$$\frac{x^2\left(5+\frac{8\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]}{\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)}\right)}{10\left(1-x^3\right)^{1/3}}$$

- **Problem 656: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\frac{1}{2x\left(1-x^3\right)^{1/3}}-\frac{3\left(1-x^3\right)^{2/3}}{2x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times2^{1/3}\sqrt{3}}-\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times2^{1/3}}$$

Result (type 6, 232 leaves) :

$$\frac{1}{5\left(1-x^3\right)^{1/3}}\left(-\frac{5}{x}+\frac{15\,x^2}{2}+\left(25\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+$$
$$\left(12\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

- **Problem 657: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^5\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 308 leaves, 14 steps) :

$$\frac{1}{2\,x^4\left(1-x^3\right)^{1/3}}-\frac{3\left(1-x^3\right)^{2/3}}{4\,x^4}-\frac{\left(1-x^3\right)^{2/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times2^{1/3}\sqrt{3}}-\frac{1}{2}\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$
$$\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times2^{1/3}}$$

Result (type 6, 234 leaves) :

$$\frac{1}{20\,x^4\left(1-x^3\right)^{1/3}}\left(-5-5\,x^3+20\,x^6+\left(25\,x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+$$
$$\left(32\,x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/$$
$$\left.\left(\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

- **Problem 658: Result unnecessarily involves higher level functions.**

$$\int\frac{x^{11}\left(a+b\,x^3\right)^{1/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 264 leaves, 8 steps) :

$$-\frac{c^3 \left(a+b x^3\right)^{1/3}}{d^4}+\frac{\left(b^2 c^2+a b c d+a^2 d^2\right) \left(a+b x^3\right)^{4/3}}{4 b^3 d^3}-\frac{(b c+2 a d) \left(a+b x^3\right)^{7/3}}{7 b^3 d^2}+\frac{\left(a+b x^3\right)^{10/3}}{10 b^3 d}-$$
$$\frac{c^3 (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{13/3}}-\frac{c^3 (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 d^{13/3}}+\frac{c^3 (b c-a d)^{1/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{13/3}}$$

Result (type 5, 193 leaves) :

$$\frac{1}{140 b^3 d^5 \left(a+b x^3\right)^{2/3}}\left(d \left(a+b x^3\right) \left(9 a^3 d^3-3 a^2 b d^2 \left(-5 c+d x^3\right)+a b^2 d \left(35 c^2-5 c d x^3+2 d^2 x^6\right)+b^3 \left(-140 c^3+35 c^2 d x^3-20 c d^2 x^6+14 d^3 x^9\right)\right)-\right.$$
$$\left.70 b^3 c^3 (b c-a d) \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]\right)$$

Problem 659: Result unnecessarily involves higher level functions.

$$\int \frac{x^8 \left(a+b x^3\right)^{1/3}}{c+d x^3} \, dx$$

Optimal (type 3, 220 leaves, 8 steps) :

$$\frac{c^2 \left(a+b x^3\right)^{1/3}}{d^3}-\frac{(b c+a d) \left(a+b x^3\right)^{4/3}}{4 b^2 d^2}+\frac{\left(a+b x^3\right)^{7/3}}{7 b^2 d}+\frac{c^2 (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{10/3}}+$$
$$\frac{c^2 (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 d^{10/3}}-\frac{c^2 (b c-a d)^{1/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{10/3}}$$

Result (type 5, 152 leaves) :

$$\frac{1}{28 b^2 d^4 \left(a+b x^3\right)^{2/3}}\left(-d \left(a+b x^3\right) \left(3 a^2 d^2+a b d \left(7 c-d x^3\right)+b^2 \left(-28 c^2+7 c d x^3-4 d^2 x^6\right)\right)+\right.$$
$$\left.14 b^2 c^2 (b c-a d) \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]\right)$$

Problem 660: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(a+b x^3\right)^{1/3}}{c+d x^3} \, dx$$

Optimal (type 3, 186 leaves, 7 steps) :

$$-\frac{c\left(a+b x^3\right)^{1/3}}{d^2}+\frac{\left(a+b x^3\right)^{4/3}}{4 b d}-\frac{c\,(b c-a d)^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2 d^{1/3}\left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{7/3}}-\frac{c\,(b c-a d)^{1/3}\,\text{Log}\left[c+d x^3\right]}{6 d^{7/3}}+\frac{c\,(b c-a d)^{1/3}\,\text{Log}\left[(b c-a d)^{1/3}+d^{1/3}\left(a+b x^3\right)^{1/3}\right]}{2 d^{7/3}}$$

Result (type 5, 113 leaves) :

$$\frac{d\left(a+b x^3\right)\left(-4 b c+a d+b d x^3\right)+2 b c\,(-b c+a d)\left(\frac{d\left(a+b x^3\right)}{b\left(c+d x^3\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]}{4 b d^3\left(a+b x^3\right)^{2/3}}$$

Problem 661: Result unnecessarily involves higher level functions.

$$\int\frac{x^2\left(a+b x^3\right)^{1/3}}{c+d x^3}\,dx$$

Optimal (type 3, 159 leaves, 6 steps) :

$$\frac{\left(a+b x^3\right)^{1/3}}{d}+\frac{(b c-a d)^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2 d^{1/3}\left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{4/3}}+\frac{(b c-a d)^{1/3}\,\text{Log}\left[c+d x^3\right]}{6 d^{4/3}}-\frac{(b c-a d)^{1/3}\,\text{Log}\left[(b c-a d)^{1/3}+d^{1/3}\left(a+b x^3\right)^{1/3}\right]}{2 d^{4/3}}$$

Result (type 5, 94 leaves) :

$$\frac{2 d\left(a+b x^3\right)+(b c-a d)\left(\frac{d\left(a+b x^3\right)}{b\left(c+d x^3\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]}{2 d^2\left(a+b x^3\right)^{2/3}}$$

Problem 662: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b x^3\right)^{1/3}}{x\left(c+d x^3\right)}\,dx$$

Optimal (type 3, 246 leaves, 10 steps) :

$$-\frac{a^{1/3}\,\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b x^3\right)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{\sqrt{3}\, c}-\frac{(b c-a d)^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2 d^{1/3}\left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c\, d^{1/3}}-\frac{a^{1/3}\,\text{Log}[x]}{2 c}-\frac{(b c-a d)^{1/3}\,\text{Log}\left[c+d x^3\right]}{6 c\, d^{1/3}}+\frac{a^{1/3}\,\text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2 c}+\frac{(b c-a d)^{1/3}\,\text{Log}\left[(b c-a d)^{1/3}+d^{1/3}\left(a+b x^3\right)^{1/3}\right]}{2 c\, d^{1/3}}$$

Result (type 6, 162 leaves) :

$$-\left(5\,b\,d\,x^3\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\Big/\left(2\,\left(c+d\,x^3\right)\,\left(5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]-3\,b\,c\,\text{AppellF1}\left[\frac{5}{3},-\frac{1}{3},2,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)$$

Problem 663: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^4\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 340 leaves, 13 steps):

$$\frac{d\,\left(a+b\,x^3\right)^{1/3}}{c^2}+\frac{\left(b\,c-3\,a\,d\right)\,\left(a+b\,x^3\right)^{1/3}}{3\,a\,c^2}-\frac{\left(a+b\,x^3\right)^{4/3}}{3\,a\,c\,x^3}-\frac{\left(b\,c-3\,a\,d\right)\,\text{ArcTan}\left[\frac{a^{1/3}+2\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{3\,\sqrt{3}\,a^{2/3}\,c^2}+\frac{d^{2/3}\,\left(b\,c-a\,d\right)^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,\left(a+b\,x^3\right)^{1/3}}{\left(b\,c-a\,d\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^2}-\frac{\left(b\,c-3\,a\,d\right)\,\text{Log}[x]}{6\,a^{2/3}\,c^2}+\frac{d^{2/3}\,\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^2}+\frac{\left(b\,c-3\,a\,d\right)\,\text{Log}\left[a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{6\,a^{2/3}\,c^2}-\frac{d^{2/3}\,\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[\left(b\,c-a\,d\right)^{1/3}+d^{1/3}\,\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^2}$$

Result (type 6, 411 leaves):

$$\left(\left(20\,a\,b\,d\,x^6\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-6\,a\,c\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[2,\frac{2}{3},2,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[2,\frac{5}{3},1,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+\left(8\,b\,d\,x^3\,\left(5\,a\,c+6\,b\,c\,x^3+2\,a\,d\,x^3+5\,b\,d\,x^6\right)\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]-5\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)\,\left(3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\Big/\left(c\,\left(-8\,b\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\right)\Big/\left(15\,x^3\,\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)\right)$$

Problem 664: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^7 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 370 leaves, 12 steps):

$$\frac{(b c+3 a d) \left(a+b x^3\right)^{1/3}}{9 a c^2 x^3}-\frac{\left(a+b x^3\right)^{4/3}}{6 a c x^6}+\frac{\left(b^2 c^2+3 a b c d-9 a^2 d^2\right) \text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{9 \sqrt{3} a^{5/3} c^3}-$$

$$\frac{d^{5/3} (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^3}+\frac{\left(b^2 c^2+3 a b c d-9 a^2 d^2\right) \text{Log}[x]}{18 a^{5/3} c^3}-\frac{d^{5/3} (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^3}-$$

$$\frac{\left(b^2 c^2+3 a b c d-9 a^2 d^2\right) \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{18 a^{5/3} c^3}+\frac{d^{5/3} (b c-a d)^{1/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 c^3}$$

Result (type 6, 371 leaves):

$$\frac{1}{90 c^2 x^6 \left(a+b x^3\right)^{2/3}}\left(\left(20 b c d (b c-6 a d) x^9 \text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\left(c+d x^3\right)\left(-6 a c \text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.\right.$$

$$\left.\left.x^3\left(3 a d \text{AppellF1}\left[2,\frac{2}{3},2,3,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[2,\frac{5}{3},1,3,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+$$

$$1/a\left(-5\left(a+b x^3\right)\left(b c x^3+3 a\left(c-2 d x^3\right)\right)+\left(16 b d\left(b^2 c^2+3 a b c d-9 a^2 d^2\right) x^9 \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\Big/\right.$$

$$\left(\left(c+d x^3\right)\left(8 b d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]-\right.\right.$$

$$\left.\left.\left.\left.3 b c \text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]-2 a d \text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)\right)\right)$$

Problem 665: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \left(a+b x^3\right)^{1/3}}{c+d x^3} \, dx$$

Optimal (type 3, 336 leaves, 6 steps):

$$-\frac{(6\,b\,c-a\,d)\,x^2\,\left(a+b\,x^3\right)^{1/3}}{18\,b\,d^2}+\frac{x^5\,\left(a+b\,x^3\right)^{1/3}}{6\,d}-\frac{\left(9\,b^2\,c^2-3\,a\,b\,c\,d-a^2\,d^2\right)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,b^{5/3}\,d^3}+\frac{c^{5/3}\,(b\,c-a\,d)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^3}-$$
$$\frac{c^{5/3}\,(b\,c-a\,d)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d^3}-\frac{\left(9\,b^2\,c^2-3\,a\,b\,c\,d-a^2\,d^2\right)\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{18\,b^{5/3}\,d^3}+\frac{c^{5/3}\,(b\,c-a\,d)^{1/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^3}$$

Result (type 6, 293 leaves):

$$\frac{1}{90\,b\,d^2\,\left(a+b\,x^3\right)^{2/3}}x^2\,\left(5\,\left(a+b\,x^3\right)\,\left(-6\,b\,c+a\,d+3\,b\,d\,x^3\right)+\right.$$
$$\left(16\,a\,c\,\left(-9\,b^2\,c^2+3\,a\,b\,c\,d+a^2\,d^2\right)\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)-$$
$$\left.5\,a\,(-6\,b\,c+a\,d)\,\left(\frac{c\,\left(a+b\,x^3\right)}{a\,\left(c+d\,x^3\right)}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b\,c+a\,d)\,x^3}{a\,\left(c+d\,x^3\right)}\right]\right)$$

- **Problem 666: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4\,\left(a+b\,x^3\right)^{1/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 276 leaves, 5 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,d}+\frac{(3\,b\,c-a\,d)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{2/3}\,d^2}-\frac{c^{2/3}\,(b\,c-a\,d)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2}+$$
$$\frac{c^{2/3}\,(b\,c-a\,d)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d^2}+\frac{(3\,b\,c-a\,d)\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{6\,b^{2/3}\,d^2}-\frac{c^{2/3}\,(b\,c-a\,d)^{1/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^2}$$

Result (type 6, 253 leaves):

$$\frac{1}{15\,d\,\left(a+b\,x^3\right)^{2/3}}$$

$$x^2\left(-\left(8\,a\,c\,(-3\,b\,c+a\,d)\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+5\left(a+b\,x^3-a\left(\frac{c\left(a+b\,x^3\right)}{a\left(c+d\,x^3\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b\,c+a\,d)\,x^3}{a\left(c+d\,x^3\right)}\right]\right)\right)$$

- **Problem 667: Result unnecessarily involves higher level functions.**

$$\int\frac{x\left(a+b\,x^3\right)^{1/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d}+\frac{(b\,c-a\,d)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{1/3}\,d}-\frac{(b\,c-a\,d)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{1/3}\,d}-\frac{b^{1/3}\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d}+\frac{(b\,c-a\,d)^{1/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{1/3}\,d}$$

Result (type 6, 164 leaves):

$$\left(5\,a\,c\,x^2\left(a+b\,x^3\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(2\left(c+d\,x^3\right)\left(5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(-3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},-\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 668: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^2\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 168 leaves, 3 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{c\,x}-\frac{(b\,c-a\,d)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{4/3}}+\frac{(b\,c-a\,d)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{4/3}}-\frac{(b\,c-a\,d)^{1/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{4/3}}$$

Result (type 5, 103 leaves):

$$\frac{-2 c \left(a+b x^3\right)+(b c-a d) x^3 \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b c+a d) x^3}{a \left(c+d x^3\right)}\right]}{2 c^2 x \left(a+b x^3\right)^{2/3}}$$

- **Problem 669: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^5 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 204 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{4 c x^4}-\frac{(b c-4 a d) \left(a+b x^3\right)^{1/3}}{4 a c^2 x}+\frac{d (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{7/3}}-$$
$$\frac{d (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^{7/3}}+\frac{d (b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{7/3}}$$

Result (type 5, 126 leaves):

$$\frac{1}{4 a c^3 x^4 \left(a+b x^3\right)^{2/3}}\left(-c \left(a+b x^3\right) \left(b c x^3+a \left(c-4 d x^3\right)\right)+2 a d (-b c+a d) x^6 \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b c+a d) x^3}{a \left(c+d x^3\right)}\right]\right)$$

- **Problem 670: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^8 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 258 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{7 c x^7}-\frac{(b c-7 a d) \left(a+b x^3\right)^{1/3}}{28 a c^2 x^4}+\frac{\left(3 b^2 c^2+7 a b c d-28 a^2 d^2\right) \left(a+b x^3\right)^{1/3}}{28 a^2 c^3 x}-$$
$$\frac{d^2 (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{10/3}}+\frac{d^2 (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^{10/3}}-\frac{d^2 (b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{10/3}}$$

Result (type 5, 165 leaves):

$$\frac{1}{28 a^2 c^4 x^7 \left(a+b x^3\right)^{2/3}}\left(-c \left(a+b x^3\right) \left(-3 b^2 c^2 x^6+a b c x^3 \left(c-7 d x^3\right)+a^2 \left(4 c^2-7 c d x^3+28 d^2 x^6\right)\right)-\right.$$
$$\left.14 a^2 d^2 (-b c+a d) x^9 \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b c+a d) x^3}{a \left(c+d x^3\right)}\right]\right)$$

Problem 671: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^{11} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{10 c x^{10}}-\frac{(b c-10 a d) \left(a+b x^3\right)^{1/3}}{70 a c^2 x^7}+\frac{\left(3 b^2 c^2+5 a b c d-35 a^2 d^2\right) \left(a+b x^3\right)^{1/3}}{140 a^2 c^3 x^4}-\frac{\left(9 b^3 c^3+15 a b^2 c^2 d+35 a^2 b c d^2-140 a^3 d^3\right) \left(a+b x^3\right)^{1/3}}{140 a^3 c^4 x}+$$

$$\frac{d^3 (b c-a d)^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{13/3}}-\frac{d^3 (b c-a d)^{1/3} \operatorname{Log}\left[c+d x^3\right]}{6 c^{13/3}}+\frac{d^3 (b c-a d)^{1/3} \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{13/3}}$$

Result (type 5, 211 leaves):

$$\frac{1}{140 a^3 c^5 x^{10} \left(a+b x^3\right)^{2/3}}$$

$$\left(-c \left(a+b x^3\right) \left(9 b^3 c^3 x^9-3 a b^2 c^2 x^6 \left(c-5 d x^3\right)+a^2 b c x^3 \left(2 c^2-5 c d x^3+35 d^2 x^6\right)+a^3 \left(14 c^3-20 c^2 d x^3+35 c d^2 x^6-140 d^3 x^9\right)\right)+\right.$$

$$\left.70 a^3 d^3 (-b c+a d) x^{12} \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b c+a d) x^3}{a \left(c+d x^3\right)}\right]\right)$$

Problem 672: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6 \left(a+b x^3\right)^{1/3}}{c+d x^3} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^7 \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{7 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 382 leaves):

$$\frac{1}{40\,b\,d^2\,(a+b\,x^3)^{2/3}}x\left(4\,(a+b\,x^3)\,(-5\,b\,c+a\,d+2\,b\,d\,x^3)+\left(16\,a^2\,c^2\,(-5\,b\,c+a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left((c+d\,x^3)\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(7\,a\,c\,(-10\,b^2\,c^2+5\,a\,b\,c\,d+a^2\,d^2)\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left((c+d\,x^3)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

■ Problem 673: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3\,(a+b\,x^3)^{1/3}}{c+d\,x^3}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4\,(a+b\,x^3)^{1/3}\,\text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,c\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Result (type 6, 427 leaves):

$$\frac{1}{8\,d\,(a+b\,x^3)^{2/3}\,(c+d\,x^3)}x\left(\left(16\,a^2\,c^2\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+\left(-7\,a\,c\,(4\,a\,c+2\,b\,c\,x^3+5\,a\,d\,x^3+4\,b\,d\,x^6)\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+4\,x^3\,(a+b\,x^3)\,(c+d\,x^3)\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\Big/\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

Problem 674: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{c+d x^3}\, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 160 leaves):

$$\left(4 a c x \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(4 a c \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(-3 a d \text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

Problem 675: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^3 \left(c+d x^3\right)}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 c x^2 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 344 leaves):

$$\frac{1}{8 x^2 \left(a+b x^3\right)^{2/3}} \left(-\frac{4 \left(a+b x^3\right)}{c}+\left(16 a (-b c+2 a d) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+ \left(7 a b d x^6 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 676: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^6 \left(c+d x^3\right)} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{5 c x^5 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 386 leaves):

$$\frac{1}{40 c^2 x^5 \left(a+b x^3\right)^{2/3}}\left(-\frac{4 \left(a+b x^3\right) \left(2 a c+b c x^3-5 a d x^3\right)}{a}+\left(16 c \left(b^2 c^2+5 a b c d-10 a^2 d^2\right) x^6 \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\left(c+d x^3\right) \left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+\left(7 b c d (b c-5 a d) x^9 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\left(c+d x^3\right) \left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 677: Result unnecessarily involves higher level functions.

$$\int \frac{x^{11} \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 266 leaves, 8 steps):

$$-\frac{c^3 \left(a+b x^3\right)^{2/3}}{2 d^4}+\frac{\left(b^2 c^2+a b c d+a^2 d^2\right) \left(a+b x^3\right)^{5/3}}{5 b^3 d^3}-\frac{(b c+2 a d) \left(a+b x^3\right)^{8/3}}{8 b^3 d^2}+\frac{\left(a+b x^3\right)^{11/3}}{11 b^3 d}-\frac{c^3 (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{14/3}}+\frac{c^3 (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 d^{14/3}}-\frac{c^3 (b c-a d)^{2/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{14/3}}$$

Result (type 5, 195 leaves):

$$\frac{1}{440\, b^3\, d^5\, \left(a+b\, x^3\right)^{1/3}}\left(d\, \left(a+b\, x^3\right)\, \left(18\, a^3\, d^3+3\, a^2\, b\, d^2\, \left(11\, c-4\, d\, x^3\right)+2\, a\, b^2\, d\, \left(44\, c^2-11\, c\, d\, x^3+5\, d^2\, x^6\right)+b^3\, \left(-220\, c^3+88\, c^2\, d\, x^3-55\, c\, d^2\, x^6+40\, d^3\, x^9\right)\right)-440\, b^3\, c^3\, (b\, c-a\, d)\, \left(\frac{d\, \left(a+b\, x^3\right)}{b\, \left(c+d\, x^3\right)}\right)^{1/3}\, \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b\, c-a\, d}{b\, c+b\, d\, x^3}\right]\right)$$

Problem 678: Result unnecessarily involves higher level functions.

$$\int \frac{x^8\, \left(a+b\, x^3\right)^{2/3}}{c+d\, x^3}\, dx$$

Optimal (type 3, 223 leaves, 8 steps):

$$\frac{c^2\, \left(a+b\, x^3\right)^{2/3}}{2\, d^3}-\frac{(b\, c+a\, d)\, \left(a+b\, x^3\right)^{5/3}}{5\, b^2\, d^2}+\frac{\left(a+b\, x^3\right)^{8/3}}{8\, b^2\, d}+\frac{c^2\, (b\, c-a\, d)^{2/3}\, \text{ArcTan}\left[\frac{1-\frac{2\, d^{1/3}\, \left(a+b\, x^3\right)^{1/3}}{(b\, c-a\, d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{11/3}}-\frac{c^2\, (b\, c-a\, d)^{2/3}\, \text{Log}\left[c+d\, x^3\right]}{6\, d^{11/3}}+\frac{c^2\, (b\, c-a\, d)^{2/3}\, \text{Log}\left[(b\, c-a\, d)^{1/3}+d^{1/3}\, \left(a+b\, x^3\right)^{1/3}\right]}{2\, d^{11/3}}$$

Result (type 5, 152 leaves):

$$\frac{1}{40\, b^2\, d^4\, \left(a+b\, x^3\right)^{1/3}}\left(-d\, \left(a+b\, x^3\right)\, \left(3\, a^2\, d^2-2\, a\, b\, d\, \left(-4\, c+d\, x^3\right)+b^2\, \left(-20\, c^2+8\, c\, d\, x^3-5\, d^2\, x^6\right)\right)+40\, b^2\, c^2\, (b\, c-a\, d)\, \left(\frac{d\, \left(a+b\, x^3\right)}{b\, \left(c+d\, x^3\right)}\right)^{1/3}\, \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b\, c-a\, d}{b\, c+b\, d\, x^3}\right]\right)$$

Problem 679: Result unnecessarily involves higher level functions.

$$\int \frac{x^5\, \left(a+b\, x^3\right)^{2/3}}{c+d\, x^3}\, dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$-\frac{c\, \left(a+b\, x^3\right)^{2/3}}{2\, d^2}+\frac{\left(a+b\, x^3\right)^{5/3}}{5\, b\, d}-\frac{c\, (b\, c-a\, d)^{2/3}\, \text{ArcTan}\left[\frac{1-\frac{2\, d^{1/3}\, \left(a+b\, x^3\right)^{1/3}}{(b\, c-a\, d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{8/3}}+\frac{c\, (b\, c-a\, d)^{2/3}\, \text{Log}\left[c+d\, x^3\right]}{6\, d^{8/3}}-\frac{c\, (b\, c-a\, d)^{2/3}\, \text{Log}\left[(b\, c-a\, d)^{1/3}+d^{1/3}\, \left(a+b\, x^3\right)^{1/3}\right]}{2\, d^{8/3}}$$

Result (type 5, 115 leaves):

$$\frac{1}{10\,b\,d^3\,(a+b\,x^3)^{1/3}}\left(d\,(a+b\,x^3)\,(-5\,b\,c+2\,a\,d+2\,b\,d\,x^3)-10\,b\,c\,(b\,c-a\,d)\left(\frac{d\,(a+b\,x^3)}{b\,(c+d\,x^3)}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b\,c-a\,d}{b\,c+b\,d\,x^3}\right]\right)$$

- **Problem 680: Result unnecessarily involves higher level functions.**

$$\int\frac{x^2\,(a+b\,x^3)^{2/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\frac{(a+b\,x^3)^{2/3}}{2\,d}+\frac{(b\,c-a\,d)^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,(a+b\,x^3)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{5/3}}-\frac{(b\,c-a\,d)^{2/3}\,\text{Log}[c+d\,x^3]}{6\,d^{5/3}}+\frac{(b\,c-a\,d)^{2/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,(a+b\,x^3)^{1/3}\right]}{2\,d^{5/3}}$$

Result (type 5, 94 leaves):

$$\frac{d\,(a+b\,x^3)+2\,(b\,c-a\,d)\left(\frac{d\,(a+b\,x^3)}{b\,(c+d\,x^3)}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b\,c-a\,d}{b\,c+b\,d\,x^3}\right]}{2\,d^2\,(a+b\,x^3)^{1/3}}$$

- **Problem 681: Result unnecessarily involves higher level functions.**

$$\int\frac{(a+b\,x^3)^{2/3}}{x\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 245 leaves, 10 steps):

$$\frac{a^{2/3}\,\text{ArcTan}\left[\frac{a^{1/3}+2\,(a+b\,x^3)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,c}-\frac{(b\,c-a\,d)^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,(a+b\,x^3)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c\,d^{2/3}}-\frac{a^{2/3}\,\text{Log}[x]}{2\,c}+$$
$$\frac{(b\,c-a\,d)^{2/3}\,\text{Log}[c+d\,x^3]}{6\,c\,d^{2/3}}+\frac{a^{2/3}\,\text{Log}\left[a^{1/3}-(a+b\,x^3)^{1/3}\right]}{2\,c}-\frac{(b\,c-a\,d)^{2/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,(a+b\,x^3)^{1/3}\right]}{2\,c\,d^{2/3}}$$

Result (type 6, 161 leaves):

$$-\left(4\,b\,d\,x^3\,(a+b\,x^3)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\Big/$$
$$\left((c+d\,x^3)\left(4\,b\,d\,x^3\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]-3\,b\,c\,\text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+\right.\right.$$
$$\left.\left.2\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)$$

Problem 682: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^4 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 347 leaves, 13 steps):

$$\frac{d \left(a+b x^3\right)^{2/3}}{2 c^2}+\frac{(2 b c-3 a d) \left(a+b x^3\right)^{2/3}}{6 a c^2}-\frac{\left(a+b x^3\right)^{5/3}}{3 a c x^3}+\frac{(2 b c-3 a d) \text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{1/3} c^2}+$$

$$\frac{d^{1/3} (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^2}-\frac{(2 b c-3 a d) \text{Log}[x]}{6 a^{1/3} c^2}-\frac{d^{1/3} (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^2}+$$

$$\frac{(2 b c-3 a d) \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{6 a^{1/3} c^2}+\frac{d^{1/3} (b c-a d)^{2/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 c^2}$$

Result (type 6, 407 leaves):

$$\left(\left(8 a b d x^6 \text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\right.$$

$$\left(-6 a c \text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[2,\frac{1}{3},2,3,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[2,\frac{4}{3},1,3,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+$$

$$\left(7 b d x^3 \left(4 a c+6 b c x^3+a d x^3+4 b d x^6\right) \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]-\right.$$

$$\left.4 \left(a+b x^3\right) \left(c+d x^3\right) \left(3 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)\Big/$$

$$\left(c \left(-7 b d x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+3 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+\right.\right.$$

$$\left.\left.\left.a d \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)\right)\Big/\left(12 x^3 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)\right)$$

Problem 683: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^7 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 370 leaves, 12 steps):

$$\frac{(b c+6 a d)\left(a+b x^3\right)^{2/3}}{18 a c^2 x^3}-\frac{\left(a+b x^3\right)^{5/3}}{6 a c x^6}-\frac{\left(b^2 c^2+6 a b c d-9 a^2 d^2\right) \text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{9 \sqrt{3}\, a^{4/3} c^3}-$$

$$\frac{d^{4/3} (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^3}+\frac{\left(b^2 c^2+6 a b c d-9 a^2 d^2\right) \text{Log}[x]}{18 a^{4/3} c^3}+\frac{d^{4/3} (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^3}-$$

$$\frac{\left(b^2 c^2+6 a b c d-9 a^2 d^2\right) \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{18 a^{4/3} c^3}-\frac{d^{4/3} (b c-a d)^{2/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 c^3}$$

Result (type 6, 370 leaves) :

$$\frac{1}{36 c^2 x^6 \left(a+b x^3\right)^{1/3}}\left(\left(8 b c d (b c-3 a d) x^9 \text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\left(c+d x^3\right)\right.\right.$$

$$\left.\left.\left(-6 a c \text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[2,\frac{1}{3},2,3,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[2,\frac{4}{3},1,3,-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+$$

$$1/a\left(2 \left(a+b x^3\right) \left(-3 a c-2 b c x^3+6 a d x^3\right)-\left(7 b d \left(b^2 c^2+6 a b c d-9 a^2 d^2\right) x^9 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\Big/\right.$$

$$\left(\left(c+d x^3\right) \left(-7 b d x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+\right.\right.$$

$$\left.\left.\left.\left.3 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)\right)\right)$$

- **Problem 684: Result unnecessarily involves higher level functions.**

$$\int \frac{x^6 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 334 leaves, 5 steps) :

$$-\frac{(3 b c-a d) x \left(a+b x^3\right)^{2/3}}{9 b d^2}+\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 d}+\frac{\left(9 b^2 c^2-6 a b c d-a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{4/3} d^3}-\frac{c^{4/3} (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^3}-$$

$$\frac{c^{4/3} (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 d^3}+\frac{c^{4/3} (b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^3}-\frac{\left(9 b^2 c^2-6 a b c d-a^2 d^2\right) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 b^{4/3} d^3}$$

Result (type 6, 553 leaves) :

$$\frac{1}{108\, b\, d^2}\left(\left(21\, a\, c\left(-9\, b^2 c^2+6\, a\, b\, c\, d+a^2 d^2\right) x^4\, \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]\right) \bigg/ \left(\left(a+b\, x^3\right)^{1/3}\left(c+d\, x^3\right)\left(-7\, a\, c\, \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]+x^3\left(3\, a\, d\, \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]+b\, c\, \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]\right)\right)\right)+\frac{1}{(b\, c-a\, d)^{1/3}}\left(-36\, b\, c\,(b\, c-a\, d)^{1/3} x\left(a+b\, x^3\right)^{2/3}+12\, a\, d\,(b\, c-a\, d)^{1/3} x\left(a+b\, x^3\right)^{2/3}+18\, b\, d\,(b\, c-a\, d)^{1/3} x^4\left(a+b\, x^3\right)^{2/3}-4\sqrt{3}\, a\, c^{1/3}(-3\, b\, c+a\, d)\, \text{ArcTan}\left[\frac{1+\frac{2\,(b\, c-a\, d)^{1/3} x}{c^{1/3}\left(b+a\, x^3\right)^{1/3}}}{\sqrt{3}}\right]+4\, a\, c^{1/3}(-3\, b\, c+a\, d)\, \text{Log}\left[c^{1/3}-\frac{(b\, c-a\, d)^{1/3} x}{\left(b+a\, x^3\right)^{1/3}}\right]+6\, a\, b\, c^{4/3}\, \text{Log}\left[c^{2/3}+\frac{(b\, c-a\, d)^{2/3} x^2}{\left(b+a\, x^3\right)^{2/3}}+\frac{c^{1/3}(b\, c-a\, d)^{1/3} x}{\left(b+a\, x^3\right)^{1/3}}\right]-2\, a^2 c^{1/3} d\, \text{Log}\left[c^{2/3}+\frac{(b\, c-a\, d)^{2/3} x^2}{\left(b+a\, x^3\right)^{2/3}}+\frac{c^{1/3}(b\, c-a\, d)^{1/3} x}{\left(b+a\, x^3\right)^{1/3}}\right]\right)\right)$$

- **Problem 685: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3\left(a+b\, x^3\right)^{2/3}}{c+d\, x^3}\, dx$$

Optimal (type 3, 272 leaves, 4 steps):

$$\frac{x\left(a+b\, x^3\right)^{2/3}}{3\, d}-\frac{(3\, b\, c-2\, a\, d)\, \text{ArcTan}\left[\frac{1+\frac{2\, b^{1/3} x}{\left(a+b\, x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\, b^{1/3} d^2}+\frac{c^{1/3}(b\, c-a\, d)^{2/3}\, \text{ArcTan}\left[\frac{1+\frac{2\,(b\, c-a\, d)^{1/3} x}{c^{1/3}\left(a+b\, x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2}+\frac{c^{1/3}(b\, c-a\, d)^{2/3}\, \text{Log}\left[c+d\, x^3\right]}{6\, d^2}-\frac{c^{1/3}(b\, c-a\, d)^{2/3}\, \text{Log}\left[\frac{(b\, c-a\, d)^{1/3} x}{c^{1/3}}-\left(a+b\, x^3\right)^{1/3}\right]}{2\, d^2}+\frac{(3\, b\, c-2\, a\, d)\, \text{Log}\left[-b^{1/3} x+\left(a+b\, x^3\right)^{1/3}\right]}{6\, b^{1/3} d^2}$$

Result (type 6, 386 leaves):

$$\frac{1}{36\,d}\left(-\left(21\,a\,c\,(-3\,b\,c+2\,a\,d)\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+1\Big/(b\,c-a\,d)^{1/3}2\left(6\,(b\,c-a\,d)^{1/3}\,x\left(a+b\,x^3\right)^{2/3}-2\sqrt{3}\,a\,c^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(b+a\,x^3)^{1/3}}}{\sqrt{3}}\right]+2\,a\,c^{1/3}\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]-a\,c^{1/3}\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)\right)$$

- **Problem 686: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{2/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{b^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d}-\frac{(b\,c-a\,d)^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,d}-\frac{(b\,c-a\,d)^{2/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,d}+\frac{(b\,c-a\,d)^{2/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{2/3}\,d}-\frac{b^{2/3}\,\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{2\,d}$$

Result (type 6, 161 leaves):

$$\left(4\,a\,c\,x\left(a+b\,x^3\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(-3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 691: Result more than twice size of optimal antiderivative.**

$$\int\frac{x^7\left(a+b\,x^3\right)^{2/3}}{c+d\,x^3}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^8 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{8 c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Result (type 6, 385 leaves):

$$\frac{1}{140 b d^2 \left(a+b x^3\right)^{1/3}} x^2 \left(5 \left(a+b x^3\right) \left(-7 b c+2 a d+4 b d x^3\right)+\left(25 a^2 c^2 (-7 b c+2 a d) \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\left(c+d x^3\right) \left(-5 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+\left(16 a c \left(-14 b^2 c^2+7 a b c d+2 a^2 d^2\right) x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\left(c+d x^3\right) \left(-8 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 692: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{5 c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Result (type 6, 425 leaves):

$$\left(x^2\left(\left(25\,a^2\,c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+\left(-8\,a\,c\,\left(5\,a\,c+b\,c\,x^3+7\,a\,d\,x^3+5\,b\,d\,x^6\right)\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+5\,x^3\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)\Big/\left(20\,d\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)\right)$$

- **Problem 693: Result more than twice size of optimal antiderivative.**

$$\int\frac{x\,\left(a+b\,x^3\right)^{2/3}}{c+d\,x^3}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,c\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}}$$

Result (type 6, 165 leaves):

$$\left(5\,a\,c\,x^2\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(2\,\left(c+d\,x^3\right)\,\left(5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\,\left(-3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 694: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+b\,x^3\right)^{2/3}}{x^2\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}}$$

Result (type 6, 341 leaves):

$$\frac{1}{10\,x\,\left(a+b\,x^3\right)^{1/3}}\left(-\frac{10\,\left(a+b\,x^3\right)}{c}+\left(25\,a\,(-2\,b\,c+a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)-\left(16\,a\,b\,d\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 695: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\,x^3\right)^{2/3}}{x^5\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,c\,x^4\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}}$$

Result (type 6, 384 leaves):

$$-\frac{1}{20\,c^2\,x^4\,\left(a+b\,x^3\right)^{1/3}}\left(\frac{5\,\left(a+b\,x^3\right)\,\left(2\,b\,c\,x^3+a\,\left(c-4\,d\,x^3\right)\right)}{a}+\left(25\,c\,\left(b^2\,c^2-4\,a\,b\,c\,d+2\,a^2\,d^2\right)\,x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(16\,b\,c\,d\,(b\,c-2\,a\,d)\,x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 696: Result unnecessarily involves higher level functions.

$$\int \frac{x^8\,\left(a+b\,x^3\right)^{4/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 251 leaves, 9 steps) :

$$-\frac{c^2 (b c-a d) \left(a+b x^3\right)^{1/3}}{d^4}+\frac{c^2 \left(a+b x^3\right)^{4/3}}{4 d^3}-\frac{(b c+a d) \left(a+b x^3\right)^{7/3}}{7 b^2 d^2}+\frac{\left(a+b x^3\right)^{10/3}}{10 b^2 d}-\frac{c^2 (b c-a d)^{4/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{13/3}}-\frac{c^2 (b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 d^{13/3}}+\frac{c^2 (b c-a d)^{4/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{13/3}}$$

Result (type 5, 196 leaves) :

$$\frac{1}{140 b^2 d^5 \left(a+b x^3\right)^{2/3}} \left(-d \left(a+b x^3\right) \left(6 a^3 d^3-2 a^2 b d^2 \left(-10 c+d x^3\right)+a b^2 d \left(-175 c^2+40 c d x^3-22 d^2 x^6\right)+b^3 \left(140 c^3-35 c^2 d x^3+20 c d^2 x^6-14 d^3 x^9\right)\right)-70 b^2 c^2 (b c-a d)^2 \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]\right)$$

Problem 697: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

Optimal (type 3, 211 leaves, 8 steps) :

$$\frac{c (b c-a d) \left(a+b x^3\right)^{1/3}}{d^3}-\frac{c \left(a+b x^3\right)^{4/3}}{4 d^2}+\frac{\left(a+b x^3\right)^{7/3}}{7 b d}+\frac{c (b c-a d)^{4/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{10/3}}+\frac{c (b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 d^{10/3}}-\frac{c (b c-a d)^{4/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{10/3}}$$

Result (type 5, 149 leaves) :

$$\frac{1}{28 b d^4 \left(a+b x^3\right)^{2/3}} \left(d \left(a+b x^3\right) \left(4 a^2 d^2+a b d \left(-35 c+8 d x^3\right)+b^2 \left(28 c^2-7 c d x^3+4 d^2 x^6\right)\right)+14 b c (b c-a d)^2 \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]\right)$$

Problem 698: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$-\frac{(b c-a d) \left(a+b x^3\right)^{1/3}}{d^2}+\frac{\left(a+b x^3\right)^{4/3}}{4 d}-\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^{7/3}}-\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 d^{7/3}}+\frac{(b c-a d)^{4/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{7/3}}$$

Result (type 5, 111 leaves):

$$\frac{d \left(a+b x^3\right) \left(-4 b c+5 a d+b d x^3\right)-2 (b c-a d)^2 \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]}{4 d^3 \left(a+b x^3\right)^{2/3}}$$

Problem 699: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 261 leaves, 11 steps):

$$\frac{b \left(a+b x^3\right)^{1/3}}{d}-\frac{a^{4/3} \text{ArcTan}\left[\frac{a^{1/3}+2 \left(a+b x^3\right)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} c}+\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c d^{4/3}}-\frac{a^{4/3} \text{Log}[x]}{2 c}+\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c d^{4/3}}+\frac{a^{4/3} \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2 c}-\frac{(b c-a d)^{4/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 c d^{4/3}}$$

Result (type 6, 327 leaves):

$$\frac{1}{5\,d\,\left(a+b\,x^3\right)^{2/3}}$$

$$b\left(5\,\left(a+b\,x^3\right)+\left(10\,a\,c\,(b\,c-2\,a\,d)\,x^3\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-6\,a\,c\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[2,\frac{2}{3},2,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[2,\frac{5}{3},1,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(8\,a^2\,d^2\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-8\,b\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 700: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{x^4\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 399 leaves, 15 steps) :

$$\frac{(4\,b\,c-3\,a\,d)\,\left(a+b\,x^3\right)^{1/3}}{3\,c^2}-\frac{(b\,c-a\,d)\,\left(a+b\,x^3\right)^{1/3}}{c^2}+\frac{d\,\left(a+b\,x^3\right)^{4/3}}{4\,c^2}+\frac{(4\,b\,c-3\,a\,d)\,\left(a+b\,x^3\right)^{4/3}}{12\,a\,c^2}-\frac{\left(a+b\,x^3\right)^{7/3}}{3\,a\,c\,x^3}-\frac{a^{1/3}\,(4\,b\,c-3\,a\,d)\,\text{ArcTan}\left[\frac{a^{1/3}+2\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{3\,\sqrt{3}\,c^2}-\frac{(b\,c-a\,d)^{4/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,\left(a+b\,x^3\right)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^2\,d^{1/3}}-\frac{a^{1/3}\,(4\,b\,c-3\,a\,d)\,\text{Log}[x]}{6\,c^2}-\frac{(b\,c-a\,d)^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^2\,d^{1/3}}+\frac{a^{1/3}\,(4\,b\,c-3\,a\,d)\,\text{Log}\left[a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{6\,c^2}+\frac{(b\,c-a\,d)^{4/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^2\,d^{1/3}}$$

Result (type 6, 419 leaves) :

$$\left(a\left(-\left(10\,b\,(3\,b\,c-2\,a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-6\,a\,c\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.\right.$$
$$\left.x^3\left(3\,a\,d\,\text{AppellF1}\left[2,\frac{2}{3},2,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[2,\frac{5}{3},1,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+$$
$$\left(8\,b\,d\,x^3\left(5\,a\,c+9\,b\,c\,x^3+2\,a\,d\,x^3+5\,b\,d\,x^6\right)\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]-\right.$$
$$\left.5\left(a+b\,x^3\right)\left(c+d\,x^3\right)\left(3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\Big/$$
$$\left(c\left(-8\,b\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+\right.\right.$$
$$\left.\left.\left.\left.2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\right)\right)\Big/\left(15\,x^3\left(a+b\,x^3\right)^{2/3}\left(c+d\,x^3\right)\right)$$

Problem 701: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{x^7\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 440 leaves, 14 steps):

$$\frac{d\,(b\,c-a\,d)\left(a+b\,x^3\right)^{1/3}}{c^3}+\frac{\left(2\,b^2\,c^2-12\,a\,b\,c\,d+9\,a^2\,d^2\right)\left(a+b\,x^3\right)^{1/3}}{9\,a\,c^3}-\frac{(b\,c-6\,a\,d)\left(a+b\,x^3\right)^{4/3}}{18\,a\,c^2\,x^3}-\frac{\left(a+b\,x^3\right)^{7/3}}{6\,a\,c\,x^6}-$$
$$\frac{\left(2\,b^2\,c^2-12\,a\,b\,c\,d+9\,a^2\,d^2\right)\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{9\,\sqrt{3}\,a^{2/3}\,c^3}+\frac{d^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\left(a+b\,x^3\right)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^3}-\frac{\left(2\,b^2\,c^2-12\,a\,b\,c\,d+9\,a^2\,d^2\right)\text{Log}[x]}{18\,a^{2/3}\,c^3}+$$
$$\frac{d^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^3}+\frac{\left(2\,b^2\,c^2-12\,a\,b\,c\,d+9\,a^2\,d^2\right)\text{Log}\left[a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{18\,a^{2/3}\,c^3}-\frac{d^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^3}$$

Result (type 6, 370 leaves):

$$\frac{1}{90\,c^2\,x^6\,\left(a+b\,x^3\right)^{2/3}}\left(5\,\left(a+b\,x^3\right)\,\left(-3\,a\,c-7\,b\,c\,x^3+6\,a\,d\,x^3\right)+\right.$$
$$\left(20\,a\,b\,c\,d\,(7\,b\,c-6\,a\,d)\,x^9\,\text{AppellF1}\left[1,\,\frac{2}{3},\,1,\,2,\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-6\,a\,c\,\text{AppellF1}\left[1,\,\frac{2}{3},\,1,\,2,\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[2,\,\frac{2}{3},\,2,\,3,\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[2,\,\frac{5}{3},\,1,\,3,\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)+$$
$$\left(16\,b\,d\,\left(2\,b^2\,c^2-12\,a\,b\,c\,d+9\,a^2\,d^2\right)\,x^9\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{a}{b\,x^3},\,-\frac{c}{d\,x^3}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-8\,b\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{a}{b\,x^3},\,-\frac{c}{d\,x^3}\right]+\right.\right.$$
$$\left.\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\,\frac{2}{3},\,2,\,\frac{11}{3},\,-\frac{a}{b\,x^3},\,-\frac{c}{d\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\,\frac{5}{3},\,1,\,\frac{11}{3},\,-\frac{a}{b\,x^3},\,-\frac{c}{d\,x^3}\right]\right)\right)\right)$$

- **Problem 702: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4\,\left(a+b\,x^3\right)^{4/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 334 leaves, 6 steps):

$$-\frac{(6\,b\,c-7\,a\,d)\,x^2\,\left(a+b\,x^3\right)^{1/3}}{18\,d^2}+\frac{b\,x^5\,\left(a+b\,x^3\right)^{1/3}}{6\,d}-\frac{\left(9\,b^2\,c^2-12\,a\,b\,c\,d+2\,a^2\,d^2\right)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,b^{2/3}\,d^3}+\frac{c^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^3}-$$
$$\frac{c^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d^3}-\frac{\left(9\,b^2\,c^2-12\,a\,b\,c\,d+2\,a^2\,d^2\right)\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{18\,b^{2/3}\,d^3}+\frac{c^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^3}$$

Result (type 6, 293 leaves):

$$\frac{1}{90\,d^2\,\left(a+b\,x^3\right)^{2/3}}x^2\left(5\,\left(a+b\,x^3\right)\,\left(-6\,b\,c+7\,a\,d+3\,b\,d\,x^3\right)-\right.$$
$$\left(16\,a\,c\,\left(9\,b^2\,c^2-12\,a\,b\,c\,d+2\,a^2\,d^2\right)\,x^3\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{2}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\,\frac{2}{3},\,2,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\,\frac{5}{3},\,1,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)-$$
$$\left.5\,a\,(-6\,b\,c+7\,a\,d)\,\left(\frac{c\,\left(a+b\,x^3\right)}{a\,\left(c+d\,x^3\right)}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,\frac{(-b\,c+a\,d)\,x^3}{a\,\left(c+d\,x^3\right)}\right]\right)$$

- **Problem 703: Result unnecessarily involves higher level functions.**

$$\int\frac{x\,\left(a+b\,x^3\right)^{4/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 277 leaves, 5 steps):

$$\frac{b x^2 \left(a+b x^3\right)^{1/3}}{3 d}+\frac{b^{1/3} (3 b c-4 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^2}-\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{1/3} d^2}+$$
$$\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{1/3} d^2}+\frac{b^{1/3} (3 b c-4 a d) \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 d^2}-\frac{(b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{1/3} d^2}$$

Result (type 6, 271 leaves):

$$\frac{1}{30 c d \left(a+b x^3\right)^{2/3}} x^2 \left(10 b c \left(a+b x^3\right)+\right.$$
$$\left(16 a b c^2 (3 b c-4 a d) x^3 \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(-8 a c \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$
$$\left.\left. x^3 \left(3 a d \text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)+$$
$$\left. 5 a (-2 b c+3 a d) \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b c+a d) x^3}{a \left(c+d x^3\right)}\right]\right)$$

- **Problem 704: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^2 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 254 leaves, 5 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{c x}-\frac{b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d}+\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{4/3} d}-$$
$$\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{4/3} d}-\frac{b^{4/3} \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 d}+\frac{(b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{4/3} d}$$

Result (type 6, 263 leaves):

$$\frac{1}{10 c^2 x \left(a+b x^3\right)^{2/3}} a \left(-\left(16 b^2 c^3 x^6 \text{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(-8 a c \text{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right] + x^3 \left(3 a d \text{AppellF1}\left[\frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right] + 2 b c \text{AppellF1}\left[\frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]\right)\right)\right) - 5 \left(2 c \left(a+b x^3\right) + (-2 b c + a d) x^3 \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-b c + a d) x^3}{a \left(c+d x^3\right)}\right]\right)\right)$$

- **Problem 705: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^5 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 201 leaves, 4 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{4 c x^4} - \frac{(5 b c - 4 a d) \left(a+b x^3\right)^{1/3}}{4 c^2 x} - \frac{(b c - a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{7/3}} + \frac{(b c - a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{7/3}} - \frac{(b c - a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}} - \left(a+b x^3\right)^{1/3}\right]}{2 c^{7/3}}$$

Result (type 5, 124 leaves):

$$\frac{1}{4 c^3 x^4 \left(a+b x^3\right)^{2/3}} \left(-c \left(a+b x^3\right) \left(5 b c x^3 + a \left(c - 4 d x^3\right)\right) + 2 (b c - a d)^2 x^6 \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-b c + a d) x^3}{a \left(c+d x^3\right)}\right]\right)$$

- **Problem 706: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^8 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{7 c x^7} - \frac{(8 b c - 7 a d) \left(a+b x^3\right)^{1/3}}{28 c^2 x^4} - \frac{\left(4 b^2 c^2 - 35 a b c d + 28 a^2 d^2\right) \left(a+b x^3\right)^{1/3}}{28 a c^3 x} + \frac{d (b c - a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{10/3}} - \frac{d (b c - a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{10/3}} + \frac{d (b c - a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}} - \left(a+b x^3\right)^{1/3}\right]}{2 c^{10/3}}$$

Result (type 5, 165 leaves):

$$\frac{1}{28\,a\,c^4\,x^7\,\left(a+b\,x^3\right)^{2/3}}\left(-c\,\left(a+b\,x^3\right)\,\left(4\,b^2\,c^2\,x^6+a\,b\,c\,x^3\,\left(8\,c-35\,d\,x^3\right)+a^2\,\left(4\,c^2-7\,c\,d\,x^3+28\,d^2\,x^6\right)\right)-\right.$$
$$\left.14\,a\,d\,(b\,c-a\,d)^2\,x^9\,\left(\frac{c\,\left(a+b\,x^3\right)}{a\,\left(c+d\,x^3\right)}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,\frac{(-b\,c+a\,d)\,x^3}{a\,\left(c+d\,x^3\right)}\right]\right)$$

- **Problem 707: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{x^{11}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$-\frac{a\,\left(a+b\,x^3\right)^{1/3}}{10\,c\,x^{10}}-\frac{(11\,b\,c-10\,a\,d)\,\left(a+b\,x^3\right)^{1/3}}{70\,c^2\,x^7}-\frac{\left(2\,b^2\,c^2-40\,a\,b\,c\,d+35\,a^2\,d^2\right)\,\left(a+b\,x^3\right)^{1/3}}{140\,a\,c^3\,x^4}+$$
$$\frac{\left(6\,b^3\,c^3+20\,a\,b^2\,c^2\,d-175\,a^2\,b\,c\,d^2+140\,a^3\,d^3\right)\,\left(a+b\,x^3\right)^{1/3}}{140\,a^2\,c^4\,x}-\frac{d^2\,(b\,c-a\,d)^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{13/3}}+$$
$$\frac{d^2\,(b\,c-a\,d)^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{13/3}}-\frac{d^2\,(b\,c-a\,d)^{4/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{13/3}}$$

Result (type 5, 212 leaves):

$$\frac{1}{140\,a^2\,c^5\,x^{10}\,\left(a+b\,x^3\right)^{2/3}}$$
$$\left(c\,\left(a+b\,x^3\right)\,\left(6\,b^3\,c^3\,x^9-2\,a\,b^2\,c^2\,x^6\,\left(c-10\,d\,x^3\right)+a^2\,b\,c\,x^3\,\left(-22\,c^2+40\,c\,d\,x^3-175\,d^2\,x^6\right)+a^3\,\left(-14\,c^3+20\,c^2\,d\,x^3-35\,c\,d^2\,x^6+140\,d^3\,x^9\right)\right)+\right.$$
$$\left.70\,a^2\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\left(\frac{c\,\left(a+b\,x^3\right)}{a\,\left(c+d\,x^3\right)}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,\frac{(-b\,c+a\,d)\,x^3}{a\,\left(c+d\,x^3\right)}\right]\right)$$

- **Problem 708: Result unnecessarily involves higher level functions.**

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{x^{14}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 392 leaves, 7 steps):

$$-\frac{a\left(a+b x^3\right)^{1/3}}{13 c x^{13}}-\frac{(14 b c-13 a d)\left(a+b x^3\right)^{1/3}}{130 c^2 x^{10}}-\frac{\left(4 b^2 c^2-143 a b c d+130 a^2 d^2\right)\left(a+b x^3\right)^{1/3}}{910 a c^3 x^7}+\frac{\left(12 b^3 c^3+26 a b^2 c^2 d-520 a^2 b c d^2+455 a^3 d^3\right)\left(a+b x^3\right)^{1/3}}{1820 a^2 c^4 x^4}-\frac{\left(36 b^4 c^4+78 a b^3 c^3 d+260 a^2 b^2 c^2 d^2-2275 a^3 b c d^3+1820 a^4 d^4\right)\left(a+b x^3\right)^{1/3}}{1820 a^3 c^5 x}+\frac{d^3 (b c-a d)^{4/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{16/3}}-\frac{d^3 (b c-a d)^{4/3} \operatorname{Log}\left[c+d x^3\right]}{6 c^{16/3}}+\frac{d^3 (b c-a d)^{4/3} \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{16/3}}$$

Result (type 5, 272 leaves) :

$$\frac{1}{1820 a^3 c^6 x^{13}\left(a+b x^3\right)^{2/3}}\left(-c\left(a+b x^3\right)\left(36 b^4 c^4 x^{12}+6 a b^3 c^3 x^9\left(-2 c+13 d x^3\right)+2 a^2 b^2 c^2 x^6\left(4 c^2-13 c d x^3+130 d^2 x^6\right)+a^3 b c x^3\left(196 c^3-286 c^2 d x^3+520 c d^2 x^6-2275 d^3 x^9\right)+a^4\left(140 c^4-182 c^3 d x^3+260 c^2 d^2 x^6-455 c d^3 x^9+1820 d^4 x^{12}\right)\right)-910 a^3 d^3 (b c-a d)^2 x^{15}\left(\frac{c\left(a+b x^3\right)}{a\left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b c+a d) x^3}{a\left(c+d x^3\right)}\right]\right)$$

- **Problem 709: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^6\left(a+b x^3\right)^{4/3}}{c+d x^3}\, dx$$

Optimal (type 6, 65 leaves, 2 steps) :

$$\frac{a x^7\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{7}{3},-\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{7 c\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 444 leaves) :

$$\frac{1}{80\,b\,d^3\,\left(a+b\,x^3\right)^{2/3}}x\left(2\,\left(a+b\,x^3\right)\,\left(2\,a^2\,d^2+3\,a\,b\,d\,\left(-8\,c+3\,d\,x^3\right)+b^2\,\left(20\,c^2-8\,c\,d\,x^3+5\,d^2\,x^6\right)\right)+\right.$$
$$\left(16\,a^2\,c^2\,\left(10\,b^2\,c^2-12\,a\,b\,c\,d+a^2\,d^2\right)\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,2,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{5}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)+$$
$$\left(7\,a\,c\,\left(20\,b^3\,c^3-30\,a\,b^2\,c^2\,d+8\,a^2\,b\,c\,d^2+a^3\,d^3\right)\,x^3\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/$$
$$\left(\left(c+d\,x^3\right)\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\,\frac{2}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\,\frac{5}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 710: Result more than twice size of optimal antiderivative.**

$$\int\frac{x^3\,\left(a+b\,x^3\right)^{4/3}}{c+d\,x^3}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{a\,x^4\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{4}{3},\,-\frac{4}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]}{4\,c\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Result (type 6, 382 leaves):

$$\frac{1}{40\,d^2\,\left(a+b\,x^3\right)^{2/3}}x\left(4\,\left(a+b\,x^3\right)\,\left(-5\,b\,c+6\,a\,d+2\,b\,d\,x^3\right)+\right.$$
$$\left(16\,a^2\,c^2\,\left(-5\,b\,c+6\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\,\frac{2}{3},\,1,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,2,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{5}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)-$$
$$\left(7\,a\,c\,\left(10\,b^2\,c^2-15\,a\,b\,c\,d+4\,a^2\,d^2\right)\,x^3\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{2}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\,\frac{2}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\,\frac{5}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 711: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{c+d x^3}\,dx$$

Optimal (type 6, 60 leaves, 2 steps):

$$\frac{a x \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 435 leaves):

$$\frac{1}{8 d \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} x \left(-\left(16 a^2 c (-b c+2 a d) \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+\left(b \left(-7 a c \left(4 a c+2 b c x^3+7 a d x^3+4 b d x^6\right) \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+4 x^3 \left(a+b x^3\right) \left(c+d x^3\right) \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right) \Big/ \left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

Problem 712: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^3 \left(c+d x^3\right)}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{2}{3},-\frac{4}{3},1,\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 c x^2 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 448 leaves):

$$\frac{1}{8 x^2 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} a \left(\left(16 a \left(-3 b c+2 a d\right) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right) + \left(7 c \left(-2 b^2 c x^6+4 a^2 \left(c+d x^3\right)+a b x^3 \left(4 c+5 d x^3\right)\right) \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right] - 4 x^3 \left(a+b x^3\right) \left(c+d x^3\right) \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right) \Big/ \left(c \left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 713: Result more than twice size of optimal antiderivative.**

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^6 \left(c+d x^3\right)} \, dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{5}{3},-\frac{4}{3},1,-\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{5 c x^5 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Result (type 6, 388 leaves):

$$\frac{1}{40 c^2 x^5 \left(a+b x^3\right)^{2/3}} \left(4 \left(a+b x^3\right) \left(-2 a c-6 b c x^3+5 a d x^3\right) - \left(16 a c \left(4 b^2 c^2-15 a b c d+10 a^2 d^2\right) x^6 \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right) + \left(7 a b c d \left(6 b c-5 a d\right) x^9 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(\left(c+d x^3\right) \left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+ x^3 \left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 714: Result unnecessarily involves higher level functions.

$$\int \frac{x^{14}}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 290 leaves, 7 steps):

$$-\frac{(b c+a d) \left(b^2 c^2+a^2 d^2\right) \left(a+b x^3\right)^{2/3}}{2 b^4 d^4}+\frac{\left(b^2 c^2+2 a b c d+3 a^2 d^2\right) \left(a+b x^3\right)^{5/3}}{5 b^4 d^3}-\frac{(b c+3 a d) \left(a+b x^3\right)^{8/3}}{8 b^4 d^2}+\frac{\left(a+b x^3\right)^{11/3}}{11 b^4 d}-\frac{c^4 \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^{14/3} (b c-a d)^{1/3}}+\frac{c^4 \text{Log}\left[c+d x^3\right]}{6 d^{14/3} (b c-a d)^{1/3}}-\frac{c^4 \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{14/3} (b c-a d)^{1/3}}$$

Result (type 5, 188 leaves):

$$\frac{1}{440 b^4 d^5 \left(a+b x^3\right)^{1/3}} \left(-d \left(a+b x^3\right) \left(81 a^3 d^3+9 a^2 b d^2 \left(11 c-6 d x^3\right)+3 a b^2 d \left(44 c^2-22 c d x^3+15 d^2 x^6\right)+b^3 \left(220 c^3-88 c^2 d x^3+55 c d^2 x^6-40 d^3 x^9\right)\right)-440 b^4 c^4 \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b c-a d}{b c+b d x^3}\right]\right)$$

Problem 715: Result unnecessarily involves higher level functions.

$$\int \frac{x^{11}}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 244 leaves, 7 steps):

$$\frac{\left(b^2 c^2+a b c d+a^2 d^2\right) \left(a+b x^3\right)^{2/3}}{2 b^3 d^3}-\frac{(b c+2 a d) \left(a+b x^3\right)^{5/3}}{5 b^3 d^2}+\frac{\left(a+b x^3\right)^{8/3}}{8 b^3 d}+\frac{c^3 \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^{11/3} (b c-a d)^{1/3}}-\frac{c^3 \text{Log}\left[c+d x^3\right]}{6 d^{11/3} (b c-a d)^{1/3}}+\frac{c^3 \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{11/3} (b c-a d)^{1/3}}$$

Result (type 5, 143 leaves):

$$\frac{1}{40 b^3 d^4 \left(a+b x^3\right)^{1/3}} \left(d \left(a+b x^3\right) \left(9 a^2 d^2-6 a b d \left(-2 c+d x^3\right)+b^2 \left(20 c^2-8 c d x^3+5 d^2 x^6\right)\right)+40 b^3 c^3 \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b c-a d}{b c+b d x^3}\right]\right)$$

Problem 716: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 203 leaves, 7 steps):

$$-\frac{(b c+a d) \left(a+b x^3\right)^{2/3}}{2 b^2 d^2}+\frac{\left(a+b x^3\right)^{5/3}}{5 b^2 d}-\frac{c^2 \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{8/3} (b c-a d)^{1/3}}+\frac{c^2 \text{Log}\left[c+d x^3\right]}{6 d^{8/3} (b c-a d)^{1/3}}-\frac{c^2 \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{8/3} (b c-a d)^{1/3}}$$

Result (type 5, 112 leaves):

$$\frac{-d \left(a+b x^3\right) \left(5 b c+3 a d-2 b d x^3\right)-10 b^2 c^2 \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b c-a d}{b c+b d x^3}\right]}{10 b^2 d^3 \left(a+b x^3\right)^{1/3}}$$

Problem 717: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 168 leaves, 6 steps):

$$\frac{\left(a+b x^3\right)^{2/3}}{2 b d}+\frac{c \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{5/3} (b c-a d)^{1/3}}-\frac{c \text{Log}\left[c+d x^3\right]}{6 d^{5/3} (b c-a d)^{1/3}}+\frac{c \text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{5/3} (b c-a d)^{1/3}}$$

Result (type 5, 91 leaves):

$$\frac{d \left(a+b x^3\right)+2 b c \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b c-a d}{b c+b d x^3}\right]}{2 b d^2 \left(a+b x^3\right)^{1/3}}$$

Problem 718: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^{2/3} (b c-a d)^{1/3}}+\frac{\text{Log}\left[c+d x^3\right]}{6 d^{2/3} (b c-a d)^{1/3}}-\frac{\text{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{2/3} (b c-a d)^{1/3}}$$

Result (type 5, 72 leaves):

$$-\frac{\left(\frac{d\,(a+b\,x^3)}{b\,(c+d\,x^3)}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b\,c-a\,d}{b\,c+b\,d\,x^3}\right]}{d\,(a+b\,x^3)^{1/3}}$$

- **Problem 719: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x\,(a+b\,x^3)^{1/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 244 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[\frac{a^{1/3}+2\,(a+b\,x^3)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{1/3}\,c}+\frac{d^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,(a+b\,x^3)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c\,(b\,c-a\,d)^{1/3}}-\frac{\text{Log}[x]}{2\,a^{1/3}\,c}-$$
$$\frac{d^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c\,(b\,c-a\,d)^{1/3}}+\frac{\text{Log}\left[a^{1/3}-(a+b\,x^3)^{1/3}\right]}{2\,a^{1/3}\,c}+\frac{d^{1/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,(a+b\,x^3)^{1/3}\right]}{2\,c\,(b\,c-a\,d)^{1/3}}$$

Result (type 6, 162 leaves):

$$\left(7\,b\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\Big/\left(4\,(a+b\,x^3)^{1/3}\,(c+d\,x^3)\right.$$
$$\left.\left(-7\,b\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)$$

- **Problem 720: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4\,(a+b\,x^3)^{1/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 296 leaves, 11 steps):

$$-\frac{(a+b\,x^3)^{2/3}}{3\,a\,c\,x^3}-\frac{(b\,c+3\,a\,d)\,\text{ArcTan}\left[\frac{a^{1/3}+2\,(a+b\,x^3)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{3\,\sqrt{3}\,a^{4/3}\,c^2}-\frac{d^{4/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,(a+b\,x^3)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^2\,(b\,c-a\,d)^{1/3}}+\frac{(b\,c+3\,a\,d)\,\text{Log}[x]}{6\,a^{4/3}\,c^2}+$$
$$\frac{d^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^2\,(b\,c-a\,d)^{1/3}}-\frac{(b\,c+3\,a\,d)\,\text{Log}\left[a^{1/3}-(a+b\,x^3)^{1/3}\right]}{6\,a^{4/3}\,c^2}-\frac{d^{4/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,(a+b\,x^3)^{1/3}\right]}{2\,c^2\,(b\,c-a\,d)^{1/3}}$$

Result (type 6, 409 leaves):

$$\left(\left(8\,b\,d\,x^6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(-6\,a\,c\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[2,\frac{1}{3},2,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[2,\frac{4}{3},1,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+\left(7\,b\,d\,x^3\left(4\,a\,c+3\,b\,c\,x^3+a\,d\,x^3+4\,b\,d\,x^6\right)\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]-4\left(a+b\,x^3\right)\left(c+d\,x^3\right)\left(3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\Big/\left(a\,c\left(-7\,b\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\right)\Big/\left(12\,x^3\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)\right)$$

- **Problem 721: Result unnecessarily involves higher level functions.**

$$\int \frac{x^6}{\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\frac{x\left(a+b\,x^3\right)^{2/3}}{3\,b\,d}-\frac{\left(3\,b\,c+a\,d\right)\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{4/3}\,d^2}+\frac{c^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2\,\left(b\,c-a\,d\right)^{1/3}}+\frac{c^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d^2\,\left(b\,c-a\,d\right)^{1/3}}-\frac{c^{4/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^2\,\left(b\,c-a\,d\right)^{1/3}}+\frac{\left(3\,b\,c+a\,d\right)\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{6\,b^{4/3}\,d^2}$$

Result (type 6, 388 leaves):

$$\frac{1}{36\,b\,d}\Bigg(\Big(21\,a\,c\,(3\,b\,c+a\,d)\,x^4\,\text{AppellF1}\Big[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]\Big)\Big/\Big(\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)\Big(-7\,a\,c\,\text{AppellF1}\Big[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]+$$

$$x^3\Big(3\,a\,d\,\text{AppellF1}\Big[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]+b\,c\,\text{AppellF1}\Big[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]\Big)\Big)\Big)+$$

$$1\Big/(b\,c-a\,d)^{1/3}2\Bigg(6\,(b\,c-a\,d)^{1/3}\,x\,\left(a+b\,x^3\right)^{2/3}-2\,\sqrt{3}\,a\,c^{1/3}\,\text{ArcTan}\Big[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(b+a\,x^3)^{1/3}}}{\sqrt{3}}\Big]+2\,a\,c^{1/3}\,\text{Log}\Big[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\Big]-$$

$$a\,c^{1/3}\,\text{Log}\Big[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\Big]\Bigg)\Bigg)$$

- **Problem 722: Result unnecessarily involves higher level functions.**

$$\int\frac{x^3}{\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{\text{ArcTan}\Big[\frac{1+\frac{2\,b^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\Big]}{\sqrt{3}\,b^{1/3}\,d}-\frac{c^{1/3}\,\text{ArcTan}\Big[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\Big]}{\sqrt{3}\,d\,(b\,c-a\,d)^{1/3}}-\frac{c^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d\,(b\,c-a\,d)^{1/3}}+\frac{c^{1/3}\,\text{Log}\Big[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\Big]}{2\,d\,(b\,c-a\,d)^{1/3}}-\frac{\text{Log}\Big[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\Big]}{2\,b^{1/3}\,d}$$

Result (type 6, 164 leaves):

$$-\Big(7\,a\,c\,x^4\,\text{AppellF1}\Big[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]\Big)\Big/\Big(4\,\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)\Big(-7\,a\,c\,\text{AppellF1}\Big[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]+$$

$$x^3\Big(3\,a\,d\,\text{AppellF1}\Big[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]+b\,c\,\text{AppellF1}\Big[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]\Big)\Big)\Big)$$

- **Problem 727: Result more than twice size of optimal antiderivative.**

$$\int\frac{x^7}{\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^8\left(1+\frac{b\,x^3}{a}\right)^{1/3}\text{AppellF1}\Big[\frac{8}{3},\frac{1}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\Big]}{8\,c\,\left(a+b\,x^3\right)^{1/3}}$$

Result (type 6, 428 leaves):

$$\left(x^2 \left(\left(25\, a^2\, c^2\, \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]\right) \Big/ \left(-5\, a\, c\, \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + x^3 \left(3\, a\, d\, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + b\, c\, \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]\right)\right) + \left(-8\, a\, c\, \left(5\, a\, c + b\, c\, x^3 + 3\, a\, d\, x^3 + 5\, b\, d\, x^6\right)\, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + 5\, x^3\, \left(a + b\, x^3\right)\, \left(c + d\, x^3\right)\, \left(3\, a\, d\, \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + b\, c\, \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]\right)\right) \Big/ \left(-8\, a\, c\, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + x^3 \left(3\, a\, d\, \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + b\, c\, \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]\right)\right)\right)\right) \Big/ \left(20\, b\, d\, \left(a + b\, x^3\right)^{1/3}\, \left(c + d\, x^3\right)\right)$$

- **Problem 728: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^4}{\left(a + b\, x^3\right)^{1/3}\, \left(c + d\, x^3\right)}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5\, \left(1 + \frac{b\, x^3}{a}\right)^{1/3}\, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]}{5\, c\, \left(a + b\, x^3\right)^{1/3}}$$

Result (type 6, 164 leaves):

$$-\left(8\, a\, c\, x^5\, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]\right) \Big/ \left(5\, \left(a + b\, x^3\right)^{1/3}\, \left(c + d\, x^3\right)\, \left(-8\, a\, c\, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + x^3 \left(3\, a\, d\, \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right] + b\, c\, \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]\right)\right)\right)$$

- **Problem 729: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{\left(a + b\, x^3\right)^{1/3}\, \left(c + d\, x^3\right)}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2\, \left(1 + \frac{b\, x^3}{a}\right)^{1/3}\, \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -\frac{b\, x^3}{a}, -\frac{d\, x^3}{c}\right]}{2\, c\, \left(a + b\, x^3\right)^{1/3}}$$

Result (type 6, 164 leaves):

$$-\left(5\,a\,c\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(2\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)\,\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

- **Problem 730: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^2\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\left(1+\frac{b\,x^3}{a}\right)^{1/3}\,\text{AppellF1}\left[-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c\,x\,\left(a+b\,x^3\right)^{1/3}}$$

Result (type 6, 342 leaves):

$$\frac{1}{10\,x\,\left(a+b\,x^3\right)^{1/3}}\left(-\frac{10\,\left(a+b\,x^3\right)}{a\,c}-\left(25\,(b\,c-a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)-\left(16\,b\,d\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 731: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^5\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\left(1+\frac{b\,x^3}{a}\right)^{1/3}\,\text{AppellF1}\left[-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,c\,x^4\,\left(a+b\,x^3\right)^{1/3}}$$

Result (type 6, 388 leaves):

$$\frac{1}{20\,a^2\,c^2\,x^4\,\left(a+b\,x^3\right)^{1/3}}\left(5\,\left(a+b\,x^3\right)\,\left(-a\,c+2\,b\,c\,x^3+4\,a\,d\,x^3\right)-\right.$$
$$\left(25\,a\,c\,\left(b^2\,c^2+2\,a\,b\,c\,d-2\,a^2\,d^2\right)\,x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]-\right.\right.$$
$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+$$
$$\left(16\,a\,b\,c\,d\,(b\,c+2\,a\,d)\,x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$\left.\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

- **Problem 732: Result unnecessarily involves higher level functions.**

$$\int\frac{x^{11}}{\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 241 leaves, 7 steps) :

$$\frac{\left(b^2\,c^2+a\,b\,c\,d+a^2\,d^2\right)\,\left(a+b\,x^3\right)^{1/3}}{b^3\,d^3}-\frac{(b\,c+2\,a\,d)\,\left(a+b\,x^3\right)^{4/3}}{4\,b^3\,d^2}+\frac{\left(a+b\,x^3\right)^{7/3}}{7\,b^3\,d}+$$
$$\frac{c^3\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,\left(a+b\,x^3\right)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{10/3}\,(b\,c-a\,d)^{2/3}}+\frac{c^3\,\text{Log}\left[c+d\,x^3\right]}{6\,d^{10/3}\,(b\,c-a\,d)^{2/3}}-\frac{c^3\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^{10/3}\,(b\,c-a\,d)^{2/3}}$$

Result (type 5, 144 leaves) :

$$\frac{1}{28\,b^3\,d^4\,\left(a+b\,x^3\right)^{2/3}}$$
$$\left(d\,\left(a+b\,x^3\right)\,\left(18\,a^2\,d^2+3\,a\,b\,d\,\left(7\,c-2\,d\,x^3\right)+b^2\,\left(28\,c^2-7\,c\,d\,x^3+4\,d^2\,x^6\right)\right)+14\,b^3\,c^3\,\left(\frac{d\,\left(a+b\,x^3\right)}{b\,\left(c+d\,x^3\right)}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b\,c-a\,d}{b\,c+b\,d\,x^3}\right]\right)$$

- **Problem 733: Result unnecessarily involves higher level functions.**

$$\int\frac{x^8}{\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 201 leaves, 7 steps) :

$$-\frac{(b\,c+a\,d)\,\left(a+b\,x^3\right)^{1/3}}{b^2\,d^2}+\frac{\left(a+b\,x^3\right)^{4/3}}{4\,b^2\,d}-\frac{c^2\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,\left(a+b\,x^3\right)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{7/3}\,(b\,c-a\,d)^{2/3}}-\frac{c^2\,\text{Log}\left[c+d\,x^3\right]}{6\,d^{7/3}\,(b\,c-a\,d)^{2/3}}+\frac{c^2\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^{7/3}\,(b\,c-a\,d)^{2/3}}$$

Result (type 5, 112 leaves):

$$-\frac{d\left(a+b x^3\right)\left(4 b c+3 a d-b d x^3\right)-2 b^2 c^2\left(\frac{d\left(a+b x^3\right)}{b\left(c+d x^3\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]}{4 b^2 d^3\left(a+b x^3\right)^{2/3}}$$

Problem 734: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)}\,dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\frac{\left(a+b x^3\right)^{1/3}}{b d}+\frac{c\,\text{ArcTan}\left[\frac{1-\frac{2 d^{1/3}\left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{4/3}(b c-a d)^{2/3}}+\frac{c\,\text{Log}\left[c+d x^3\right]}{6 d^{4/3}(b c-a d)^{2/3}}-\frac{c\,\text{Log}\left[(b c-a d)^{1/3}+d^{1/3}\left(a+b x^3\right)^{1/3}\right]}{2 d^{4/3}(b c-a d)^{2/3}}$$

Result (type 5, 91 leaves):

$$\frac{2 d\left(a+b x^3\right)+b c\left(\frac{d\left(a+b x^3\right)}{b\left(c+d x^3\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]}{2 b d^2\left(a+b x^3\right)^{2/3}}$$

Problem 735: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)}\,dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2 d^{1/3}\left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{1/3}(b c-a d)^{2/3}}-\frac{\text{Log}\left[c+d x^3\right]}{6 d^{1/3}(b c-a d)^{2/3}}+\frac{\text{Log}\left[(b c-a d)^{1/3}+d^{1/3}\left(a+b x^3\right)^{1/3}\right]}{2 d^{1/3}(b c-a d)^{2/3}}$$

Result (type 5, 74 leaves):

$$-\frac{\left(\frac{d\left(a+b x^3\right)}{b\left(c+d x^3\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{b c-a d}{b c+b d x^3}\right]}{2 d\left(a+b x^3\right)^{2/3}}$$

Problem 736: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)}\,dx$$

Optimal (type 3, 245 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b x^3\right)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{\sqrt{3}\, a^{2/3} c}+\frac{d^{2/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3}\left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c\,(b c-a d)^{2/3}}-\frac{\text{Log}[x]}{2 a^{2/3} c}+\frac{d^{2/3} \text{Log}\left[c+d x^3\right]}{6 c\,(b c-a d)^{2/3}}+\frac{\text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2 a^{2/3} c}-\frac{d^{2/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3}\left(a+b x^3\right)^{1/3}\right]}{2 c\,(b c-a d)^{2/3}}$$

Result (type 6, 163 leaves) :

$$\left(8 b d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\Big/\left(5\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)\left(-8 b d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+3 b c \text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+2 a d \text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)$$

- **Problem 737: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)}\, dx$$

Optimal (type 3, 299 leaves, 11 steps) :

$$-\frac{\left(a+b x^3\right)^{1/3}}{3 a c x^3}+\frac{(2 b c+3 a d) \text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b x^3\right)^{1/3}}{\sqrt{3}\, a^{1/3}}\right]}{3 \sqrt{3}\, a^{5/3} c^2}-\frac{d^{5/3} \text{ArcTan}\left[\frac{1-\frac{2 d^{1/3}\left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^2 (b c-a d)^{2/3}}+\frac{(2 b c+3 a d) \text{Log}[x]}{6 a^{5/3} c^2}-\frac{d^{5/3} \text{Log}\left[c+d x^3\right]}{6 c^2 (b c-a d)^{2/3}}-\frac{(2 b c+3 a d) \text{Log}\left[a^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{6 a^{5/3} c^2}+\frac{d^{5/3} \text{Log}\left[(b c-a d)^{1/3}+d^{1/3}\left(a+b x^3\right)^{1/3}\right]}{2 c^2 (b c-a d)^{2/3}}$$

Result (type 6, 413 leaves) :

$$\left(\left(20\,b\,d\,x^6\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\right.$$
$$\left(-6\,a\,c\,\text{AppellF1}\left[1,\frac{2}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[2,\frac{2}{3},2,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[2,\frac{5}{3},1,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+$$
$$\left(8\,b\,d\,x^3\left(5\,a\,c+3\,b\,c\,x^3+2\,a\,d\,x^3+5\,b\,d\,x^6\right)\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]-\right.$$
$$5\left(a+b\,x^3\right)\left(c+d\,x^3\right)\left(3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\Big)\Big/$$
$$\left(a\,c\left(-8\,b\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+\right.\right.$$
$$\left.\left.\left.2\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\right)\Big/\left(15\,x^3\left(a+b\,x^3\right)^{2/3}\left(c+d\,x^3\right)\right)$$

Problem 738: Result unnecessarily involves higher level functions.

$$\int\frac{x^7}{\left(a+b\,x^3\right)^{2/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 279 leaves, 5 steps):

$$\frac{x^2\left(a+b\,x^3\right)^{1/3}}{3\,b\,d}+\frac{\left(3\,b\,c+2\,a\,d\right)\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}\,d^2}-\frac{c^{5/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2\,\left(b\,c-a\,d\right)^{2/3}}+$$
$$\frac{c^{5/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d^2\,\left(b\,c-a\,d\right)^{2/3}}+\frac{\left(3\,b\,c+2\,a\,d\right)\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{6\,b^{5/3}\,d^2}-\frac{c^{5/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^2\,\left(b\,c-a\,d\right)^{2/3}}$$

Result (type 6, 257 leaves):

$$\frac{1}{15\,b\,d\,\left(a+b\,x^3\right)^{2/3}}$$
$$x^2\left(\left(8\,a\,c\,\left(3\,b\,c+2\,a\,d\right)x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\right.\right.\right.$$
$$\left.\left.\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+$$
$$\left.5\left(a+b\,x^3-a\left(\frac{c\left(a+b\,x^3\right)}{a\left(c+d\,x^3\right)}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{\left(-b\,c+a\,d\right)x^3}{a\left(c+d\,x^3\right)}\right]\right)\right)$$

Problem 739: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{c^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d\, (b c-a d)^{2/3}}-\frac{c^{2/3} \text{Log}\left[c+d x^3\right]}{6 d\, (b c-a d)^{2/3}}-\frac{\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3} d}+\frac{c^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d\, (b c-a d)^{2/3}}$$

Result (type 6, 165 leaves):

$$-\left(8 a c x^5 \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(5 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right) \left(-8 a c \text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$
$$\left.\left.x^3 \left(3 a d \text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

Problem 740: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 149 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{1/3} (b c-a d)^{2/3}}+\frac{\text{Log}\left[c+d x^3\right]}{6 c^{1/3} (b c-a d)^{2/3}}-\frac{\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{1/3} (b c-a d)^{2/3}}$$

Result (type 5, 80 leaves):

$$\frac{x^2 \left(\frac{c \left(a+b x^3\right)}{a \left(c+d x^3\right)}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(-b c+a d) x^3}{a \left(c+d x^3\right)}\right]}{2 c \left(a+b x^3\right)^{2/3}}$$

Problem 741: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 173 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{a c x}+\frac{d \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{4/3} (b c-a d)^{2/3}}-\frac{d \text{Log}\left[c+d x^3\right]}{6 c^{4/3} (b c-a d)^{2/3}}+\frac{d \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{4/3} (b c-a d)^{2/3}}$$

Result (type 5, 101 leaves):

$$\frac{-2\,c\,\left(a+b\,x^3\right)-a\,d\,x^3\,\left(\frac{c\,\left(a+b\,x^3\right)}{a\,\left(c+d\,x^3\right)}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,\frac{\left(-b\,c+a\,d\right)\,x^3}{a\,\left(c+d\,x^3\right)}\right]}{2\,a\,c^2\,x\,\left(a+b\,x^3\right)^{2/3}}$$

Problem 742: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5\,\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 215 leaves, 4 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{4\,a\,c\,x^4}+\frac{\left(3\,b\,c+4\,a\,d\right)\,\left(a+b\,x^3\right)^{1/3}}{4\,a^2\,c^2\,x}-\frac{d^2\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{7/3}\,\left(b\,c-a\,d\right)^{2/3}}+\frac{d^2\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{7/3}\,\left(b\,c-a\,d\right)^{2/3}}-\frac{d^2\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{7/3}\,\left(b\,c-a\,d\right)^{2/3}}$$

Result (type 5, 123 leaves):

$$\frac{c\,\left(a+b\,x^3\right)\,\left(-a\,c+3\,b\,c\,x^3+4\,a\,d\,x^3\right)+2\,a^2\,d^2\,x^6\,\left(\frac{c\,\left(a+b\,x^3\right)}{a\,\left(c+d\,x^3\right)}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,\frac{2}{3},\,\frac{5}{3},\,\frac{\left(-b\,c+a\,d\right)\,x^3}{a\,\left(c+d\,x^3\right)}\right]}{4\,a^2\,c^3\,x^4\,\left(a+b\,x^3\right)^{2/3}}$$

Problem 743: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6}{\left(a+b\,x^3\right)^{2/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^7\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{AppellF1}\left[\frac{7}{3},\,\frac{2}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]}{7\,c\,\left(a+b\,x^3\right)^{2/3}}$$

Result (type 6, 430 leaves):

$$\left(x\left(\left(16 a^2 c^2 \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-4 a c \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+\left(-7 a c\left(4 a c+2 b c x^3+3 a d x^3+4 b d x^6\right) \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+4 x^3\left(a+b x^3\right)\left(c+d x^3\right)\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\Big/\left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)\Big/\left(8 b d\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)\right)$$

- **Problem 744: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4\left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 c\left(a+b x^3\right)^{2/3}}$$

Result (type 6, 165 leaves):

$$-\left(7 a c x^4 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(4\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)\left(-7 a c \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3\left(3 a d \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+2 b c \text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)$$

- **Problem 745: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)}\, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c\left(a+b x^3\right)^{2/3}}$$

Result (type 6, 161 leaves):

$$-\left(4\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(a+b\,x^3\right)^{2/3}\left(c+d\,x^3\right)\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)$$

Problem 746: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\left(a+b\,x^3\right)^{2/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\left(1+\frac{b\,x^3}{a}\right)^{2/3}\text{AppellF1}\left[-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,c\,x^2\left(a+b\,x^3\right)^{2/3}}$$

Result (type 6, 344 leaves):

$$\frac{1}{8\,x^2\left(a+b\,x^3\right)^{2/3}}\left(-\frac{4\left(a+b\,x^3\right)}{a\,c}+\left(16\left(b\,c+2\,a\,d\right)x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-4\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(7\,b\,d\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{2}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{5}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 747: Result unnecessarily involves higher level functions.

$$\int \frac{x^{14}}{\left(a+b\,x^3\right)^{4/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 347 leaves, 11 steps):

$$-\frac{a^4}{b^4\left(b\,c-a\,d\right)\left(a+b\,x^3\right)^{1/3}}+\frac{a^2\left(a+b\,x^3\right)^{2/3}}{2\,b^4\,d}+\frac{a\left(b\,c+a\,d\right)\left(a+b\,x^3\right)^{2/3}}{2\,b^4\,d^2}+\frac{\left(b^2\,c^2+a\,b\,c\,d+a^2\,d^2\right)\left(a+b\,x^3\right)^{2/3}}{2\,b^4\,d^3}-\frac{2\,a\left(a+b\,x^3\right)^{5/3}}{5\,b^4\,d}-\frac{\left(b\,c+a\,d\right)\left(a+b\,x^3\right)^{5/3}}{5\,b^4\,d^2}+\frac{\left(a+b\,x^3\right)^{8/3}}{8\,b^4\,d}+\frac{c^4\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\left(a+b\,x^3\right)^{1/3}}{\left(b\,c-a\,d\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{11/3}\left(b\,c-a\,d\right)^{4/3}}-\frac{c^4\,\text{Log}\left[c+d\,x^3\right]}{6\,d^{11/3}\left(b\,c-a\,d\right)^{4/3}}+\frac{c^4\,\text{Log}\left[\left(b\,c-a\,d\right)^{1/3}+d^{1/3}\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^{11/3}\left(b\,c-a\,d\right)^{4/3}}$$

Result (type 5, 179 leaves) :

$$\frac{1}{40\,(a+b\,x^3)^{1/3}}\left(\frac{(a+b\,x^3)\left(\frac{41\,a^2}{d}+\frac{40\,a^4}{(-b\,c+a\,d)\,(a+b\,x^3)}+\frac{2\,a\,b\,(16\,c-7\,d\,x^3)}{d^2}+\frac{b^2\,(20\,c^2-8\,c\,d\,x^3+5\,d^2\,x^6)}{d^3}\right)}{b^4}+\frac{40\,c^4\left(\frac{d\,(a+b\,x^3)}{b\,(c+d\,x^3)}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b\,c-a\,d}{b\,c+b\,d\,x^3}\right]}{d^4\,(b\,c-a\,d)}\right)$$

Problem 748: Result unnecessarily involves higher level functions.

$$\int\frac{x^{11}}{(a+b\,x^3)^{4/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 253 leaves, 9 steps) :

$$\frac{a^3}{b^3\,(b\,c-a\,d)\,(a+b\,x^3)^{1/3}}-\frac{a\,(a+b\,x^3)^{2/3}}{2\,b^3\,d}-\frac{(b\,c+a\,d)\,(a+b\,x^3)^{2/3}}{2\,b^3\,d^2}+\frac{(a+b\,x^3)^{5/3}}{5\,b^3\,d}-\frac{c^3\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,(a+b\,x^3)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{8/3}\,(b\,c-a\,d)^{4/3}}+\frac{c^3\,\text{Log}\left[c+d\,x^3\right]}{6\,d^{8/3}\,(b\,c-a\,d)^{4/3}}-\frac{c^3\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,(a+b\,x^3)^{1/3}\right]}{2\,d^{8/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 5, 163 leaves) :

$$\frac{1}{3}\,(a+b\,x^3)^{2/3}\left(-\frac{3\,(5\,b\,c+8\,a\,d)}{10\,b^3\,d^2}+\frac{3\,x^3}{5\,b^2\,d}+\frac{3\,a^3}{b^3\,(b\,c-a\,d)\,(a+b\,x^3)}\right)+\frac{c^3\left(\frac{d\,(a+b\,x^3)}{b\,(c+d\,x^3)}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{\left(a-\frac{b\,c}{d}\right)\,d}{b\,(c+d\,x^3)}\right]}{d^3\,(-b\,c+a\,d)\,(a+b\,x^3)^{1/3}}$$

Problem 749: Result unnecessarily involves higher level functions.

$$\int\frac{x^{8}}{(a+b\,x^3)^{4/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 203 leaves, 7 steps) :

$$-\frac{a^2}{b^2\,(b\,c-a\,d)\,(a+b\,x^3)^{1/3}}+\frac{(a+b\,x^3)^{2/3}}{2\,b^2\,d}+\frac{c^2\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,(a+b\,x^3)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^{5/3}\,(b\,c-a\,d)^{4/3}}-\frac{c^2\,\text{Log}\left[c+d\,x^3\right]}{6\,d^{5/3}\,(b\,c-a\,d)^{4/3}}+\frac{c^2\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,(a+b\,x^3)^{1/3}\right]}{2\,d^{5/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 5, 124 leaves) :

$$\frac{d\left(-3\,a^2\,d+b^2\,c\,x^3+a\,b\,(c-d\,x^3)\right)+2\,b^2\,c^2\left(\frac{d\,(a+b\,x^3)}{b\,(c+d\,x^3)}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{b\,c-a\,d}{b\,c+b\,d\,x^3}\right]}{2\,b^2\,d^2\,(b\,c-a\,d)\,(a+b\,x^3)^{1/3}}$$

Problem 750: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 174 leaves, 6 steps):

$$\frac{a}{b (b c-a d) \left(a+b x^3\right)^{1/3}}-\frac{c \operatorname{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^{2/3} (b c-a d)^{4/3}}+\frac{c \operatorname{Log}\left[c+d x^3\right]}{6 d^{2/3} (b c-a d)^{4/3}}-\frac{c \operatorname{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 d^{2/3} (b c-a d)^{4/3}}$$

Result (type 5, 92 leaves):

$$\frac{a d-b c \left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b c-a d}{b c+b d x^3}\right]}{b d (b c-a d) \left(a+b x^3\right)^{1/3}}$$

Problem 751: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 167 leaves, 6 steps):

$$-\frac{1}{(b c-a d) \left(a+b x^3\right)^{1/3}}+\frac{d^{1/3} \operatorname{ArcTan}\left[\frac{1-\frac{2 d^{1/3} \left(a+b x^3\right)^{1/3}}{(b c-a d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c-a d)^{4/3}}-\frac{d^{1/3} \operatorname{Log}\left[c+d x^3\right]}{6 (b c-a d)^{4/3}}+\frac{d^{1/3} \operatorname{Log}\left[(b c-a d)^{1/3}+d^{1/3} \left(a+b x^3\right)^{1/3}\right]}{2 (b c-a d)^{4/3}}$$

Result (type 5, 81 leaves):

$$\frac{-1+\left(\frac{d \left(a+b x^3\right)}{b \left(c+d x^3\right)}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b c-a d}{b c+b d x^3}\right]}{(b c-a d) \left(a+b x^3\right)^{1/3}}$$

Problem 752: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 271 leaves, 11 steps):

$$\frac{b}{a\,(b\,c-a\,d)\,\left(a+b\,x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{a^{1/3}+2\,\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{4/3}\,c}-\frac{d^{4/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\,\left(a+b\,x^3\right)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c\,(b\,c-a\,d)^{4/3}}-$$
$$\frac{\text{Log}[x]}{2\,a^{4/3}\,c}+\frac{d^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c\,(b\,c-a\,d)^{4/3}}+\frac{\text{Log}\left[a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,a^{4/3}\,c}-\frac{d^{4/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\,\left(a+b\,x^3\right)^{1/3}\right]}{2\,c\,(b\,c-a\,d)^{4/3}}$$

Result (type 6, 396 leaves) :

$$\left(b\left(\left(8\,c\,d\,x^3\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right/\right.$$
$$\left(-6\,a\,c\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[2,\frac{1}{3},2,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[2,\frac{4}{3},1,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)+$$
$$\left(7\,d\,x^3\left(3\,b\,c+a\,d+4\,b\,d\,x^3\right)\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]-\right.$$
$$\left.4\left(c+d\,x^3\right)\left(3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\Big/$$
$$\left(a\left(-7\,b\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+\right.\right.$$
$$\left.\left.\left.a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\right)\Big/\left(4\,(-b\,c+a\,d)\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)\right)$$

- **Problem 753: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^4\left(a+b\,x^3\right)^{4/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 357 leaves, 13 steps) :

$$-\frac{d^2}{c^2\,(b\,c-a\,d)\left(a+b\,x^3\right)^{1/3}}-\frac{4\,b\,c+3\,a\,d}{3\,a^2\,c^2\left(a+b\,x^3\right)^{1/3}}-\frac{1}{3\,a\,c\,x^3\left(a+b\,x^3\right)^{1/3}}-\frac{(4\,b\,c+3\,a\,d)\,\text{ArcTan}\left[\frac{a^{1/3}+2\left(a+b\,x^3\right)^{1/3}}{\sqrt{3}\,a^{1/3}}\right]}{3\sqrt{3}\,a^{7/3}\,c^2}+\frac{d^{7/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,d^{1/3}\left(a+b\,x^3\right)^{1/3}}{(b\,c-a\,d)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^2\,(b\,c-a\,d)^{4/3}}+$$
$$\frac{(4\,b\,c+3\,a\,d)\,\text{Log}[x]}{6\,a^{7/3}\,c^2}-\frac{d^{7/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^2\,(b\,c-a\,d)^{4/3}}-\frac{(4\,b\,c+3\,a\,d)\,\text{Log}\left[a^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{6\,a^{7/3}\,c^2}+\frac{d^{7/3}\,\text{Log}\left[(b\,c-a\,d)^{1/3}+d^{1/3}\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^2\,(b\,c-a\,d)^{4/3}}$$

Result (type 6, 491 leaves) :

$$\left(\left(8\,a\,b\,d\,(-4\,b\,c+a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left((-b\,c+a\,d)\left(-6\,a\,c\,\text{AppellF1}\left[1,\frac{1}{3},1,2,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[2,\frac{1}{3},2,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[2,\frac{4}{3},1,3,-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(7\,b\,d\,x^3\left(-a^2\,d\left(4\,c+d\,x^3\right)+4\,b^2\,c\,x^3\left(3\,c+4\,d\,x^3\right)+a\,b\left(4\,c^2+c\,d\,x^3-4\,d^2\,x^6\right)\right)\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+4\left(c+d\,x^3\right)\left(a^2\,d-4\,b^2\,c\,x^3+a\,b\left(-c+d\,x^3\right)\right)\left(3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\Big/\left(c\,(b\,c-a\,d)\left(-7\,b\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{a}{b\,x^3},-\frac{c}{d\,x^3}\right]\right)\right)\right)\Big/\left(12\,a^2\,x^3\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)\right)$$

- **Problem 754: Result unnecessarily involves higher level functions.**

$$\int \frac{x^9}{\left(a+b\,x^3\right)^{4/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 322 leaves, 5 steps):

$$\frac{a\,x^4}{b\,(b\,c-a\,d)\left(a+b\,x^3\right)^{1/3}}+\frac{(b\,c-4\,a\,d)\,x\left(a+b\,x^3\right)^{2/3}}{3\,b^2\,d\,(b\,c-a\,d)}-\frac{(3\,b\,c+4\,a\,d)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{7/3}\,d^2}+\frac{c^{7/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2\,(b\,c-a\,d)^{4/3}}+\frac{c^{7/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d^2\,(b\,c-a\,d)^{4/3}}-\frac{c^{7/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d^2\,(b\,c-a\,d)^{4/3}}+\frac{(3\,b\,c+4\,a\,d)\,\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{6\,b^{7/3}\,d^2}$$

Result (type 6, 578 leaves):

$$\frac{1}{36\,b^2\,d\,\left(a+b\,x^3\right)^{1/3}}\left(\left(21\,a\,c\,(3\,b\,c+4\,a\,d)\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\frac{1}{(b\,c-a\,d)^{4/3}}\,2\left(6\,a\,b\,c\,(b\,c-a\,d)^{1/3}\,x-24\,a^2\,d\,(b\,c-a\,d)^{1/3}\,x+6\,b^2\,c\,(b\,c-a\,d)^{1/3}\,x^4-6\,a\,b\,d\,(b\,c-a\,d)^{1/3}\,x^4+2\,\sqrt{3}\,a\,c^{1/3}\,(-b\,c+4\,a\,d)\left(a+b\,x^3\right)^{1/3}\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(b+a\,x^3\right)^{1/3}}}{\sqrt{3}}\right]+2\,a\,c^{1/3}\,(b\,c-4\,a\,d)\left(a+b\,x^3\right)^{1/3}\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]-a\,b\,c^{4/3}\left(a+b\,x^3\right)^{1/3}\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]+4\,a^2\,c^{1/3}\,d\left(a+b\,x^3\right)^{1/3}\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(b+a\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(b+a\,x^3\right)^{1/3}}\right]\right)$$

- **Problem 755: Result unnecessarily involves higher level functions.**

$$\int\frac{x^6}{\left(a+b\,x^3\right)^{4/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 260 leaves, 4 steps):

$$\frac{a\,x}{b\,(b\,c-a\,d)\left(a+b\,x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{4/3}\,d}-\frac{c^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d\,(b\,c-a\,d)^{4/3}}-\frac{c^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d\,(b\,c-a\,d)^{4/3}}+\frac{c^{4/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d\,(b\,c-a\,d)^{4/3}}-\frac{\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{4/3}\,d}$$

Result (type 6, 393 leaves):

$$\frac{1}{12\,b}a\left(\frac{12\,x}{(b\,c-a\,d)\,\left(a+b\,x^3\right)^{1/3}}-\right.$$

$$\left(21\,c\,x^4\,\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{3},\,1,\,\frac{7}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$

$$\left.\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{7}{3},\,\frac{1}{3},\,2,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{7}{3},\,\frac{4}{3},\,1,\,\frac{10}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)-$$

$$\left.\frac{4\,\sqrt{3}\,c^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(b+a\,x^3)^{1/3}}}{\sqrt{3}}\right]}{(b\,c-a\,d)^{4/3}}+\frac{4\,c^{1/3}\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]}{(b\,c-a\,d)^{4/3}}-\frac{2\,c^{1/3}\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(b+a\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(b+a\,x^3)^{1/3}}\right]}{(b\,c-a\,d)^{4/3}}\right)$$

Problem 761: Result more than twice size of optimal antiderivative.

$$\int\frac{x^{10}}{\left(a+b\,x^3\right)^{4/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^{11}\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}\,\text{AppellF1}\left[\frac{11}{3},\,\frac{4}{3},\,1,\,\frac{14}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]}{11\,a\,c\,\left(a+b\,x^3\right)^{1/3}}$$

Result (type 6, 498 leaves):

$$\left(x^2\,\left(\left(25\,a^2\,c^2\,(b\,c-5\,a\,d)\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{3},\,1,\,\frac{5}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{3},\,1,\,\frac{5}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.\right.$$

$$\left.x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,2,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{4}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)+$$

$$\left(8\,a\,c\,\left(5\,a^2\,d\,\left(5\,c+3\,d\,x^3\right)-b^2\,c\,x^3\,\left(c+5\,d\,x^3\right)+a\,b\,\left(-5\,c^2+2\,c\,d\,x^3+5\,d^2\,x^6\right)\right)\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+5\,x^3\,\left(c+d\,x^3\right)\right.$$

$$\left.\left(-5\,a^2\,d+b^2\,c\,x^3+a\,b\,\left(c-d\,x^3\right)\right)\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\,\frac{1}{3},\,2,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\,\frac{4}{3},\,1,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\Big/$$

$$\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\,\frac{1}{3},\,2,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+\right.\right.$$

$$\left.\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{8}{3},\,\frac{4}{3},\,1,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)\Big/\left(20\,b^2\,d\,(b\,c-a\,d)\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)\right)$$

Problem 762: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^8 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{8 a c \left(a+b x^3\right)^{1/3}}$$

Result (type 6, 419 leaves):

$$\left(a x^2 \left(\left(25 a c^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \middle/ \left(-5 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.\right.$$
$$\left.x^3 \left(3 a d \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+$$
$$\left(-8 c \left(5 a c+b c x^3+3 a d x^3\right) \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.$$
$$\left.5 x^3 \left(c+d x^3\right) \left(3 a d \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right) \Big/$$
$$\left(-8 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+x^3 \left(3 a d \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+\right.\right.$$
$$\left.\left.\left.\left.b c \text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right) \Big/ \left(5 b (b c-a d) \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)\right)$$

Problem 763: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^5 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{5 a c \left(a+b x^3\right)^{1/3}}$$

Result (type 6, 332 leaves):

$$\frac{1}{5\,(-b\,c+a\,d)\,\left(a+b\,x^3\right)^{1/3}}x^2\left(5+\left(25\,a\,c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+\left(8\,a\,c\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 764: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b\,x^3\right)^{4/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^2\left(1+\frac{b\,x^3}{a}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{4}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a\,c\,\left(a+b\,x^3\right)^{1/3}}$$

Result (type 6, 343 leaves):

$$\left(x^2\left(-\left(25\,c\,(b\,c+a\,d)\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+2\,b\left(-\frac{5}{a}-\left(8\,c\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+x^3\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)\right)\Big/\left(10\,(-b\,c+a\,d)\,\left(a+b\,x^3\right)^{1/3}\right)$$

Problem 765: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\left(a+b\,x^3\right)^{4/3}\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{\left(1+\frac{b\,x^3}{a}\right)^{1/3}\text{AppellF1}\left[-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a\,c\,x\,\left(a+b\,x^3\right)^{1/3}}$$

Result (type 6, 409 leaves):

$$\frac{1}{10\,a^2\,x\,\left(a+b\,x^3\right)^{1/3}}\left(-\left(25\,a\,\left(2\,b^2\,c^2-a\,b\,c\,d+a^2\,d^2\right)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{3},\,1,\,\frac{5}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left((b\,c-a\,d)\,\left(c+d\,x^3\right)\,\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{3},\,1,\,\frac{5}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,2,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{4}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)+1/(-b\,c+a\,d)\,2\,\left(\frac{5\,\left(-a^2\,d+2\,b^2\,c\,x^3+a\,b\,\left(c-d\,x^3\right)\right)}{c}-\left(8\,a\,b\,d\,(-2\,b\,c+a\,d)\,x^6\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\,\frac{1}{3},\,2,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\,\frac{4}{3},\,1,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)\right)$$

- **Problem 766: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{x^5\,\left(a+b\,x^3\right)^{4/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$-\frac{\left(1+\frac{b\,x^3}{a}\right)^{1/3}\,\text{AppellF1}\left[-\frac{4}{3},\,\frac{4}{3},\,1,\,-\frac{1}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]}{4\,a\,c\,x^4\,\left(a+b\,x^3\right)^{1/3}}$$

Result (type 6, 467 leaves):

$$\frac{1}{20\,a^3\,c^2\,(b\,c-a\,d)\,x^4\,\left(a+b\,x^3\right)^{1/3}}\left(50\,b^3\,c^2\,x^6+5\,a^3\,d\,\left(c-4\,d\,x^3\right)+5\,a\,b^2\,c\,x^3\,\left(5\,c-2\,d\,x^3\right)-5\,a^2\,b\,\left(c^2+c\,d\,x^3+4\,d^2\,x^6\right)+\left(25\,a\,c\,\left(5\,b^3\,c^3-a\,b^2\,c^2\,d-2\,a^2\,b\,c\,d^2+2\,a^3\,d^3\right)\,x^6\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{3},\,1,\,\frac{5}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-5\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{3},\,1,\,\frac{5}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,2,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{4}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)-\left(16\,a\,b\,c\,d\,\left(-5\,b^2\,c^2+a\,b\,c\,d+2\,a^2\,d^2\right)\,x^9\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\Big/\left(\left(c+d\,x^3\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{3},\,1,\,\frac{8}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+x^3\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{8}{3},\,\frac{1}{3},\,2,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{8}{3},\,\frac{4}{3},\,1,\,\frac{11}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 791: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^4}}{x \left(a+b x^4\right)} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\sqrt{c} \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{2 a}+\frac{\sqrt{b c-a d} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{2 a \sqrt{b}}$$

Result (type 6, 162 leaves):

$$-\left(3 b d x^4 \sqrt{c+d x^4} \operatorname{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right) \Big/ \left(2 \left(a+b x^4\right) \left(3 b d x^4 \operatorname{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]-2 a d \operatorname{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right)$$

Problem 793: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^4}}{x^5 \left(a+b x^4\right)} \, dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^4}}{4 a x^4}+\frac{(2 b c-a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{4 a^2 \sqrt{c}}-\frac{\sqrt{b} \sqrt{b c-a d} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{2 a^2}$$

Result (type 6, 407 leaves):

$$\frac{1}{12\,x^4\,\left(a+b\,x^4\right)\,\sqrt{c+d\,x^4}}\left(\left(6\,b\,c\,d\,x^8\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\Big/\right.$$

$$\left(-4\,a\,c\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+x^4\,\left(2\,b\,c\,\text{AppellF1}\left[2,\,\frac{1}{2},\,2,\,3,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[2,\,\frac{3}{2},\,1,\,3,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)+$$

$$\left(5\,b\,d\,x^4\,\left(3\,a\,c+b\,c\,x^4+4\,a\,d\,x^4+3\,b\,d\,x^8\right)\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]-\right.$$

$$\left.3\,\left(a+b\,x^4\right)\,\left(c+d\,x^4\right)\,\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{2},\,2,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{3}{2},\,1,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]\right)\right)\Big/$$

$$\left(a\,\left(-5\,b\,d\,x^4\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{2},\,2,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]+\right.\right.$$

$$\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{3}{2},\,1,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]\right)\right)\right)$$

- **Problem 795: Result unnecessarily involves higher level functions.**

$$\int\frac{x^6\,\sqrt{c+d\,x^4}}{a+b\,x^4}\,dx$$

Optimal (type 4, 857 leaves, 13 steps) :

$$\frac{x^3\sqrt{c+d\,x^4}}{5\,b}+\frac{(2\,b\,c-5\,a\,d)\,x\,\sqrt{c+d\,x^4}}{5\,b^2\,\sqrt{d}\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{a\,\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,b^2}-$$

$$\frac{a\,\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,b^2}-\frac{c^{1/4}\,(2\,b\,c-5\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{5\,b^2\,d^{3/4}\,\sqrt{c+d\,x^4}}+$$

$$\frac{c^{1/4}\,\left(b^2\,c^2+a\,b\,c\,d-5\,a^2\,d^2\right)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{5\,b^2\,d^{3/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}+$$

$$\left(a\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,(b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,b^{5/2}\,c^{1/4}\,\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}-a\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)-$$

$$\left(a\,\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\,(b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\sqrt{c}\,\left(\sqrt{b}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{c}}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,b^{5/2}\,c^{1/4}\,\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 428 leaves) :

$$\frac{1}{35\,b\,\left(a+b\,x^4\right)\,\sqrt{c+d\,x^4}}x^3\,\left(\left(49\,a^2\,c^2\,\text{AppellF1}\left[\frac{3}{4},\,\frac{1}{2},\,1,\,\frac{7}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\Big/\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\,\frac{1}{2},\,1,\,\frac{7}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+\right.\right.$$

$$\left.\left.2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,2,\,\frac{11}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\,\frac{3}{2},\,1,\,\frac{11}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)+$$

$$\left(-11\,a\,c\,\left(7\,a\,c+9\,b\,c\,x^4+2\,a\,d\,x^4+7\,b\,d\,x^8\right)\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,1,\,\frac{11}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+\right.$$

$$\left.14\,x^4\,\left(a+b\,x^4\right)\,\left(c+d\,x^4\right)\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\,\frac{1}{2},\,2,\,\frac{15}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\,\frac{3}{2},\,1,\,\frac{15}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)\Big/$$

$$\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\,\frac{1}{2},\,1,\,\frac{11}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+\right.$$

$$\left.\left.2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\,\frac{1}{2},\,2,\,\frac{15}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\,\frac{3}{2},\,1,\,\frac{15}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 796: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4\,\sqrt{c+d\,x^4}}{a+b\,x^4}\,dx$$

Optimal (type 4, 700 leaves, 10 steps) :

$$\frac{x\sqrt{c+d\,x^4}}{3\,b}-\frac{(b\,c-a\,d)\,\text{ArcTan}\Big[\frac{\sqrt{\frac{\sqrt{-a}\,\left(\frac{b\,c}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\Big]}{4\,b^2\,\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}-\frac{(b\,c-a\,d)\,\text{ArcTan}\Big[\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\Big]}{4\,b^2\,\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}+$$

$$\frac{c^{3/4}\,(b\,c-2\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{d^{1/4}\,x}{c^{1/4}}\Big],\,\frac{1}{2}\Big]}{3\,b\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\,(b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticPi}\Big[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\Big[\frac{d^{1/4}\,x}{c^{1/4}}\Big],\,\frac{1}{2}\Big]\right)\Big/$$

$$\left(8\,b^2\,c^{1/4}\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,(b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticPi}\Big[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\Big[\frac{d^{1/4}\,x}{c^{1/4}}\Big],\,\frac{1}{2}\Big]\right)\Big/$$

$$\left(8\,b^2\,c^{1/4}\,\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 426 leaves) :

$$\frac{1}{15\,b\,\left(a+b\,x^4\right)\,\sqrt{c+d\,x^4}}x\left(\left(25\,a^2\,c^2\,\text{AppellF1}\Big[\frac{1}{4},\,\frac{1}{2},\,1,\,\frac{5}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\Big[\frac{1}{4},\,\frac{1}{2},\,1,\,\frac{5}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]+\right.\right.$$

$$\left.\left.2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,2,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]+a\,d\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{3}{2},\,1,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]\right)\right)\right)+$$

$$\left(-9\,a\,c\,\left(5\,a\,c+7\,b\,c\,x^4+2\,a\,d\,x^4+5\,b\,d\,x^8\right)\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,1,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]+\right.$$

$$\left.10\,x^4\,\left(a+b\,x^4\right)\,\left(c+d\,x^4\right)\,\left(2\,b\,c\,\text{AppellF1}\Big[\frac{9}{4},\,\frac{1}{2},\,2,\,\frac{13}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]+a\,d\,\text{AppellF1}\Big[\frac{9}{4},\,\frac{3}{2},\,1,\,\frac{13}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]\right)\right)\Big/$$

$$\left(-9\,a\,c\,\text{AppellF1}\Big[\frac{5}{4},\,\frac{1}{2},\,1,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]+\right.$$

$$\left.\left.2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\Big[\frac{9}{4},\,\frac{1}{2},\,2,\,\frac{13}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]+a\,d\,\text{AppellF1}\Big[\frac{9}{4},\,\frac{3}{2},\,1,\,\frac{13}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\Big]\right)\right)\right)$$

Problem 797: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 \sqrt{c+d x^4}}{a+b x^4} \, dx$$

Optimal (type 4, 786 leaves, 11 steps):

$$\frac{\sqrt{d}\, x \sqrt{c+d x^4}}{b \left(\sqrt{c}+\sqrt{d}\, x^2\right)}+\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\, \sqrt{b}}}\, \text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\, \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 b}+\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\, \sqrt{b}}}\, \text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\, \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 b}-$$
$$\frac{c^{1/4} d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]}{b \sqrt{c+d x^4}}+\frac{a\, c^{1/4} d^{5/4} \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]}{b (b c+a d) \sqrt{c+d x^4}}-$$
$$\left(\left(\sqrt{b}\, \sqrt{c}-\sqrt{-a}\, \sqrt{d}\right) (b c-a d) \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticPi}\left[\frac{\left(\sqrt{b}\, \sqrt{c}+\sqrt{-a}\, \sqrt{d}\right)^2}{4 \sqrt{-a}\, \sqrt{b}\, \sqrt{c}\, \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(8 b^{3/2} c^{1/4} \left(\sqrt{-a}\, \sqrt{b}\, \sqrt{c}-a \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)+$$
$$\left(\left(\sqrt{b}\, \sqrt{c}+\sqrt{-a}\, \sqrt{d}\right) (b c-a d) \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticPi}\left[-\frac{\sqrt{c} \left(\sqrt{b}-\frac{\sqrt{-a}\, \sqrt{d}}{\sqrt{c}}\right)^2}{4 \sqrt{-a}\, \sqrt{b}\, \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(8 b^{3/2} c^{1/4} \left(\sqrt{-a}\, \sqrt{b}\, \sqrt{c}+a \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)$$

Result (type 6, 165 leaves):

$$\left(7 a c x^3 \sqrt{c+d x^4}\, \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \left(3 \left(a+b x^4\right) \left(7 a c\, \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+\right.\right.$$
$$\left.\left.2 x^4 \left(-2 b c\, \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d\, \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right)$$

Problem 798: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^4}}{a+b x^4} \, dx$$

Optimal (type 4, 679 leaves, 9 steps):

$$\frac{(b\,c-a\,d)\,\text{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\,\left(\frac{b\,c}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,a\,b\,\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}+\frac{(b\,c-a\,d)\,\text{ArcTan}\left[\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,a\,b\,\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}+\frac{c^{3/4}\,d^{3/4}\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{(b\,c+a\,d)\,\sqrt{c+d\,x^4}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\,(b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a\,b\,c^{1/4}\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,(b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a\,b\,c^{1/4}\,\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 161 leaves):

$$\left(5\,a\,c\,x\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(\left(a+b\,x^4\right)\,\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.\right.$$

$$\left.\left.2\,x^4\,\left(-2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 799: Result unnecessarily involves higher level functions.**

$$\int\frac{\sqrt{c+d\,x^4}}{x^2\,\left(a+b\,x^4\right)}\,dx$$

Optimal (type 4, 809 leaves, 13 steps):

$$-\frac{\sqrt{c+d x^4}}{a x}+\frac{\sqrt{d}\, x \sqrt{c+d x^4}}{a\left(\sqrt{c}+\sqrt{d}\, x^2\right)}-\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a}-\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a}-$$

$$\frac{c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{a\sqrt{c+d x^4}}+\frac{b\, c^{5/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{a\,(b c+a d)\sqrt{c+d x^4}}-$$

$$\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\sqrt{b}\, c^{1/4}\left((-a)^{3/2}\sqrt{b}\sqrt{c}+a^2\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)-$$

$$\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 a\sqrt{b}\, c^{1/4}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 343 leaves) :

$$\frac{1}{21 x\sqrt{c+d x^4}}$$

$$\left(-\frac{21\left(c+d x^4\right)}{a}+\left(49 c\,(b c-2 a d)\, x^4\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right)\left(-7 a c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+\right.\right.\right.$$

$$\left.\left.2 x^4\left(2 b c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)-$$

$$\left(33 b c d x^8\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right)\left(-11 a c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+\right.\right.$$

$$\left.\left.\left.2 x^4\left(2 b c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)$$

Problem 800: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^4}}{x^4 \left(a+b x^4\right)} \, dx$$

Optimal (type 4, 703 leaves, 10 steps):

$$-\frac{\sqrt{c+d x^4}}{3 a x^3}-\frac{(b c-a d) \operatorname{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a} \left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{-\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}}-\frac{(b c-a d) \operatorname{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}}-$$

$$\frac{d^{3/4} (2 b c-a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]}{3 a c^{1/4} (b c+a d) \sqrt{c+d x^4}}-$$

$$\left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) (b c-a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^2 c^{1/4} \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)-$$

$$\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) (b c-a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^2 c^{1/4} \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{15\,x^3\,\sqrt{c+d\,x^4}}\left(-\frac{5\,\left(c+d\,x^4\right)}{a}+\left(25\,c\,\left(3\,b\,c-2\,a\,d\right)\,x^4\,\text{AppellF1}\left[\frac{1}{4},\,\frac{1}{2},\,1,\,\frac{5}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\Big/\left(\left(a+b\,x^4\right)\,\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\,\frac{1}{2},\,1,\,\frac{5}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\,\frac{1}{2},\,2,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\,\frac{3}{2},\,1,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)\right)+\left(9\,b\,c\,d\,x^8\,\text{AppellF1}\left[\frac{5}{4},\,\frac{1}{2},\,1,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\Big/\left(\left(a+b\,x^4\right)\,\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\,\frac{1}{2},\,1,\,\frac{9}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\,\frac{1}{2},\,2,\,\frac{13}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\,\frac{3}{2},\,1,\,\frac{13}{4},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)$$

- **Problem 801: Result more than twice size of optimal antiderivative.**

$$\int\frac{(e\,x)^{3/2}\,\sqrt{c+d\,x^4}}{a+b\,x^4}\,dx$$

Optimal (type 6, 71 leaves, 3 steps):

$$\frac{2\,(e\,x)^{5/2}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{5}{8},\,1,\,-\frac{1}{2},\,\frac{13}{8},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]}{5\,a\,e\,\sqrt{1+\frac{d\,x^4}{c}}}$$

Result (type 6, 170 leaves):

$$\left(26\,a\,c\,x\,(e\,x)^{3/2}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{5}{8},\,-\frac{1}{2},\,1,\,\frac{13}{8},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\Big/\left(5\,\left(a+b\,x^4\right)\,\left(13\,a\,c\,\text{AppellF1}\left[\frac{5}{8},\,-\frac{1}{2},\,1,\,\frac{13}{8},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+4\,x^4\,\left(-2\,b\,c\,\text{AppellF1}\left[\frac{13}{8},\,-\frac{1}{2},\,2,\,\frac{21}{8},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{13}{8},\,\frac{1}{2},\,1,\,\frac{21}{8},\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 802: Result more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{e\,x}\,\sqrt{c+d\,x^4}}{a+b\,x^4}\,dx$$

Optimal (type 6, 71 leaves, 3 steps):

$$\frac{2\,(e\,x)^{3/2}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{3}{8},\,1,\,-\frac{1}{2},\,\frac{11}{8},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]}{3\,a\,e\,\sqrt{1+\frac{d\,x^4}{c}}}$$

Result (type 6, 170 leaves):

$$\left(22\,a\,c\,x\,\sqrt{e\,x}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{3}{8},-\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(3\,\left(a+b\,x^4\right)\,\left(11\,a\,c\,\text{AppellF1}\left[\frac{3}{8},-\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+4\,x^4\,\left(-2\,b\,c\,\text{AppellF1}\left[\frac{11}{8},-\frac{1}{2},2,\frac{19}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},1,\frac{19}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 803: Result more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{c+d\,x^4}}{\sqrt{e\,x}\,\left(a+b\,x^4\right)}\,dx$$

Optimal (type 6, 69 leaves, 3 steps):

$$\frac{2\,\sqrt{e\,x}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a\,e\,\sqrt{1+\frac{d\,x^4}{c}}}$$

Result (type 6, 168 leaves):

$$\left(18\,a\,c\,x\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{1}{8},-\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(\sqrt{e\,x}\,\left(a+b\,x^4\right)\,\left(9\,a\,c\,\text{AppellF1}\left[\frac{1}{8},-\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+4\,x^4\,\left(-2\,b\,c\,\text{AppellF1}\left[\frac{9}{8},-\frac{1}{2},2,\frac{17}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 804: Result more than twice size of optimal antiderivative.**

$$\int\frac{\sqrt{c+d\,x^4}}{(e\,x)^{3/2}\,\left(a+b\,x^4\right)}\,dx$$

Optimal (type 6, 69 leaves, 3 steps):

$$-\frac{2\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a\,e\,\sqrt{e\,x}\,\sqrt{1+\frac{d\,x^4}{c}}}$$

Result (type 6, 348 leaves):

$$\frac{1}{35\,(e\,x)^{3/2}\,\sqrt{c+d\,x^4}}\,2\,x\left(-\frac{35\,(c+d\,x^4)}{a}+\left(75\,c\,(b\,c-4\,a\,d)\,x^4\,\text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left((a+b\,x^4)\left(-15\,a\,c\,\text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+4\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},2,\frac{23}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{15}{8},\frac{3}{2},1,\frac{23}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)-\left(161\,b\,c\,d\,x^8\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left((a+b\,x^4)\left(-23\,a\,c\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+4\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{23}{8},\frac{1}{2},2,\frac{31}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{23}{8},\frac{3}{2},1,\frac{31}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)$$

- **Problem 808: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x\,(a+b\,x^4)\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^4}}{\sqrt{c}}\right]}{2\,a\,\sqrt{c}}+\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^4}}{\sqrt{b\,c-a\,d}}\right]}{2\,a\,\sqrt{b\,c-a\,d}}$$

Result (type 6, 162 leaves):

$$\left(5\,b\,d\,x^4\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\Big/\left(6\,(a+b\,x^4)\,\sqrt{c+d\,x^4}\left(-5\,b\,d\,x^4\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\right)$$

- **Problem 809: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int\frac{1}{x^5\,(a+b\,x^4)\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d\,x^4}}{4\,a\,c\,x^4}+\frac{(2\,b\,c+a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^4}}{\sqrt{c}}\right]}{4\,a^2\,c^{3/2}}-\frac{b^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^4}}{\sqrt{b\,c-a\,d}}\right]}{2\,a^2\,\sqrt{b\,c-a\,d}}$$

Result (type 6, 409 leaves):

$$\frac{1}{12\,x^4\,\left(a+b\,x^4\right)\,\sqrt{c+d\,x^4}}\left(\left(6\,b\,d\,x^8\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\Big/\right.$$

$$\left(-4\,a\,c\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+x^4\left(2\,b\,c\,\text{AppellF1}\left[2,\,\frac{1}{2},\,2,\,3,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[2,\,\frac{3}{2},\,1,\,3,\,-\frac{d\,x^4}{c},\,-\frac{b\,x^4}{a}\right]\right)\right)+$$

$$\left(5\,b\,d\,x^4\left(3\,a\,c+b\,c\,x^4+2\,a\,d\,x^4+3\,b\,d\,x^8\right)\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]-\right.$$

$$\left.3\left(a+b\,x^4\right)\left(c+d\,x^4\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{2},\,2,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{3}{2},\,1,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]\right)\right)\Big/$$

$$\left(a\,c\left(-5\,b\,d\,x^4\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{2},\,2,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]+\right.\right.$$

$$\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{3}{2},\,1,\,\frac{7}{2},\,-\frac{c}{d\,x^4},\,-\frac{a}{b\,x^4}\right]\right)\right)\right)$$

- **Problem 815: Result unnecessarily involves higher level functions.**

$$\int\frac{x^8}{\left(a+b\,x^4\right)\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 872 leaves, 10 steps) :

$$\frac{x\sqrt{c+d\,x^4}}{3\,b\,d}-\frac{(-a)^{5/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{4\,b^{7/4}\,\sqrt{b\,c-a\,d}}-\frac{(-a)^{5/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{4\,b^{7/4}\,\sqrt{-b\,c+a\,d}}+$$

$$\frac{a^2\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{4\,b^2\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}+$$

$$\frac{a\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\,\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{4\,b^2\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}-$$

$$\frac{(b\,c+3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{6\,b^2\,c^{1/4}\,d^{5/4}\,\sqrt{c+d\,x^4}}+$$

$$\left(a\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,b^2\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(a\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,b^2\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 429 leaves) :

$$\frac{1}{15\,b\,d\,\left(a+b\,x^4\right)\sqrt{c+d\,x^4}}\,x\,\Bigg(\Bigg(25\,a^2\,c^2\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\Bigg)\Bigg/\Bigg(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+$$
$$2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Bigg)\Bigg)+$$
$$\Bigg(-9\,a\,c\,\left(5\,a\,c+4\,b\,c\,x^4+2\,a\,d\,x^4+5\,b\,d\,x^8\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+$$
$$10\,x^4\left(a+b\,x^4\right)\left(c+d\,x^4\right)\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Bigg)\Bigg/$$
$$\Bigg(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+$$
$$2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Bigg)\Bigg)$$

- **Problem 816: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4}{\left(a+b\,x^4\right)\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 638 leaves, 9 steps) :

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b\,c}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,b\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,b\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}+\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{2\,d^{1/4}\,(b\,c+a\,d)\sqrt{c+d\,x^4}}-$$
$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/$$
$$\left(8\,b\,c^{1/4}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)d^{1/4}\sqrt{c+d\,x^4}\right)-$$
$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/$$
$$\left(8\,b\,c^{1/4}\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)d^{1/4}\sqrt{c+d\,x^4}\right)$$

Result (type 6, 165 leaves) :

$$-\left(9\,a\,c\,x^5\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(5\,\left(a+b\,x^4\right)\sqrt{c+d\,x^4}\,\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

Problem 817: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^4\right)\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 638 leaves, 7 steps) :

$$\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\,\left(\frac{b\,c}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,a\,\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,a\,\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}}+\frac{d^{3/4}\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{2\,c^{1/4}\,\left(b\,c+a\,d\right)\sqrt{c+d\,x^4}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a\,c^{1/4}\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)d^{1/4}\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a\,c^{1/4}\,\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)d^{1/4}\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 161 leaves) :

$$-\left(5\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(\left(a+b\,x^4\right)\sqrt{c+d\,x^4}\,\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

Problem 818: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4 \left(a+b x^4\right) \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 677 leaves, 10 steps):

$$-\frac{\sqrt{c+d x^4}}{3 a c x^3}-\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{-\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}}-\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}}-\frac{d^{3/4} (4 b c+a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]}{6 a c^{5/4} (b c+a d) \sqrt{c+d x^4}}-$$

$$\left(b \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^2 c^{1/4} \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^2 c^{1/4} \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{15 x^3 \sqrt{c+d x^4}}$$

$$\left(-\frac{5 \left(c+d x^4\right)}{a c}+\left(25 (3 b c+a d) x^4 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \left(\left(a+b x^4\right) \left(-5 a c \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+\right.\right.\right.$$

$$\left.\left.\left. 2 x^4 \left(2 b c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right)+$$

$$\left(9 b d x^8 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \left(\left(a+b x^4\right) \left(-9 a c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+\right.\right.$$

$$\left.\left.\left. 2 x^4 \left(2 b c \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right)\right)$$

Problem 819: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a+b x^4\right)\sqrt{c+d x^4}}\,dx$$

Optimal (type 4, 804 leaves, 11 steps):

$$\frac{x\sqrt{c+d x^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{a\sqrt{-\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 b (b c-a d)}-$$

$$\frac{a\sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 b (b c-a d)}-\frac{c^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]}{b\,d^{3/4}\sqrt{c+d x^4}}+$$

$$\frac{c^{1/4}(b c+2 a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]}{2 b\,d^{3/4}(b c+a d)\sqrt{c+d x^4}}+$$

$$\left(a\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{3/2} c^{1/4}\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}-a\sqrt{d}\right)d^{1/4}\sqrt{c+d x^4}\right)-$$

$$\left(a\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{3/2} c^{1/4}\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\sqrt{d}\right)d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 165 leaves):

$$-\left(11\,a\,c\,x^7\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(7\,(a+b\,x^4)\,\sqrt{c+d\,x^4}\,\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 820: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2}{(a+b\,x^4)\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 656 leaves, 7 steps):

$$\frac{\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,(b\,c-a\,d)}+\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d\,x^4}}\right]}{4\,(b\,c-a\,d)}-\frac{c^{1/4}\,d^{1/4}\,(\sqrt{c}+\sqrt{d}\,x^2)\,\sqrt{\frac{c+d\,x^4}{(\sqrt{c}+\sqrt{d}\,x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{2\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{(\sqrt{c}+\sqrt{d}\,x^2)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,\sqrt{b}\,c^{1/4}\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}-a\,\sqrt{d}\right)d^{1/4}\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{(\sqrt{c}+\sqrt{d}\,x^2)^2}}\,\text{EllipticPi}\left[-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{c}}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,\sqrt{b}\,c^{1/4}\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\,\sqrt{d}\right)d^{1/4}\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 165 leaves):

$$-\left(7\,a\,c\,x^3\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(3\,(a+b\,x^4)\,\sqrt{c+d\,x^4}\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

- **Problem 821: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2\,(a+b\,x^4)\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 833 leaves, 13 steps):

$$-\frac{\sqrt{c+d x^4}}{a c x}+\frac{\sqrt{d}\, x \sqrt{c+d x^4}}{a c \left(\sqrt{c}+\sqrt{d}\, x^2\right)}-\frac{b \sqrt{-\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a (b c-a d)}-$$

$$\frac{b \sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a (b c-a d)}-\frac{d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{a c^{3/4}\sqrt{c+d x^4}}+$$

$$\frac{d^{1/4}(2 b c+a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{2 a c^{3/4}(b c+a d)\sqrt{c+d x^4}}+$$

$$\left(\sqrt{b}\left(\frac{\sqrt{b}\, c^{1/4}}{d^{1/4}}-\frac{\sqrt{-a}\, d^{1/4}}{c^{1/4}}\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\bigg/$$

$$\left(8 a\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}-a\sqrt{d}\right)\sqrt{c+d x^4}\right)-$$

$$\left(\sqrt{b}\left(\frac{\sqrt{b}\, c^{1/4}}{d^{1/4}}+\frac{\sqrt{-a}\, d^{1/4}}{c^{1/4}}\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\bigg/$$

$$\left(8 a\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\sqrt{d}\right)\sqrt{c+d x^4}\right)$$

Result (type 6, 344 leaves) :

$$\frac{1}{21\,x\,\sqrt{c+d\,x^4}}\left(-\frac{21\,(c+d\,x^4)}{a\,c}+\left(49\,(b\,c-a\,d)\,x^4\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left((a+b\,x^4)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)-\left(33\,b\,d\,x^8\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left((a+b\,x^4)\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)$$

- **Problem 826: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x\,(a+b\,x^4)^2\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b\,\sqrt{c+d\,x^4}}{4\,a\,(b\,c-a\,d)\,(a+b\,x^4)}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^4}}{\sqrt{c}}\right]}{2\,a^2\,\sqrt{c}}+\frac{\sqrt{b}\,(2\,b\,c-3\,a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^4}}{\sqrt{b\,c-a\,d}}\right]}{4\,a^2\,(b\,c-a\,d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(b\left(\left(6\,c\,d\,x^4\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+x^4\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)+\left(5\,d\,x^4\,(2\,a\,d+b\,(c+3\,d\,x^4))\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]-3\,(c+d\,x^4)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\right)\Big/\left(a\left(-5\,b\,d\,x^4\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\right)\right)\right)\Big/\left(12\,(-b\,c+a\,d)\,(a+b\,x^4)\,\sqrt{c+d\,x^4}\right)$$

Problem 827: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b (2 b c-a d) \sqrt{c+d x^4}}{4 a^2 c (b c-a d) \left(a+b x^4\right)}-\frac{\sqrt{c+d x^4}}{4 a c x^4 \left(a+b x^4\right)}+\frac{(4 b c+a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{4 a^3 c^{3/2}}-\frac{b^{3/2} (4 b c-5 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{4 a^3 (b c-a d)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6 a b d (-2 b c+a d) x^8 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right) \Big/ \left((-b c+a d) \left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+x^4 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)+\left(5 b d x^4 \left(-a^2 d \left(3 c+2 d x^4\right)+2 b^2 c x^4 \left(c+3 d x^4\right)+3 a b \left(c^2+c d x^4-d^2 x^8\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+3 \left(c+d x^4\right) \left(a^2 d-2 b^2 c x^4+a b \left(-c+d x^4\right)\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right) \Big/ \left(c (b c-a d) \left(-5 b d x^4 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right)\right) \Big/ \left(12 a^2 x^4 \left(a+b x^4\right) \sqrt{c+d x^4}\right)$$

Problem 834: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 996 leaves, 10 steps):

$$\frac{a\,x\,\sqrt{c+d\,x^4}}{4\,b\,(b\,c-a\,d)\,\left(a+b\,x^4\right)}-\frac{(-a)^{1/4}\,(5\,b\,c-3\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,b^{7/4}\,(b\,c-a\,d)^{3/2}}+$$

$$\frac{(-a)^{1/4}\,(5\,b\,c-3\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,b^{7/4}\,(-b\,c+a\,d)^{3/2}}+\frac{(4\,b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,b^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^4}}-$$

$$\frac{a\,\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\,d^{1/4}\,(5\,b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]}{16\,b^2\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}-$$

$$\left(\sqrt{-a}\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,(5\,b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(16\,b^2\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\,(5\,b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,b^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\,(5\,b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,b^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 420 leaves) :

$$\left(a\,x\left(\left(25\,a\,c^2\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.\right.\right.$$
$$\left.2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)+$$
$$\left(-9\,c\,\left(5\,a\,c+4\,b\,c\,x^4+2\,a\,d\,x^4\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.$$
$$\left.10\,x^4\left(c+d\,x^4\right)\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\Big/$$
$$\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.\right.$$
$$\left.\left.\left.\left.a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)\Big/\left(20\,b\,(b\,c-a\,d)\left(a+b\,x^4\right)\sqrt{c+d\,x^4}\right)$$

- **Problem 835: Result unnecessarily involves higher level functions.**

$$\int\frac{x^4}{\left(a+b\,x^4\right)^2\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 908 leaves, 10 steps):

$$-\frac{x\sqrt{c+d x^4}}{4(b c-a d)\left(a+b x^4\right)}-\frac{(b c+a d)\,\text{ArcTan}\left[\frac{\sqrt{b c-a d}\,x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16(-a)^{3/4} b^{3/4}(b c-a d)^{3/2}}+\frac{(b c+a d)\,\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\,x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16(-a)^{3/4} b^{3/4}(-b c+a d)^{3/2}}+$$

$$\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right) d^{1/4}\left(\sqrt{c}+\sqrt{d} x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{16 b c^{1/4}(b c-a d)\sqrt{c+d x^4}}+$$

$$\frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right) d^{1/4}\left(\sqrt{c}+\sqrt{d} x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{16 a b c^{1/4}(b c-a d)\sqrt{c+d x^4}}-$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d} x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{8 b c^{1/4}(b c-a d)\sqrt{c+d x^4}}+$$

$$\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d} x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(32 a b c^{1/4} d^{1/4}(b c-a d)\sqrt{c+d x^4}\right)+$$

$$\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d} x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(32 a b c^{1/4} d^{1/4}(b c-a d)\sqrt{c+d x^4}\right)$$

Result (type 6, 331 leaves) :

$$\left(x \left(5 \left(c+d x^4\right)+\left(25 a c^2 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \left(-5 a c \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+\right.\right.\right.$$

$$2 x^4 \left(2 b c \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\Big)-$$

$$\left(9 a c d x^4 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \left(-9 a c \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+2 x^4\right.$$

$$\left.\left.\left.\left(2 b c \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right)\right) \Big/ \left(20 (-b c+a d) \left(a+b x^4\right) \sqrt{c+d x^4}\right)$$

- **Problem 836: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 983 leaves, 10 steps) :

$$\frac{b\,x\,\sqrt{c+d\,x^4}}{4\,a\,(b\,c-a\,d)\,\left(a+b\,x^4\right)}+\frac{b^{1/4}\,(3\,b\,c-5\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,(-a)^{7/4}\,(b\,c-a\,d)^{3/2}}-$$

$$\frac{b^{1/4}\,(3\,b\,c-5\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,(-a)^{7/4}\,(-b\,c+a\,d)^{3/2}}+\frac{d^{3/4}\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{8\,a\,c^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^4}}+$$

$$\frac{\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\,d^{1/4}\,(3\,b\,c-5\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{16\,a\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,(3\,b\,c-5\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$
$$\left(16\,(-a)^{3/2}\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\,(3\,b\,c-5\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$
$$\left(32\,a^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\,(3\,b\,c-5\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$
$$\left(32\,a^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 341 leaves) :

$$\left(x\left(-\frac{5\,b\,\left(c+d\,x^4\right)}{a}+\left(25\,c\,(3\,b\,c-4\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)+\left(9\,b\,c\,d\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)\Big/\left(20\,(-b\,c+a\,d)\,\left(a+b\,x^4\right)\sqrt{c+d\,x^4}\right)$$

- **Problem 837: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4\,\left(a+b\,x^4\right)^2\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 1046 leaves, 11 steps) :

$$-\frac{(7 b c-4 a d) \sqrt{c+d x^4}}{12 a^2 c (b c-a d) x^3}+\frac{b \sqrt{c+d x^4}}{4 a (b c-a d) x^3 \left(a+b x^4\right)}+\frac{b^{5/4} (7 b c-9 a d) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{11/4} (b c-a d)^{3/2}}-$$

$$\frac{b^{5/4} (7 b c-9 a d) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{11/4} (-b c+a d)^{3/2}}-\frac{d^{3/4} (7 b c-4 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]}{24 a^2 c^{5/4} (b c-a d) \sqrt{c+d x^4}}+$$

$$\left(b \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) d^{1/4} (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(16 (-a)^{5/2} c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) d^{1/4} (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(16 (-a)^{5/2} c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\, \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(32 a^3 c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\, \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(32 a^3 c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)$$

Result (type 6, 399 leaves) :

$$\left(\frac{5\left(c+d x^4\right)\left(-4 a^2 d+7 b^2 c x^4+4 a b\left(c-d x^4\right)\right)}{c}+\right.$$
$$\left(25 a\left(-21 b^2 c^2+20 a b c d+4 a^2 d^2\right) x^4 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+\right.$$
$$\left.2 x^4\left(2 b c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)+$$
$$\left(9 a b d(-7 b c+4 a d) x^8 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+\right.$$
$$\left.\left.2 x^4\left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\Big/\left(60 a^2(-b\right.$$
$$\left.c+a d) x^3\left(a+b x^4\right)\sqrt{c+d x^4}\right)$$

- **Problem 838: Result unnecessarily involves higher level functions.**

$$\int \frac{x^6}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}}\, dx$$

Optimal (type 4, 1146 leaves, 13 steps):

$$\frac{\sqrt{d}\,x\,\sqrt{c+d\,x^4}}{4\,b\,(b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{x^3\,\sqrt{c+d\,x^4}}{4\,(b\,c-a\,d)\,\left(a+b\,x^4\right)}+\frac{(3\,b\,c-a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,(-a)^{1/4}\,b^{5/4}\,(b\,c-a\,d)^{3/2}}+\frac{(3\,b\,c-a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,(-a)^{1/4}\,b^{5/4}\,(-b\,c+a\,d)^{3/2}}-$$

$$\frac{c^{1/4}\,d^{1/4}\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{4\,b\,(b\,c-a\,d)\,\sqrt{c+d\,x^4}}+\frac{c^{1/4}\,d^{1/4}\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{8\,b\,(b\,c-a\,d)\,\sqrt{c+d\,x^4}}-$$

$$\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)\,d^{1/4}\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{16\,b\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}-$$

$$\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)\,d^{1/4}\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{16\,b\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,\sqrt{-a}\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,\sqrt{-a}\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 333 leaves) :

$$\left(x^3 \left(7 \left(c+d x^4\right)+\left(49 a c^2 \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \left(-7 a c \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+\right.\right.\right.$$
$$\left. 2 x^4 \left(2 b c \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)+$$
$$\left(11 a c d x^4 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \left(-11 a c \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+2 x^4 \left(2 b c\right.\right.$$
$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right)\right) \Big/ \left(28 (-b c+a d) \left(a+b x^4\right) \sqrt{c+d x^4}\right)$$

- **Problem 839: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 1144 leaves, 13 steps) :

$$-\frac{\sqrt{d}\,x\sqrt{c+d x^4}}{4 a (b c-a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}+\frac{b x^3\sqrt{c+d x^4}}{4 a (b c-a d)\left(a+b x^4\right)}-\frac{(b c-3 a d)\,\mathrm{ArcTan}\left[\frac{\sqrt{b c-a d}\,x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16 (-a)^{5/4} b^{1/4} (b c-a d)^{3/2}}-\frac{(b c-3 a d)\,\mathrm{ArcTan}\left[\frac{\sqrt{-b c+a d}\,x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16 (-a)^{5/4} b^{1/4} (-b c+a d)^{3/2}}+$$

$$\frac{c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{4 a (b c-a d)\sqrt{c+d x^4}}-\frac{c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{8 a (b c-a d)\sqrt{c+d x^4}}-$$

$$\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4} (b c-3 a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{16 a c^{1/4} (b c-a d) (b c+a d)\sqrt{c+d x^4}}-$$

$$\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4} (b c-3 a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{16 a c^{1/4} (b c-a d) (b c+a d)\sqrt{c+d x^4}}-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2 (b c-3 a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\mathrm{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32 (-a)^{3/2}\sqrt{b}\,c^{1/4} d^{1/4} (b c-a d) (b c+a d)\sqrt{c+d x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2 (b c-3 a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\mathrm{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32 (-a)^{3/2}\sqrt{b}\,c^{1/4} d^{1/4} (b c-a d) (b c+a d)\sqrt{c+d x^4}\right)$$

Result (type 6, 342 leaves) :

$$\left(x^3\left(-\frac{21\,b\,\left(c+d\,x^4\right)}{a}+\left(49\,c\,(b\,c-4\,a\,d)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)-\left(33\,b\,c\,d\,x^4\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)\Big/\left(84\,(-b\,c+a\,d)\,\left(a+b\,x^4\right)\sqrt{c+d\,x^4}\right)$$

- **Problem 840: Result unnecessarily involves higher level functions.**

$$\int\frac{1}{x^2\,\left(a+b\,x^4\right)^2\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 1225 leaves, 14 steps) :

$$-\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{\sqrt{d}\,(5bc-4ad)\,x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}-\frac{b^{3/4}(5bc-7ad)\,\mathrm{ArcTan}\left[\frac{\sqrt{bc-ad}\,x}{(-a)^{1/4}b^{1/4}\sqrt{c+dx^4}}\right]}{16(-a)^{9/4}(bc-ad)^{3/2}}-$$

$$\frac{b^{3/4}(5bc-7ad)\,\mathrm{ArcTan}\left[\frac{\sqrt{-bc+ad}\,x}{(-a)^{1/4}b^{1/4}\sqrt{c+dx^4}}\right]}{16(-a)^{9/4}(-bc+ad)^{3/2}}-\frac{d^{1/4}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]}{4a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}}+$$

$$\frac{d^{1/4}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}}+$$

$$\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)d^{1/4}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]}{16a^2c^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^4}}+$$

$$\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)d^{1/4}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]}{16a^2c^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^4}}-$$

$$\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticPi}\left[-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32(-a)^{5/2}c^{1/4}d^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^4}\right)+$$

$$\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\mathrm{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32(-a)^{5/2}c^{1/4}d^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^4}\right)$$

Result (type 6, 399 leaves) :

$$\left(\frac{21\left(c+d x^4\right)\left(-4 a^2 d+5 b^2 c x^4+4 a b\left(c-d x^4\right)\right)}{c}-\left(49 a\left(5 b^2 c^2-12 a b c d+4 a^2 d^2\right) x^4 \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-7 a c \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)+\left(33 a b d(5 b c-4 a d) x^8 \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-11 a c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\Big/\left(84 a^2(-b c+a d) x\left(a+b x^4\right)\sqrt{c+d x^4}\right)$$

- **Problem 844: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^m}{\left(a+b x^4\right)\sqrt{c+d x^4}}\,dx$$

Optimal (type 6, 81 leaves, 2 steps):

$$\frac{(e x)^{1+m}\sqrt{1+\frac{d x^4}{c}}\,\text{AppellF1}\left[\frac{1+m}{4},1,\frac{1}{2},\frac{5+m}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]}{a e(1+m)\sqrt{c+d x^4}}$$

Result (type 6, 282 leaves):

$$\frac{1}{(1+m)\sqrt{c+d x^4}}x(e x)^m\left(-\left(a b c(5+m)\left(c+d x^4\right)\text{AppellF1}\left[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left((-b c+a d)\left(a+b x^4\right)\left(a c(5+m)\text{AppellF1}\left[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(-2 b c \text{AppellF1}\left[\frac{5+m}{4},-\frac{1}{2},2,\frac{9+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{5+m}{4},\frac{1}{2},1,\frac{9+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)-\frac{d\sqrt{1+\frac{d x^4}{c}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{d x^4}{c}\right]}{b c-a d}\right)$$

Problem 845: Result more than twice size of optimal antiderivative.

$$\int \frac{(e\,x)^m}{\left(a+b\,x^4\right)^2\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 6, 81 leaves, 2 steps):

$$\frac{(e\,x)^{1+m}\sqrt{1+\frac{d\,x^4}{c}}\;\text{AppellF1}\left[\frac{1+m}{4},2,\frac{1}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a^2\,e\,(1+m)\sqrt{c+d\,x^4}}$$

Result (type 6, 488 leaves):

$$\frac{1}{(1+m)\sqrt{c+d\,x^4}}\,x\,(e\,x)^m\left(-\left(a\,b\,c\,d\,(5+m)\left(c+d\,x^4\right)\text{AppellF1}\left[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left((b\,c-a\,d)^2\left(a+b\,x^4\right)\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(-2\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},-\frac{1}{2},2,\frac{9+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5+m}{4},\frac{1}{2},1,\frac{9+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)-\left(a\,b\,c\,(5+m)\left(c+d\,x^4\right)\text{AppellF1}\left[\frac{1+m}{4},2,-\frac{1}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((-b\,c+a\,d)\left(a+b\,x^4\right)^2\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},2,-\frac{1}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+2\,x^4\left(a\,d\,\text{AppellF1}\left[\frac{5+m}{4},2,\frac{1}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]-4\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},3,-\frac{1}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+\frac{d^2\sqrt{1+\frac{d\,x^4}{c}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{d\,x^4}{c}\right]}{(b\,c-a\,d)^2}\right)$$

Problem 846: Result more than twice size of optimal antiderivative.

$$\int \frac{(e\,x)^m}{\left(a+b\,x^4\right)^3\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 6, 81 leaves, 2 steps):

$$\frac{(e\,x)^{1+m}\sqrt{1+\frac{d\,x^4}{c}}\;\text{AppellF1}\left[\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a^3\,e\,(1+m)\,\sqrt{c+d\,x^4}}$$

Result (type 6, 209 leaves) :

$$-\left(a\,c\,(5+m)\,x\,(e\,x)^m\,\text{AppellF1}\left[\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/$$
$$\left((1+m)\,\left(a+b\,x^4\right)^3\sqrt{c+d\,x^4}\,\left(-a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+\right.\right.$$
$$\left.\left.2\,x^4\left(a\,d\,\text{AppellF1}\left[\frac{5+m}{4},3,\frac{3}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+6\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},4,\frac{1}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

- **Problem 850: Result more than twice size of optimal antiderivative.**

$$\int\frac{(e\,x)^m}{\left(a+b\,x^4\right)\left(c+d\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 84 leaves, 2 steps) :

$$\frac{(e\,x)^{1+m}\sqrt{1+\frac{d\,x^4}{c}}\;\text{AppellF1}\left[\frac{1+m}{4},1,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a\,c\,e\,(1+m)\,\sqrt{c+d\,x^4}}$$

Result (type 6, 329 leaves) :

$$\left(x\,(e\,x)^m\left(\left(a\,b^2\,c\,(5+m)\,(c+d\,x^4)\,\text{AppellF1}\left[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\right.\right.$$

$$\left(\left(a+b\,x^4\right)\left(a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.\right.$$

$$\left.\left.\left.2\,x^4\left(-2\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},\frac{1}{2},2,\frac{9+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5+m}{4},\frac{1}{2},1,\frac{9+m}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)-\right.$$

$$\left.\left.b\,d\,\sqrt{1+\frac{d\,x^4}{c}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{d\,x^4}{c}\right]-\frac{d\,(b\,c-a\,d)\,\sqrt{1+\frac{d\,x^4}{c}}\,\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{d\,x^4}{c}\right]}{c}\right)\right)\Big/$$

$$\left((b\,c-a\,d)^2\,(1+m)\,\sqrt{c+d\,x^4}\right)$$

Problem 851: Result more than twice size of optimal antiderivative.

$$\int\frac{(e\,x)^m}{\left(a+b\,x^4\right)^2\left(c+d\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 84 leaves, 2 steps):

$$\frac{(e\,x)^{1+m}\,\sqrt{1+\frac{d\,x^4}{c}}\,\text{AppellF1}\left[\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a^2\,c\,e\,(1+m)\,\sqrt{c+d\,x^4}}$$

Result (type 6, 210 leaves):

$$-\left(a\,c\,(5+m)\,x\,(e\,x)^m\,\text{AppellF1}\left[\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/$$

$$\left((1+m)\,\left(a+b\,x^4\right)^2\left(c+d\,x^4\right)^{3/2}\left(-a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+\right.\right.$$

$$\left.\left.2\,x^4\left(3\,a\,d\,\text{AppellF1}\left[\frac{5+m}{4},2,\frac{5}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+4\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},3,\frac{3}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

Problem 852: Result more than twice size of optimal antiderivative.

$$\int\frac{(e\,x)^m}{\left(a+b\,x^4\right)^3\left(c+d\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 84 leaves, 2 steps):

$$\frac{(e\,x)^{1+m}\sqrt{1+\frac{d\,x^4}{c}}\;\text{AppellF1}\left[\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a^3\,c\,e\,(1+m)\sqrt{c+d\,x^4}}$$

Result (type 6, 209 leaves):

$$-\left(a\,c\,(5+m)\,x\,(e\,x)^m\,\text{AppellF1}\left[\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((1+m)\,\left(a+b\,x^4\right)^3\,\left(c+d\,x^4\right)^{3/2}\left(-a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+6\,x^4\left(a\,d\,\text{AppellF1}\left[\frac{5+m}{4},3,\frac{5}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},4,\frac{3}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

Problem 856: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\,\left(a+b\,x^6\right)\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^6}}{\sqrt{c}}\right]}{3\,a\,\sqrt{c}}+\frac{\sqrt{b}\;\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^6}}{\sqrt{b\,c-a\,d}}\right]}{3\,a\,\sqrt{b\,c-a\,d}}$$

Result (type 6, 162 leaves):

$$\left(5\,b\,d\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\Big/\left(9\,\left(a+b\,x^6\right)\sqrt{c+d\,x^6}\left(-5\,b\,d\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\right)$$

Problem 857: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^7\,\left(a+b\,x^6\right)\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d\,x^6}}{6\,a\,c\,x^6}+\frac{(2\,b\,c+a\,d)\;\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^6}}{\sqrt{c}}\right]}{6\,a^2\,c^{3/2}}-\frac{b^{3/2}\;\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^6}}{\sqrt{b\,c-a\,d}}\right]}{3\,a^2\,\sqrt{b\,c-a\,d}}$$

Result (type 6, 410 leaves):

$$\frac{1}{18 x^6 \left(a+b x^6\right) \sqrt{c+d x^6}}\left(\left(6 b d x^{12} \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\Big/\right.$$
$$\left(-4 a c \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + x^6 \left(2 b c \text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right) +$$
$$\left(5 b d x^6 \left(a \left(3 c + 2 d x^6\right) + b x^6 \left(c + 3 d x^6\right)\right) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] -\right.$$
$$3 \left(a+b x^6\right) \left(c+d x^6\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] + b c \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right]\right)\Big)\Big/$$
$$\left(a c \left(-5 b d x^6 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] + 2 a d \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] +\right.\right.$$
$$\left.\left.\left. b c \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right]\right)\right)\right)$$

- **Problem 863: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^4}{\left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5 \sqrt{1+\frac{d x^6}{c}} \text{AppellF1}\left[\frac{5}{6}, 1, \frac{1}{2}, \frac{11}{6}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{5 a \sqrt{c+d x^6}}$$

Result (type 6, 165 leaves):

$$-\left(11 a c x^5 \text{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\Big/\left(5 \left(a+b x^6\right) \sqrt{c+d x^6} \left(-11 a c \text{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] +\right.\right.$$
$$\left.\left. 3 x^6 \left(2 b c \text{AppellF1}\left[\frac{11}{6}, \frac{1}{2}, 2, \frac{17}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{AppellF1}\left[\frac{11}{6}, \frac{3}{2}, 1, \frac{17}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right)$$

- **Problem 864: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{\left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^6}{c}} \text{AppellF1}\left[\frac{2}{3}, 1, \frac{1}{2}, \frac{5}{3}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{4 a \sqrt{c+d x^6}}$$

Result (type 6, 165 leaves):

$$-\left(5\,a\,c\,x^4\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\bigg/\left(2\,\left(a+b\,x^6\right)\,\sqrt{c+d\,x^6}\,\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)$$

- **Problem 865: Result more than twice size of optimal antiderivative.**

$$\int\frac{x}{\left(a+b\,x^6\right)\,\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^2\,\sqrt{1+\frac{d\,x^6}{c}}\,\text{AppellF1}\left[\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{b\,x^6}{a},-\frac{d\,x^6}{c}\right]}{2\,a\,\sqrt{c+d\,x^6}}$$

Result (type 6, 163 leaves):

$$-\left(4\,a\,c\,x^2\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\bigg/\left(\left(a+b\,x^6\right)\,\sqrt{c+d\,x^6}\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)$$

- **Problem 866: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\left(a+b\,x^6\right)\,\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^6}{c}}\,\text{AppellF1}\left[\frac{1}{6},1,\frac{1}{2},\frac{7}{6},-\frac{b\,x^6}{a},-\frac{d\,x^6}{c}\right]}{a\,\sqrt{c+d\,x^6}}$$

Result (type 6, 161 leaves):

$$-\left(7\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\bigg/\left(\left(a+b\,x^6\right)\,\sqrt{c+d\,x^6}\,\left(-7\,a\,c\,\text{AppellF1}\left[\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{6},\frac{1}{2},2,\frac{13}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{6},\frac{3}{2},1,\frac{13}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)$$

Problem 867: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[-\frac{1}{6},1,\frac{1}{2},\frac{5}{6},-\frac{b x^6}{a},-\frac{d x^6}{c}\right]}{a x \sqrt{c+d x^6}}$$

Result (type 6, 344 leaves):

$$\frac{1}{55 x \sqrt{c+d x^6}} \left(-\frac{55 \left(c+d x^6\right)}{a c}+\left(121 (b c-2 a d) x^6 \text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(\left(a+b x^6\right) \left(-11 a c \text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},2,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{11}{6},\frac{3}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)-\left(170 b d x^{12} \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(\left(a+b x^6\right) \left(-17 a c \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{AppellF1}\left[\frac{17}{6},\frac{1}{2},2,\frac{23}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{17}{6},\frac{3}{2},1,\frac{23}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\right)$$

Problem 868: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{b x^6}{a},-\frac{d x^6}{c}\right]}{2 a x^2 \sqrt{c+d x^6}}$$

Result (type 6, 345 leaves):

$$\frac{1}{20\,x^2\,\sqrt{c+d\,x^6}}\left(-\frac{10\,\left(c+d\,x^6\right)}{a\,c}+\left(25\,\left(2\,b\,c-a\,d\right)\,x^6\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\Big/\left(\left(a+b\,x^6\right)\,\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{2},\,1,\,\frac{5}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+3\,x^6\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,2,\,\frac{8}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\,\frac{3}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\right)\right)-\left(16\,b\,d\,x^{12}\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\Big/\left(\left(a+b\,x^6\right)\,\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\,\frac{1}{2},\,1,\,\frac{8}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+3\,x^6\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\,\frac{1}{2},\,2,\,\frac{11}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\,\frac{3}{2},\,1,\,\frac{11}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\right)\right)\right)$$

- **Problem 869: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^5\,\left(a+b\,x^6\right)\,\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d\,x^6}{c}}\;\text{AppellF1}\left[-\frac{2}{3},\,1,\,\frac{1}{2},\,\frac{1}{3},\,-\frac{b\,x^6}{a},\,-\frac{d\,x^6}{c}\right]}{4\,a\,x^4\,\sqrt{c+d\,x^6}}$$

Result (type 6, 344 leaves):

$$\frac{1}{16\,x^4\,\sqrt{c+d\,x^6}}\left(-\frac{4\,\left(c+d\,x^6\right)}{a\,c}+\left(16\,\left(4\,b\,c+a\,d\right)\,x^6\,\text{AppellF1}\left[\frac{1}{3},\,\frac{1}{2},\,1,\,\frac{4}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\Big/\left(\left(a+b\,x^6\right)\,\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\,\frac{1}{2},\,1,\,\frac{4}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+3\,x^6\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{2},\,2,\,\frac{7}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\,\frac{3}{2},\,1,\,\frac{7}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\right)\right)+\left(7\,b\,d\,x^{12}\,\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{2},\,1,\,\frac{7}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\Big/\left(\left(a+b\,x^6\right)\,\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{2},\,1,\,\frac{7}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+3\,x^6\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\,\frac{1}{2},\,2,\,\frac{10}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\,\frac{3}{2},\,1,\,\frac{10}{3},\,-\frac{d\,x^6}{c},\,-\frac{b\,x^6}{a}\right]\right)\right)\right)\right)$$

Problem 873: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b \sqrt{c+d x^6}}{6 a (b c-a d) \left(a+b x^6\right)}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^6}}{\sqrt{c}}\right]}{3 a^2 \sqrt{c}}+\frac{\sqrt{b} (2 b c-3 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^6}}{\sqrt{b c-a d}}\right]}{6 a^2 (b c-a d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(b \left(\left(6 c d x^6 \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(-4 a c \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + x^6 \left(2 b c \text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right) + \left(5 d x^6 \left(2 a d + b \left(c + 3 d x^6\right)\right) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] - 3 \left(c + d x^6\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] + b c \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right]\right)\right) \Big/ \left(a \left(-5 b d x^6 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] + 2 a d \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right] + b c \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^6}, -\frac{a}{b x^6}\right]\right)\right)\right)\right) \Big/ \left(18 (-b c + a d) \left(a + b x^6\right) \sqrt{c + d x^6}\right)$$

Problem 874: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^7 \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b (2 b c-a d) \sqrt{c+d x^6}}{6 a^2 c (b c-a d) \left(a+b x^6\right)}-\frac{\sqrt{c+d x^6}}{6 a c x^6 \left(a+b x^6\right)}+\frac{(4 b c+a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^6}}{\sqrt{c}}\right]}{6 a^3 c^{3/2}}-\frac{b^{3/2} (4 b c-5 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^6}}{\sqrt{b c-a d}}\right]}{6 a^3 (b c-a d)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6\,a\,b\,d\,(-2\,b\,c+a\,d)\,x^{12}\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left((-b\,c+a\,d)\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+x^6\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)+\left(5\,b\,d\,x^6\left(-a^2\,d\left(3\,c+2\,d\,x^6\right)+2\,b^2\,c\,x^6\left(c+3\,d\,x^6\right)+3\,a\,b\left(c^2+c\,d\,x^6-d^2\,x^{12}\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+3\left(c+d\,x^6\right)\left(a^2\,d-2\,b^2\,c\,x^6+a\,b\left(-c+d\,x^6\right)\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\right)\Big/\left(c\,(b\,c-a\,d)\left(-5\,b\,d\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\right)\right)\Big/\left(18\,a^2\,x^6\left(a+b\,x^6\right)\sqrt{c+d\,x^6}\right)$$

Problem 880: Result more than twice size of optimal antiderivative.

$$\int\frac{x^4}{\left(a+b\,x^6\right)^2\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5\sqrt{1+\frac{d\,x^6}{c}}\;\text{AppellF1}\left[\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{b\,x^6}{a},-\frac{d\,x^6}{c}\right]}{5\,a^2\sqrt{c+d\,x^6}}$$

Result (type 6, 342 leaves):

$$\left(x^5\left(-\frac{55\,b\left(c+d\,x^6\right)}{a}+\left(121\,c\,(b\,c-6\,a\,d)\,\text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left(-11\,a\,c\,\text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},2,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{6},\frac{3}{2},1,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)-\left(170\,b\,c\,d\,x^6\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left(-17\,a\,c\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\left(2\,b\,c\,\text{AppellF1}\left[\frac{17}{6},\frac{1}{2},2,\frac{23}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{17}{6},\frac{3}{2},1,\frac{23}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)\right)\Big/\left(330\,(-b\,c+a\,d)\left(a+b\,x^6\right)\sqrt{c+d\,x^6}\right)$$

Problem 881: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3}{\left(a+b\,x^6\right)^2\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{b x^6}{a},-\frac{d x^6}{c}\right]}{4 a^2 \sqrt{c+d x^6}}$$

Result (type 6, 342 leaves):

$$\left(x^4 \left(-\frac{5 b \left(c+d x^6\right)}{a}+\left(25 c\ (b c-3 a d)\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-10 a c\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+\right.\right.\right.$$
$$\left.3 x^6 \left(2 b c\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d\ \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)-$$
$$\left(8 b c d x^6\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-16 a c\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\right.$$
$$\left.\left.\left.\left(2 b c\ \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d\ \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\right)\Big/\left(30\ (-b c+a d)\ \left(a+b x^6\right) \sqrt{c+d x^6}\right)$$

Problem 882: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^6\right)^2 \sqrt{c+d x^6}}\, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^2 \sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{b x^6}{a},-\frac{d x^6}{c}\right]}{2 a^2 \sqrt{c+d x^6}}$$

Result (type 6, 343 leaves):

$$\left(x^2 \left(-\frac{4 b \left(c+d x^6\right)}{a}+\left(32 c\ (2 b c-3 a d)\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-8 a c\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+\right.\right.\right.$$
$$\left.3 x^6 \left(2 b c\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d\ \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)+$$
$$\left(7 b c d x^6\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-14 a c\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\right.$$
$$\left.\left.\left.\left(2 b c\ \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d\ \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\right)\Big/\left(24\ (-b c+a d)\ \left(a+b x^6\right) \sqrt{c+d x^6}\right)$$

Problem 883: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^6}{c}} \, \text{AppellF1}\left[\frac{1}{6}, 2, \frac{1}{2}, \frac{7}{6}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{a^2 \sqrt{c+d x^6}}$$

Result (type 6, 341 leaves):

$$\left(x \left(-\frac{7 b \left(c+d x^6\right)}{a} + \left(49 c (5 b c - 6 a d) \, \text{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(-7 a c \, \text{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \, \text{AppellF1}\left[\frac{7}{6}, \frac{1}{2}, 2, \frac{13}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \, \text{AppellF1}\left[\frac{7}{6}, \frac{3}{2}, 1, \frac{13}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right) + \left(26 b c d x^6 \, \text{AppellF1}\left[\frac{7}{6}, \frac{1}{2}, 1, \frac{13}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(-13 a c \, \text{AppellF1}\left[\frac{7}{6}, \frac{1}{2}, 1, \frac{13}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \, \text{AppellF1}\left[\frac{13}{6}, \frac{1}{2}, 2, \frac{19}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \, \text{AppellF1}\left[\frac{13}{6}, \frac{3}{2}, 1, \frac{19}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right)\right) \Big/ \left(42 (-b c + a d) \left(a + b x^6\right) \sqrt{c + d x^6}\right)$$

Problem 884: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}} \, \text{AppellF1}\left[-\frac{1}{6}, 2, \frac{1}{2}, \frac{5}{6}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{a^2 x \sqrt{c+d x^6}}$$

Result (type 6, 399 leaves):

$$\left(\frac{55\left(c+d x^6\right)\left(-6 a^2 d+7 b^2 c x^6+6 a b\left(c-d x^6\right)\right)}{c}-\left(121 a\left(7 b^2 c^2-24 a b c d+12 a^2 d^2\right) x^6 \text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-11 a c \text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\left(2 b c \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},2,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{11}{6},\frac{3}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)+\left(170 a b d (7 b c-6 a d) x^{12} \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-17 a c \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\left(2 b c \text{AppellF1}\left[\frac{17}{6},\frac{1}{2},2,\frac{23}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{17}{6},\frac{3}{2},1,\frac{23}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\Big/\left(330 a^2 (-b c+a d) x \left(a+b x^6\right)\sqrt{c+d x^6}\right)$$

- **Problem 885: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^3\left(a+b x^6\right)^2\sqrt{c+d x^6}}\,dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{b x^6}{a},-\frac{d x^6}{c}\right]}{2 a^2 x^2\sqrt{c+d x^6}}$$

Result (type 6, 399 leaves):

$$\left(\frac{10\left(c+d x^6\right)\left(-3 a^2 d+4 b^2 c x^6+3 a b\left(c-d x^6\right)\right)}{c}-\left(25 a\left(8 b^2 c^2-15 a b c d+3 a^2 d^2\right) x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\left(2 b c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)+\left(16 a b d (4 b c-3 a d) x^{12} \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-16 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\left(2 b c \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\Big/\left(60 a^2 (-b c+a d) x^2 \left(a+b x^6\right)\sqrt{c+d x^6}\right)$$

- **Problem 886: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^5 \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}} \, \text{AppellF1}\left[-\frac{2}{3}, 2, \frac{1}{2}, \frac{1}{3}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{4 a^2 x^4 \sqrt{c+d x^6}}$$

Result (type 6, 399 leaves):

$$\left(\frac{4 \left(c+d x^6\right) \left(-3 a^2 d+5 b^2 c x^6+3 a b \left(c-d x^6\right)\right)}{c}+\left(16 a \left(-20 b^2 c^2+21 a b c d+3 a^2 d^2\right) x^6 \, \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \bigg/ \left(-8 a c \, \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+a d \, \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)+\left(7 a b d (-5 b c+3 a d) x^{12} \, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \bigg/ \left(-14 a c \, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \, \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+a d \, \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right) \bigg/ \left(48 a^2 (-b c+a d) x^4 \left(a+b x^6\right) \sqrt{c+d x^6}\right)$$

- **Problem 890: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x \left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^8}}{\sqrt{c}}\right]}{4 a \sqrt{c}}+\frac{\sqrt{b} \, \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^8}}{\sqrt{b c-a d}}\right]}{4 a \sqrt{b c-a d}}$$

Result (type 6, 162 leaves):

$$\left(5\,b\,d\,x^8\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]\right)\Big/\left(12\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}\right.$$
$$\left.\left(-5\,b\,d\,x^8\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]\right)\right)$$

- **Problem 891: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^9\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d\,x^8}}{8\,a\,c\,x^8}+\frac{(2\,b\,c+a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^8}}{\sqrt{c}}\right]}{8\,a^2\,c^{3/2}}-\frac{b^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^8}}{\sqrt{b\,c-a\,d}}\right]}{4\,a^2\,\sqrt{b\,c-a\,d}}$$

Result (type 6, 410 leaves):

$$\frac{1}{24\,x^8\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}}\left(\left(6\,b\,d\,x^{16}\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\right.$$
$$\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+x^8\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)+$$
$$\left(5\,b\,d\,x^8\,\left(a\,\left(3\,c+2\,d\,x^8\right)+b\,x^8\,\left(c+3\,d\,x^8\right)\right)\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]-\right.$$
$$\left.3\,\left(a+b\,x^8\right)\,\left(c+d\,x^8\right)\,\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]\right)\right)\Big/$$
$$\left(a\,c\,\left(-5\,b\,d\,x^8\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+\right.\right.$$
$$\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]\right)\right)\right)$$

- **Problem 897: Result unnecessarily involves higher level functions.**

$$\int \frac{x^9}{\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 4, 851 leaves, 10 steps):

$$-\frac{(-a)^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{3/4}\,\sqrt{b\,c-a\,d}}-\frac{(-a)^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{3/4}\,\sqrt{-b\,c+a\,d}}+\frac{\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{4\,b\,c^{1/4}\,d^{1/4}\,\sqrt{c+d\,x^8}}-$$

$$\frac{a\left(\frac{\sqrt{b}\;\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}}-$$

$$\frac{\left(\sqrt{-a}\;\sqrt{b}\;\sqrt{c}+a\,\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}}-$$

$$\left(\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)^2}{4\,\sqrt{-a}\;\sqrt{b}\;\sqrt{c}\;\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16\,b\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$
$$\left(\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)^2}{4\,\sqrt{-a}\;\sqrt{b}\;\sqrt{c}\;\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16\,b\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 165 leaves) :

$$-\left(9\,a\,c\,x^{10}\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(10\,\left(a+b\,x^8\right)\sqrt{c+d\,x^8}\,\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.\right.$$
$$\left.\left.2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)$$

- **Problem 898: Result unnecessarily involves higher level functions.**

$$\int\frac{x}{\left(a+b\,x^8\right)\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 4, 754 leaves, 8 steps) :

$$-\frac{b^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,(-a)^{3/4}\,\sqrt{b\,c-a\,d}}-\frac{b^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,(-a)^{3/4}\,\sqrt{-b\,c+a\,d}}+$$

$$\frac{\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}}+$$

$$\frac{\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\,\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,a\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16\,a\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16\,a\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 165 leaves) :

$$-\left(5\,a\,c\,x^2\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(2\,\left(a+b\,x^8\right)\sqrt{c+d\,x^8}\,\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.\right.$$
$$\left.\left.2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)$$

- **Problem 899: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^7\,\left(a+b\,x^8\right)\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 4, 878 leaves, 11 steps) :

$$-\frac{\sqrt{c+d x^8}}{6 a c x^6}-\frac{b^{5/4} \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{7/4} \sqrt{b c-a d}}-\frac{b^{5/4} \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{7/4} \sqrt{-b c+a d}}-\frac{d^{3/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{12 a c^{5/4} \sqrt{c+d x^8}}-$$

$$\frac{b \left(\frac{\sqrt{b} \sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8 a c^{1/4} (b c+a d) \sqrt{c+d x^8}}-$$

$$\frac{b \left(\sqrt{-a} \sqrt{b} \sqrt{c}+a \sqrt{d}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8 a^2 c^{1/4} (b c+a d) \sqrt{c+d x^8}}-$$

$$\left(b \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}},2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right) \Big/$$

$$\left(16 a^2 c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}},2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right) \Big/$$

$$\left(16 a^2 c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 344 leaves) :

$$\frac{1}{30\,x^6\,\sqrt{c+d\,x^8}}\left(-\frac{5\,(c+d\,x^8)}{a\,c}+\left(25\,(3\,b\,c+a\,d)\,x^8\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.\right.\right.$$
$$\left.\left.2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)+$$
$$\left(9\,b\,d\,x^{16}\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.\right.$$
$$\left.\left.\left.2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

- **Problem 900: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{13}}{(a+b\,x^8)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 4, 1005 leaves, 12 steps) :

$$\frac{x^2\sqrt{c+d\,x^8}}{2\,b\,\sqrt{d}\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{(-a)^{3/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{5/4}\,\sqrt{b\,c-a\,d}}-\frac{(-a)^{3/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{5/4}\,\sqrt{-b\,c+a\,d}}-$$

$$\frac{c^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{2\,b\,d^{3/4}\,\sqrt{c+d\,x^8}}+\frac{c^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{4\,b\,d^{3/4}\,\sqrt{c+d\,x^8}}+$$

$$\frac{a\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}}+$$

$$\frac{a\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}}+$$

$$\left(\sqrt{-a}\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$
$$\left(\sqrt{-a}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(16\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 165 leaves) :

$$-\left(11\,a\,c\,x^{14}\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(14\,\left(a+b\,x^8\right)\sqrt{c+d\,x^8}\,\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.\right.$$
$$\left.\left.2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)$$

- **Problem 901: Result unnecessarily involves higher level functions.**

$$\int \frac{x^5}{\left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 768 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{1/4} b^{1/4} \sqrt{b c-a d}}-\frac{\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{1/4} b^{1/4} \sqrt{-b c+a d}}-\frac{\left(\sqrt{c}-\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{8 c^{1/4} (b c+a d) \sqrt{c+d x^8}}-$$

$$\frac{\left(\sqrt{c}+\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{8 c^{1/4} (b c+a d) \sqrt{c+d x^8}}+$$

$$\left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(16 \sqrt{-a} \sqrt{b}\, c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)-$$

$$\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\, \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(16 \sqrt{-a} \sqrt{b}\, c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 165 leaves):

$$-\left(7 a c x^6 \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right) \Big/ \left(6 \left(a+b x^8\right) \sqrt{c+d x^8} \left(-7 a c \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]+\right.\right.$$

$$\left.\left.2 x^8 \left(2 b c \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right)\right)\right)$$

- **Problem 902: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 \left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 1032 leaves, 14 steps):

$$-\frac{\sqrt{c+d x^8}}{2 a c x^2}+\frac{\sqrt{d}\, x^2 \sqrt{c+d x^8}}{2 a c \left(\sqrt{c}+\sqrt{d}\, x^4\right)}+\frac{b^{3/4}\,\text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{5/4} \sqrt{b c-a d}}-\frac{b^{3/4}\,\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{5/4} \sqrt{-b c+a d}}-$$

$$\frac{d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{2 a c^{3/4}\sqrt{c+d x^8}}+\frac{d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{4 a c^{3/4}\sqrt{c+d x^8}}+$$

$$\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8 a c^{1/4} (b c+a d)\sqrt{c+d x^8}}+$$

$$\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8 a c^{1/4} (b c+a d)\sqrt{c+d x^8}}+$$

$$\left(\sqrt{b}\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\, 2\,\text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(16 (-a)^{3/2} c^{1/4} d^{1/4} (b c+a d)\sqrt{c+d x^8}\right)-$$

$$\left(\sqrt{b}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\, 2\,\text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(16 (-a)^{3/2} c^{1/4} d^{1/4} (b c+a d)\sqrt{c+d x^8}\right)$$

Result (type 6, 344 leaves) :

$$\frac{1}{42\,x^2\,\sqrt{c+d\,x^8}}\left(-\frac{21\,(c+d\,x^8)}{a\,c}+\left(49\,(b\,c-a\,d)\,x^8\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)-\left(33\,b\,d\,x^{16}\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

Problem 903: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{(a+b\,x^8)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5\,\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{5\,a\,\sqrt{c+d\,x^8}}$$

Result (type 6, 165 leaves):

$$-\left(13\,a\,c\,x^5\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(5\,(a+b\,x^8)\,\sqrt{c+d\,x^8}\left(-13\,a\,c\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},2,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)$$

Problem 904: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a+b\,x^8)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^3\,\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{3\,a\,\sqrt{c+d\,x^8}}$$

Result (type 6, 165 leaves):

$$-\left(11\,a\,c\,x^3\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\bigg/\left(3\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}\,\left(-11\,a\,c\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},2,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{8},\frac{3}{2},1,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)$$

- **Problem 905: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{a\,\sqrt{c+d\,x^8}}$$

Result (type 6, 161 leaves):

$$-\left(9\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\bigg/\left(\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}\,\left(-9\,a\,c\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{8},\frac{3}{2},1,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)$$

- **Problem 906: Result more than twice size of optimal antiderivative.**

$$\int\frac{1}{x^2\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{a\,x\,\sqrt{c+d\,x^8}}$$

Result (type 6, 344 leaves):

$$\frac{1}{35\,x\,\sqrt{c+d\,x^8}}\left(-\frac{35\,(c+d\,x^8)}{a\,c}+\left(75\,(b\,c-3\,a\,d)\,x^8\,\text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-15\,a\,c\,\text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},2,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{15}{8},\frac{3}{2},1,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)-\left(161\,b\,d\,x^{16}\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-23\,a\,c\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{23}{8},\frac{1}{2},2,\frac{31}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{23}{8},\frac{3}{2},1,\frac{31}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

- **Problem 907: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^4\,(a+b\,x^8)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^8}{c}}\;\text{AppellF1}\left[-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{3\,a\,x^3\,\sqrt{c+d\,x^8}}$$

Result (type 6, 345 leaves):

$$\frac{1}{195\,x^3\,\sqrt{c+d\,x^8}}\left(-\frac{65\,(c+d\,x^8)}{a\,c}+\left(169\,(3\,b\,c-a\,d)\,x^8\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-13\,a\,c\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},2,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)-\left(105\,b\,d\,x^{16}\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-21\,a\,c\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{21}{8},\frac{1}{2},2,\frac{29}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{21}{8},\frac{3}{2},1,\frac{29}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

- **Problem 911: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b \sqrt{c+d x^8}}{8 a (b c-a d) \left(a+b x^8\right)}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^8}}{\sqrt{c}}\right]}{4 a^2 \sqrt{c}}+\frac{\sqrt{b} (2 b c-3 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^8}}{\sqrt{b c-a d}}\right]}{8 a^2 (b c-a d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(b \left(\left(6 c d x^8 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\right.\right.$$
$$\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+x^8 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+$$
$$\left(5 d x^8 \left(2 a d+b \left(c+3 d x^8\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]-\right.$$
$$\left.3 \left(c+d x^8\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\right)\right)\Big/$$
$$\left(a \left(-5 b d x^8 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+\right.\right.$$
$$\left.\left.\left.\left.b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\right)\right)\right)\right)\Big/\left(24 (-b c+a d) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

- **Problem 912: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{x^9 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b (2 b c-a d) \sqrt{c+d x^8}}{8 a^2 c (b c-a d) \left(a+b x^8\right)}-\frac{\sqrt{c+d x^8}}{8 a c x^8 \left(a+b x^8\right)}+\frac{(4 b c+a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^8}}{\sqrt{c}}\right]}{8 a^3 c^{3/2}}-\frac{b^{3/2} (4 b c-5 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^8}}{\sqrt{b c-a d}}\right]}{8 a^3 (b c-a d)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6\,a\,b\,d\,(-2\,b\,c+a\,d)\,x^{16}\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((-b\,c+a\,d)\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+x^8\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)+\left(5\,b\,d\,x^8\left(-a^2\,d\left(3\,c+2\,d\,x^8\right)+2\,b^2\,c\,x^8\left(c+3\,d\,x^8\right)+3\,a\,b\left(c^2+c\,d\,x^8-d^2\,x^{16}\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+3\left(c+d\,x^8\right)\left(a^2\,d-2\,b^2\,c\,x^8+a\,b\left(-c+d\,x^8\right)\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]\right)\right)\Big/\left(c\,(b\,c-a\,d)\left(-5\,b\,d\,x^8\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^8},-\frac{a}{b\,x^8}\right]\right)\right)\right)\Big/\left(24\,a^2\,x^8\left(a+b\,x^8\right)\sqrt{c+d\,x^8}\right)$$

- **Problem 918: Result unnecessarily involves higher level functions.**

$$\int \frac{x^9}{\left(a+b\,x^8\right)^2\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 4, 924 leaves, 11 steps):

$$-\frac{x^2\sqrt{c+d x^8}}{8(b c-a d)(a+b x^8)}-\frac{(b c+a d)\,\text{ArcTan}\left[\frac{\sqrt{b c-a d}\,x^2}{(-a)^{1/4}b^{1/4}\sqrt{c+d x^8}}\right]}{32(-a)^{3/4}b^{3/4}(b c-a d)^{3/2}}+\frac{(b c+a d)\,\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\,x^2}{(-a)^{1/4}b^{1/4}\sqrt{c+d x^8}}\right]}{32(-a)^{3/4}b^{3/4}(-b c+a d)^{3/2}}+$$

$$\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{32\,b\,c^{1/4}(b c-a d)\sqrt{c+d x^8}}+$$

$$\frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{32\,a\,b\,c^{1/4}(b c-a d)\sqrt{c+d x^8}}-$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{16\,b\,c^{1/4}(b c-a d)\sqrt{c+d x^8}}+$$

$$\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(64\,a\,b\,c^{1/4}d^{1/4}(b c-a d)\sqrt{c+d x^8}\right)+$$

$$\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(64\,a\,b\,c^{1/4}d^{1/4}(b c-a d)\sqrt{c+d x^8}\right)$$

Result (type 6, 333 leaves) :

$$\left(x^2\left(5\left(c+d x^8\right)+\left(25 a c^2 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\bigg/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+\right.\right.\right.$$
$$\left.2 x^8\left(2 b c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)-$$
$$\left(9 a c d x^8 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\bigg/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8\right.$$
$$\left.\left.\left.\left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\bigg/\left(40(-b c+a d)\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

- **Problem 919: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{\left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 999 leaves, 11 steps):

$$\frac{b x^2 \sqrt{c+d x^8}}{8 a (b c-a d) (a+b x^8)}+\frac{b^{1/4} (3 b c-5 a d) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\ x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{7/4} (b c-a d)^{3/2}}-$$

$$\frac{b^{1/4} (3 b c-5 a d) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\ x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{7/4} (-b c+a d)^{3/2}}+\frac{d^{3/4} \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{16 a c^{1/4} (b c-a d) \sqrt{c+d x^8}}+$$

$$\frac{\left(\frac{\sqrt{b} \sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right) d^{1/4} (3 b c-5 a d) \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{32 a c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}}+$$

$$\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) d^{1/4} (3 b c-5 a d) \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(32 (-a)^{3/2} c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)+$$

$$\left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 (3 b c-5 a d) \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(64 a^2 c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)+$$

$$\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 (3 b c-5 a d) \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(64 a^2 c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 343 leaves) :

$$\left(x^2\left(-\frac{5\,b\,(c+d\,x^8)}{a}+\left(25\,c\,(3\,b\,c-4\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)+\left(9\,b\,c\,d\,x^8\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)\Big/\left(40\,(-b\,c+a\,d)\,(a+b\,x^8)\,\sqrt{c+d\,x^8}\right)$$

- **Problem 920: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^7\,(a+b\,x^8)^2\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 4, 1060 leaves, 12 steps) :

$$-\frac{(7\,b\,c-4\,a\,d)\,\sqrt{c+d\,x^8}}{24\,a^2\,c\,(b\,c-a\,d)\,x^6}+\frac{b\,\sqrt{c+d\,x^8}}{8\,a\,(b\,c-a\,d)\,x^6\,\left(a+b\,x^8\right)}+\frac{b^{5/4}\,(7\,b\,c-9\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{11/4}\,(b\,c-a\,d)^{3/2}}-$$

$$\frac{b^{5/4}\,(7\,b\,c-9\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{11/4}\,(-b\,c+a\,d)^{3/2}}-\frac{d^{3/4}\,(7\,b\,c-4\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\,\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{48\,a^2\,c^{5/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^8}}+$$

$$\left(b\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,(7\,b\,c-9\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\,\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,(-a)^{5/2}\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(b\,\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\,d^{1/4}\,(7\,b\,c-9\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\,\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,(-a)^{5/2}\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(b\,\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\,(7\,b\,c-9\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\,\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,a^3\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(b\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\,(7\,b\,c-9\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\,\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,a^3\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 399 leaves) :

$$\left(\frac{5\left(c+d x^8\right)\left(-4 a^2 d+7 b^2 c x^8+4 a b\left(c-d x^8\right)\right)}{c}+\right.$$

$$\left(25 a\left(-21 b^2 c^2+20 a b c d+4 a^2 d^2\right) x^8 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+\right.$$

$$\left.2 x^8\left(2 b c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+$$

$$\left(9 a b d(-7 b c+4 a d) x^{16} \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+\right.$$

$$\left.\left.2 x^8\left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\Big/$$

$$\left(120 a^2(-b c+a d) x^6\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

- **Problem 921: Result unnecessarily involves higher level functions.**

$$\int \frac{x^{13}}{\left(a+b x^8\right)^2\sqrt{c+d x^8}}\,dx$$

Optimal (type 4, 1164 leaves, 14 steps) :

$$\frac{\sqrt{d}\, x^2 \sqrt{c+d x^8}}{8 b (b c-a d) \left(\sqrt{c}+\sqrt{d} x^4\right)}-\frac{x^6 \sqrt{c+d x^8}}{8 (b c-a d) \left(a+b x^8\right)}+\frac{(3 b c-a d) \operatorname{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{1/4} b^{5/4} (b c-a d)^{3/2}}+\frac{(3 b c-a d) \operatorname{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{1/4} b^{5/4} (-b c+a d)^{3/2}}-$$

$$\frac{c^{1/4} d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\ \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{8 b (b c-a d) \sqrt{c+d x^8}}+\frac{c^{1/4} d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\ \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{16 b (b c-a d) \sqrt{c+d x^8}}-$$

$$\frac{\left(\sqrt{c}-\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} (3 b c-a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\ \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{32 b c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}}-$$

$$\frac{\left(\sqrt{c}+\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} (3 b c-a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\ \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{32 b c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}}+$$

$$\left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 (3 b c-a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\ \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(64 \sqrt{-a}\, b^{3/2} c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)-$$

$$\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 (3 b c-a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\ \operatorname{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(64 \sqrt{-a}\, b^{3/2} c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 333 leaves) :

$$\left(x^6 \left(7 \left(c+d x^8\right)+\left(49 a c^2 \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-7 a c \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+\right.\right.\right.$$

$$\left.2 x^8 \left(2 b c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+$$

$$\left(11 a c d x^8 \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-11 a c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c\right.\right.$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(56 (-b c+a d) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

- **Problem 922: Result unnecessarily involves higher level functions.**

$$\int \frac{x^5}{\left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 1162 leaves, 14 steps) :

$$-\frac{\sqrt{d}\,x^2\sqrt{c+d\,x^8}}{8\,a\,(b\,c-a\,d)\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{b\,x^6\sqrt{c+d\,x^8}}{8\,a\,(b\,c-a\,d)\left(a+b\,x^8\right)}-\frac{(b\,c-3\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{5/4}\,b^{1/4}\,(b\,c-a\,d)^{3/2}}-\frac{(b\,c-3\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{5/4}\,b^{1/4}\,(-b\,c+a\,d)^{3/2}}+$$

$$\frac{c^{1/4}\,d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,a\,(b\,c-a\,d)\sqrt{c+d\,x^8}}-\frac{c^{1/4}\,d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{16\,a\,(b\,c-a\,d)\sqrt{c+d\,x^8}}-$$

$$\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)d^{1/4}\,(b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{32\,a\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}}-$$

$$\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)d^{1/4}\,(b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{32\,a\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}}-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2(b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,(-a)^{3/2}\sqrt{b}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2(b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,(-a)^{3/2}\sqrt{b}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}\right)$$

Result (type 6, 342 leaves) :

$$\left(x^6 \left(-\frac{21 b \left(c+d x^8\right)}{a} + \left(49 c (b c-4 a d) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right) \Big/ \left(-7 a c \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + 2 x^8 \left(2 b c \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + a d \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right)\right) - \left(33 b c d x^8 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right) \Big/ \left(-11 a c \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + 2 x^8 \left(2 b c \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + a d \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right)\right)\right)\right) \Big/ \left(168 (-b c+a d) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

- **Problem 923: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 1243 leaves, 15 steps) :

$$-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{\sqrt{d}\,(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)}-\frac{b^{3/4}(5bc-7ad)\,\mathrm{ArcTan}\left[\frac{\sqrt{bc-ad}\,x^2}{(-a)^{1/4}b^{1/4}\sqrt{c+dx^8}}\right]}{32(-a)^{9/4}(bc-ad)^{3/2}}-$$

$$\frac{b^{3/4}(5bc-7ad)\,\mathrm{ArcTan}\left[\frac{\sqrt{-bc+ad}\,x^2}{(-a)^{1/4}b^{1/4}\sqrt{c+dx^8}}\right]}{32(-a)^{9/4}(-bc+ad)^{3/2}}-\frac{d^{1/4}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}+$$

$$\frac{d^{1/4}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{16a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}+$$

$$\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)d^{1/4}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{32a^2c^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}}+$$

$$\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)d^{1/4}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{32a^2c^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}}-$$

$$\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\mathrm{EllipticPi}\left[-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64(-a)^{5/2}c^{1/4}d^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}\right)+$$

$$\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\mathrm{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\mathrm{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64(-a)^{5/2}c^{1/4}d^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}\right)$$

Result (type 6, 399 leaves) :

$$\left(\frac{21\left(c+d x^8\right)\left(-4 a^2 d+5 b^2 c x^8+4 a b\left(c-d x^8\right)\right)}{c}-\left(49 a\left(5 b^2 c^2-12 a b c d+4 a^2 d^2\right) x^8 \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-7 a c \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8\left(2 b c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(33 a b d(5 b c-4 a d) x^{16} \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-11 a c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8\left(2 b c \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\Big/\left(168 a^2(-b c+a d) x^2\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

Problem 924: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\left(a+b x^8\right)^2\sqrt{c+d x^8}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5\sqrt{1+\frac{d x^8}{c}}\ \text{AppellF1}\left[\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{5 a^2\sqrt{c+d x^8}}$$

Result (type 6, 343 leaves):

$$\left(x^5\left(-\frac{65 b\left(c+d x^8\right)}{a}+\left(169 c(3 b c-8 a d) \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-13 a c \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8\left(2 b c \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},2,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)-\left(105 b c d x^8 \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-21 a c \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8\left(2 b c \text{AppellF1}\left[\frac{21}{8},\frac{1}{2},2,\frac{29}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{21}{8},\frac{3}{2},1,\frac{29}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(520(-b c+a d)\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

Problem 925: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\left(a+b x^8\right)^2\sqrt{c+d x^8}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^3 \sqrt{1+\frac{d x^8}{c}}\ \text{AppellF1}\left[\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{3 a^2 \sqrt{c+d x^8}}$$

Result (type 6, 343 leaves):

$$\left(x^3\left(-\frac{33 b\left(c+d x^8\right)}{a}+\left(121 c\,(5 b c-8 a d)\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-11 a c\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8\left(2 b c\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},2,\frac{19}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\,\text{AppellF1}\left[\frac{11}{8},\frac{3}{2},1,\frac{19}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(57 b c d x^8\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},1,\frac{19}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-19 a c\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},1,\frac{19}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8\left(2 b c\,\text{AppellF1}\left[\frac{19}{8},\frac{1}{2},2,\frac{27}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\,\text{AppellF1}\left[\frac{19}{8},\frac{3}{2},1,\frac{27}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(264\,(-b c+a d)\,\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

Problem 926: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^8\right)^2 \sqrt{c+d x^8}}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^8}{c}}\ \text{AppellF1}\left[\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{a^2 \sqrt{c+d x^8}}$$

Result (type 6, 341 leaves):

$$\left(x\left(-\frac{3 b\left(c+d x^8\right)}{a}+\left(27 c\,(7 b c-8 a d)\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-9 a c\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8\left(2 b c\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\,\text{AppellF1}\left[\frac{9}{8},\frac{3}{2},1,\frac{17}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(17 b c d x^8\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-17 a c\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8\left(2 b c\,\text{AppellF1}\left[\frac{17}{8},\frac{1}{2},2,\frac{25}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\,\text{AppellF1}\left[\frac{17}{8},\frac{3}{2},1,\frac{25}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(24\,(-b c+a d)\,\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

Problem 927: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^8}{c}} \text{AppellF1}\left[-\frac{1}{8}, 2, \frac{1}{2}, \frac{7}{8}, -\frac{b x^8}{a}, -\frac{d x^8}{c}\right]}{a^2 x \sqrt{c+d x^8}}$$

Result (type 6, 399 leaves):

$$\left(\frac{35 \left(c+d x^8\right) \left(-8 a^2 d+9 b^2 c x^8+8 a b \left(c-d x^8\right)\right)}{c} - \left(75 a \left(9 b^2 c^2-40 a b c d+24 a^2 d^2\right) x^8 \text{AppellF1}\left[\frac{7}{8}, \frac{1}{2}, 1, \frac{15}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right) \Big/ \left(-15 a c \text{AppellF1}\left[\frac{7}{8}, \frac{1}{2}, 1, \frac{15}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + 4 x^8 \left(2 b c \text{AppellF1}\left[\frac{15}{8}, \frac{1}{2}, 2, \frac{23}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + a d \text{AppellF1}\left[\frac{15}{8}, \frac{3}{2}, 1, \frac{23}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right)\right) + \left(161 a b d (9 b c-8 a d) x^{16} \text{AppellF1}\left[\frac{15}{8}, \frac{1}{2}, 1, \frac{23}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right) \Big/ \left(-23 a c \text{AppellF1}\left[\frac{15}{8}, \frac{1}{2}, 1, \frac{23}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + 4 x^8 \left(2 b c \text{AppellF1}\left[\frac{23}{8}, \frac{1}{2}, 2, \frac{31}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right] + a d \text{AppellF1}\left[\frac{23}{8}, \frac{3}{2}, 1, \frac{31}{8}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right)\right)\right) \Big/ \left(280 a^2 (-b c+a d) x \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

Problem 928: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^8}{c}} \text{AppellF1}\left[-\frac{3}{8}, 2, \frac{1}{2}, \frac{5}{8}, -\frac{b x^8}{a}, -\frac{d x^8}{c}\right]}{3 a^2 x^3 \sqrt{c+d x^8}}$$

Result (type 6, 399 leaves):

$$\left(\frac{65\,(c+d\,x^8)\,(-8\,a^2\,d+11\,b^2\,c\,x^8+8\,a\,b\,(c-d\,x^8))}{c}-\right.$$

$$\left(169\,a\,(33\,b^2\,c^2-56\,a\,b\,c\,d+8\,a^2\,d^2)\,x^8\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-13\,a\,c\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.$$

$$\left.4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},2,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)+$$

$$\left(105\,a\,b\,d\,(11\,b\,c-8\,a\,d)\,x^{16}\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-21\,a\,c\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.$$

$$\left.\left.4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{21}{8},\frac{1}{2},2,\frac{29}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{21}{8},\frac{3}{2},1,\frac{29}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\Big/$$

$$\left(1560\,a^2\,(-b\,c+a\,d)\,x^3\,(a+b\,x^8)\,\sqrt{c+d\,x^8}\right)$$

- **Problem 986: Result more than twice size of optimal antiderivative.**

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q(e\,x)^m\,dx$$

Optimal (type 6, 105 leaves, 4 steps):

$$\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}(e\,x)^{1+m}\,\text{AppellF1}\left[\frac{1}{2}(-1-m),-p,-q,\frac{1-m}{2},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]}{e\,(1+m)}$$

Result (type 6, 284 leaves):

$$\left(b\,d\,(3+m-2\,p-2\,q)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,(e\,x)^m\,\text{AppellF1}\left[\frac{1}{2}(1+m-2\,p-2\,q),-p,-q,\frac{1}{2}(3+m-2\,p-2\,q),-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\Big/$$

$$\left((1+m-2\,p-2\,q)\left(b\,d\,(3+m-2\,p-2\,q)\,\text{AppellF1}\left[\frac{1}{2}(1+m-2\,p-2\,q),-p,-q,\frac{1}{2}(3+m-2\,p-2\,q),-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+\right.\right.$$

$$2\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{1}{2}(3+m-2\,p-2\,q),1-p,-q,\frac{1}{2}(5+m-2\,p-2\,q),-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+\right.$$

$$\left.\left.\left.b\,c\,q\,\text{AppellF1}\left[\frac{1}{2}(3+m-2\,p-2\,q),-p,1-q,\frac{1}{2}(5+m-2\,p-2\,q),-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

- **Problem 987: Result more than twice size of optimal antiderivative.**

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$\frac{1}{5}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}x^5\,\text{AppellF1}\left[-\frac{5}{2},-p,-q,-\frac{3}{2},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]$$

Result (type 6, 254 leaves):

$$\left(b\,d\,(-7+2\,p+2\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^5\,\text{AppellF1}\left[\frac{5}{2}-p-q,\,-p,\,-q,\,\frac{7}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-5+2\,p+2\,q)\,\left(b\,d\,(7-2\,p-2\,q)\,\text{AppellF1}\left[\frac{5}{2}-p-q,\,-p,\,-q,\,\frac{7}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.2\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{7}{2}-p-q,\,1-p,\,-q,\,\frac{9}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{7}{2}-p-q,\,-p,\,1-q,\,\frac{9}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 988: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

Optimal (type 6, 100 leaves, 3 steps):

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{b\,c-a\,d}\right)^{-q}\text{AppellF1}\left[1+p,\,-q,\,3,\,2+p,\,-\frac{d\left(a+\frac{b}{x^2}\right)}{b\,c-a\,d},\,\frac{a+\frac{b}{x^2}}{a}\right]}{2\,a^3\,(1+p)}$$

Result (type 6, 229 leaves):

$$\left(b\,d\,(-3+p+q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^4\,\text{AppellF1}\left[2-p-q,\,-p,\,-q,\,3-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left(2\,(-2+p+q)\,\left(-b\,d\,(-3+p+q)\,\text{AppellF1}\left[2-p-q,\,-p,\,-q,\,3-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.x^2\left(a\,d\,p\,\text{AppellF1}\left[3-p-q,\,1-p,\,-q,\,4-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[3-p-q,\,-p,\,1-q,\,4-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 989: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$\frac{1}{3}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}x^3\,\text{AppellF1}\left[-\frac{3}{2},\,-p,\,-q,\,-\frac{1}{2},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]$$

Result (type 6, 254 leaves):

$$\left(b\,d\,(-5+2\,p+2\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^3\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,-p,\,-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-3+2\,p+2\,q)\,\left(b\,d\,(5-2\,p-2\,q)\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,-p,\,-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.2\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{5}{2}-p-q,\,1-p,\,-q,\,\frac{7}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{5}{2}-p-q,\,-p,\,1-q,\,\frac{7}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

- **Problem 990: Result more than twice size of optimal antiderivative.**

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,dx$$

Optimal (type 6, 98 leaves, 3 steps):

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{b\,c-a\,d}\right)^{-q}\text{AppellF1}\left[1+p,\,-q,\,2,\,2+p,\,-\frac{d\left(a+\frac{b}{x^2}\right)}{b\,c-a\,d},\,\frac{a+\frac{b}{x^2}}{a}\right]}{2\,a^2\,(1+p)}$$

Result (type 6, 229 leaves):

$$\left(b\,d\,(-2+p+q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2\,\text{AppellF1}\left[1-p-q,\,-p,\,-q,\,2-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left(2\,(-1+p+q)\,\left(-b\,d\,(-2+p+q)\,\text{AppellF1}\left[1-p-q,\,-p,\,-q,\,2-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.x^2\left(a\,d\,p\,\text{AppellF1}\left[2-p-q,\,1-p,\,-q,\,3-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[2-p-q,\,-p,\,1-q,\,3-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

- **Problem 991: Result more than twice size of optimal antiderivative.**

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Optimal (type 6, 79 leaves, 4 steps):

$$\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q} x\,\text{AppellF1}\left[-\frac{1}{2},\,-p,\,-q,\,\frac{1}{2},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]$$

Result (type 6, 252 leaves):

$$\left(b\,d\,(-3+2\,p+2\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,\text{AppellF1}\left[\frac{1}{2}-p-q,\,-p,\,-q,\,\frac{3}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-1+2\,p+2\,q)\,\left(b\,d\,(3-2\,p-2\,q)\,\text{AppellF1}\left[\frac{1}{2}-p-q,\,-p,\,-q,\,\frac{3}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.2\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,1-p,\,-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,-p,\,1-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 992: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx$$

Optimal (type 6, 97 leaves, 3 steps):

$$\frac{\left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b \left(c+\frac{d}{x^2}\right)}{b c-a d}\right)^{-q} \text{AppellF1}\left[1+p,\ -q,\ 1,\ 2+p,\ -\frac{d \left(a+\frac{b}{x^2}\right)}{b c-a d},\ \frac{a+\frac{b}{x^2}}{a}\right]}{2 a (1+p)}$$

Result (type 6, 223 leaves):

$$-\left(b d (-1+p+q) \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \text{AppellF1}\left[-p-q,\ -p,\ -q,\ 1-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right]\right) \Big/ \left(2 (p+q) \left(b d (-1+p+q) \text{AppellF1}\left[-p-q,\ -p,\ -q,\ 1-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right] - x^2 \left(a d p \text{AppellF1}\left[1-p-q,\ 1-p,\ -q,\ 2-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right] + b c q \text{AppellF1}\left[1-p-q,\ -p,\ 1-q,\ 2-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 993: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} \, dx$$

Optimal (type 6, 82 leaves, 4 steps):

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c x^2}\right)^{-q} \text{AppellF1}\left[\frac{1}{2},\ -p,\ -q,\ \frac{3}{2},\ -\frac{b}{a x^2},\ -\frac{d}{c x^2}\right]}{x}$$

Result (type 6, 254 leaves):

$$\left(b d (-1+2 p+2 q) \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \text{AppellF1}\left[-\frac{1}{2}-p-q,\ -p,\ -q,\ \frac{1}{2}-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right]\right) \Big/ \left((1+2 p+2 q)\, x \left(b d (1-2 p-2 q) \text{AppellF1}\left[-\frac{1}{2}-p-q,\ -p,\ -q,\ \frac{1}{2}-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right] + 2 x^2 \left(a d p \text{AppellF1}\left[\frac{1}{2}-p-q,\ 1-p,\ -q,\ \frac{3}{2}-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right] + b c q \text{AppellF1}\left[\frac{1}{2}-p-q,\ -p,\ 1-q,\ \frac{3}{2}-p-q,\ -\frac{a x^2}{b},\ -\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 995: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c x^2}\right)^{-q} \text{AppellF1}\left[\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{a x^2},-\frac{d}{c x^2}\right]}{3 x^3}$$

Result (type 6, 255 leaves):

$$\left(b d (1+2 p+2 q) \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \text{AppellF1}\left[-\frac{3}{2}-p-q,-p,-q,-\frac{1}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right) \Big/ \left((3+2 p+2 q) x^3 \left(-b d (1+2 p+2 q) \text{AppellF1}\left[-\frac{3}{2}-p-q,-p,-q,-\frac{1}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+2 x^2 \left(a d p \text{AppellF1}\left[-\frac{1}{2}-p-q,1-p,-q,\frac{1}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+b c q \text{AppellF1}\left[-\frac{1}{2}-p-q,-p,1-q,\frac{1}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\right)\right)$$

- **Problem 996: Result more than twice size of optimal antiderivative.**

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (e x)^{5/2} \, dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$\frac{2 \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c x^2}\right)^{-q} (e x)^{7/2} \text{AppellF1}\left[-\frac{7}{4},-p,-q,-\frac{3}{4},-\frac{b}{a x^2},-\frac{d}{c x^2}\right]}{7 e}$$

Result (type 6, 260 leaves):

$$\left(2 b d (-11+4 p+4 q) \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x (e x)^{5/2} \text{AppellF1}\left[\frac{7}{4}-p-q,-p,-q,\frac{11}{4}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right) \Big/ \left((-7+4 p+4 q) \left(b d (11-4 p-4 q) \text{AppellF1}\left[\frac{7}{4}-p-q,-p,-q,\frac{11}{4}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+4 x^2 \left(a d p \text{AppellF1}\left[\frac{11}{4}-p-q,1-p,-q,\frac{15}{4}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+b c q \text{AppellF1}\left[\frac{11}{4}-p-q,-p,1-q,\frac{15}{4}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\right)\right)$$

- **Problem 997: Result more than twice size of optimal antiderivative.**

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (e x)^{3/2} \, dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$\frac{2 \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c x^2}\right)^{-q} (e x)^{5/2} \text{AppellF1}\left[-\frac{5}{4},-p,-q,-\frac{1}{4},-\frac{b}{a x^2},-\frac{d}{c x^2}\right]}{5 e}$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-9+4\,p+4\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,(e\,x)^{3/2}\,\text{AppellF1}\left[\frac{5}{4}-p-q,\,-p,\,-q,\,\frac{9}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-5+4\,p+4\,q)\,\left(b\,d\,(9-4\,p-4\,q)\,\text{AppellF1}\left[\frac{5}{4}-p-q,\,-p,\,-q,\,\frac{9}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{9}{4}-p-q,\,1-p,\,-q,\,\frac{13}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{9}{4}-p-q,\,-p,\,1-q,\,\frac{13}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

- **Problem 998: Result more than twice size of optimal antiderivative.**

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\sqrt{e\,x}\,dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$\frac{2\,\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}(e\,x)^{3/2}\,\text{AppellF1}\left[-\frac{3}{4},\,-p,\,-q,\,\frac{1}{4},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]}{3\,e}$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-7+4\,p+4\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,\sqrt{e\,x}\,\text{AppellF1}\left[\frac{3}{4}-p-q,\,-p,\,-q,\,\frac{7}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-3+4\,p+4\,q)\,\left(b\,d\,(7-4\,p-4\,q)\,\text{AppellF1}\left[\frac{3}{4}-p-q,\,-p,\,-q,\,\frac{7}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{7}{4}-p-q,\,1-p,\,-q,\,\frac{11}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{7}{4}-p-q,\,-p,\,1-q,\,\frac{11}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

- **Problem 999: Result more than twice size of optimal antiderivative.**

$$\int\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{e\,x}}\,dx$$

Optimal (type 6, 89 leaves, 4 steps):

$$\frac{2\,\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}\sqrt{e\,x}\,\text{AppellF1}\left[-\frac{1}{4},\,-p,\,-q,\,\frac{3}{4},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]}{e}$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-5+4\,p+4\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,\text{AppellF1}\left[\frac{1}{4}-p-q,\,-p,\,-q,\,\frac{5}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-1+4\,p+4\,q)\,\sqrt{e\,x}\,\left(b\,d\,(5-4\,p-4\,q)\,\text{AppellF1}\left[\frac{1}{4}-p-q,\,-p,\,-q,\,\frac{5}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$\left.\left.4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{5}{4}-p-q,\,1-p,\,-q,\,\frac{9}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{5}{4}-p-q,\,-p,\,1-q,\,\frac{9}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 1000: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{3/2}}\,dx$$

Optimal (type 6, 89 leaves, 4 steps):

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a\,x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c\,x^2}\right)^{-q} \text{AppellF1}\left[\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]}{e\sqrt{e\,x}}$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-3+4\,p+4\,q)\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,\text{AppellF1}\left[-\frac{1}{4}-p-q,-p,-q,\frac{3}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\Big/ \left((1+4\,p+4\,q)\,(e\,x)^{3/2}\left(b\,d\,(3-4\,p-4\,q)\,\text{AppellF1}\left[-\frac{1}{4}-p-q,-p,-q,\frac{3}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+ 4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{3}{4}-p-q,1-p,-q,\frac{7}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{3}{4}-p-q,-p,1-q,\frac{7}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 1001: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{5/2}}\,dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a\,x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c\,x^2}\right)^{-q} \text{AppellF1}\left[\frac{3}{4},-p,-q,\frac{7}{4},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]}{3\,e\,(e\,x)^{3/2}}$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-1+4\,p+4\,q)\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,\text{AppellF1}\left[-\frac{3}{4}-p-q,-p,-q,\frac{1}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\Big/ \left((3+4\,p+4\,q)\,(e\,x)^{5/2}\left(b\,d\,(1-4\,p-4\,q)\,\text{AppellF1}\left[-\frac{3}{4}-p-q,-p,-q,\frac{1}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+ 4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{1}{4}-p-q,1-p,-q,\frac{5}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{1}{4}-p-q,-p,1-q,\frac{5}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 1014: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx$$

Optimal (type 3, 8 leaves, 2 steps):

$2\,\text{ArcCosh}\left[\sqrt{x}\right]$

Result (type 3, 20 leaves):

$$4\,\text{ArcSinh}\left[\frac{\sqrt{-1+\sqrt{x}}}{\sqrt{2}}\right]$$

Problem 1051: Result more than twice size of optimal antiderivative.

$$\int x^{13}\,(b+c\,x)^{13}\,(b+2\,c\,x)\,dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{1}{14}\,x^{14}\,(b+c\,x)^{14}$$

Result (type 1, 172 leaves):

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13}{2}\,b^{12}\,c^2\,x^{16}+26\,b^{11}\,c^3\,x^{17}+\frac{143}{2}\,b^{10}\,c^4\,x^{18}+143\,b^9\,c^5\,x^{19}+\frac{429}{2}\,b^8\,c^6\,x^{20}+\frac{1716}{7}\,b^7\,c^7\,x^{21}+\frac{429}{2}\,b^6\,c^8\,x^{22}+143\,b^5\,c^9\,x^{23}+\frac{143}{2}\,b^4\,c^{10}\,x^{24}+26\,b^3\,c^{11}\,x^{25}+\frac{13}{2}\,b^2\,c^{12}\,x^{26}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

Problem 1052: Result more than twice size of optimal antiderivative.

$$\int x^{27}\,\left(b+c\,x^2\right)^{13}\,\left(b+2\,c\,x^2\right)\,dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{1}{28}\,x^{28}\,\left(b+c\,x^2\right)^{14}$$

Result (type 1, 182 leaves):

$$\frac{b^{14}\,x^{28}}{28}+\frac{1}{2}\,b^{13}\,c\,x^{30}+\frac{13}{4}\,b^{12}\,c^2\,x^{32}+13\,b^{11}\,c^3\,x^{34}+\frac{143}{4}\,b^{10}\,c^4\,x^{36}+\frac{143}{2}\,b^9\,c^5\,x^{38}+\frac{429}{4}\,b^8\,c^6\,x^{40}+\frac{858}{7}\,b^7\,c^7\,x^{42}+\frac{429}{4}\,b^6\,c^8\,x^{44}+\frac{143}{2}\,b^5\,c^9\,x^{46}+\frac{143}{4}\,b^4\,c^{10}\,x^{48}+13\,b^3\,c^{11}\,x^{50}+\frac{13}{4}\,b^2\,c^{12}\,x^{52}+\frac{1}{2}\,b\,c^{13}\,x^{54}+\frac{c^{14}\,x^{56}}{28}$$

Problem 1053: Result more than twice size of optimal antiderivative.

$$\int x^{41}\,\left(b+c\,x^3\right)^{13}\,\left(b+2\,c\,x^3\right)\,dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{1}{42}\,x^{42}\,\left(b+c\,x^3\right)^{14}$$

Result (type 1, 186 leaves):

$$\frac{b^{14} x^{42}}{42} + \frac{1}{3} b^{13} c x^{45} + \frac{13}{6} b^{12} c^2 x^{48} + \frac{26}{3} b^{11} c^3 x^{51} + \frac{143}{6} b^{10} c^4 x^{54} + \frac{143}{3} b^9 c^5 x^{57} + \frac{143}{2} b^8 c^6 x^{60} + \frac{572}{7} b^7 c^7 x^{63} + \frac{143}{2} b^6 c^8 x^{66} + \frac{143}{3} b^5 c^9 x^{69} + \frac{143}{6} b^4 c^{10} x^{72} + \frac{26}{3} b^3 c^{11} x^{75} + \frac{13}{6} b^2 c^{12} x^{78} + \frac{1}{3} b c^{13} x^{81} + \frac{c^{14} x^{84}}{42}$$

- **Problem 1063: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^{-1-7 n} (b + 2 c x^n)}{(b + c x^n)^8} \, dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$-\frac{x^{-7 n}}{7 n (b + c x^n)^7}$$

Result (type 3, 127 leaves):

$$-\frac{1}{7 b^{14} n (b + c x^n)^7} x^{-7 n} \left(b^{14} + 1716 b^7 c^7 x^{7 n} + 12\,012 b^6 c^8 x^{8 n} + 36\,036 b^5 c^9 x^{9 n} + 60\,060 b^4 c^{10} x^{10 n} + 60\,060 b^3 c^{11} x^{11 n} + 36\,036 b^2 c^{12} x^{12 n} + 12\,012 b c^{13} x^{13 n} + 1716 c^{14} x^{14 n}\right)$$

Test results for the 46 problems in "1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r.m"

- **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^m (A + B x^n) (c + d x^n)}{(a + b x^n)^3} \, dx$$

Optimal (type 5, 228 leaves, 3 steps):

$$-\frac{(A b (b c (1 + m - 2 n) - a d (1 + m - n)) - a B (b c (1 + m) - a d (1 + m + n))) (e x)^{1+m}}{2 a^2 b^2 e n^2 (a + b x^n)} + \frac{(A b - a B) (e x)^{1+m} (c + d x^n)}{2 a b e n (a + b x^n)^2} - \frac{1}{2 a^3 b^2 e (1 + m) n^2} (b c (a B (1 + m) - A b (1 + m - 2 n)) (1 + m - n) + a d (1 + m) (A b (1 + m - n) - a B (1 + m + n))) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1 + m}{n}, \frac{1 + m + n}{n}, -\frac{b x^n}{a}\right]$$

Result (type 5, 1153 leaves):

$$\frac{1}{2 a^3 b^2 (1+m) n^2 (a+b x^n)^2} x (e x)^m \Big(a^2 A b^2 c (1+m) n - a^3 b B c (1+m) n - a^3 A b d (1+m) n + a^4 B d (1+m) n - a A b^2 c (1+m) (a+b x^n) + a^2 b B c (1+m) (a+b x^n) + a^2 A b d (1+m) (a+b x^n) - a^3 B d (1+m) (a+b x^n) - a A b^2 c m (1+m) (a+b x^n) + a^2 b B c m (1+m) (a+b x^n) + a^2 A b d m (1+m) (a+b x^n) - a^3 B d m (1+m) (a+b x^n) + 2 a A b^2 c (1+m) n (a+b x^n) - 2 a^3 B d (1+m) n (a+b x^n) + A b^2 c (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - a b B c (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - a A b d (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a^2 B d (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + 2 A b^2 c m (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - 2 a b B c m (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - 2 a A b d m (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + 2 a^2 B d m (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + A b^2 c m^2 (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - a b B c m^2 (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - a A b d m^2 (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a^2 B d m^2 (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - 3 A b^2 c n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a b B c n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a A b d n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a^2 B d n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - 3 A b^2 c m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a b B c m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a A b d m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a^2 B d m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + 2 A b^2 c n^2 (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]\Big)$$

- **Problem 14: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^m (A + B x^n) (c + d x^n)^2}{(a + b x^n)^3} \, dx$$

Optimal (type 5, 322 leaves, 4 steps):

$$\frac{d\,(b\,c\,(1+m)-a\,d\,(1+m+n))\,(A\,b\,(1+m)-a\,B\,(1+m+2\,n))\,(e\,x)^{1+m}}{2\,a^2\,b^3\,e\,(1+m)\,n^2}+\frac{(A\,b-a\,B)\,(e\,x)^{1+m}\,(c+d\,x^n)^2}{2\,a\,b\,e\,n\,(a+b\,x^n)^2}+$$
$$\frac{(b\,c-a\,d)\,(e\,x)^{1+m}\,(c\,(a\,B\,(1+m)-A\,b\,(1+m-2\,n))-d\,(A\,b\,(1+m)-a\,B\,(1+m+2\,n))\,x^n)}{2\,a^2\,b^2\,e\,n^2\,(a+b\,x^n)}+\frac{1}{2\,a^3\,b^3\,e\,(1+m)\,n^2}$$
$$(b\,c\,(a\,B\,(1+m)-A\,b\,(1+m-2\,n))\,(a\,d\,(1+m)-b\,c\,(1+m-n))-a\,d\,(b\,c\,(1+m)-a\,d\,(1+m+n))\,(A\,b\,(1+m)-a\,B\,(1+m+2\,n)))$$
$$(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]$$

Result (type 5, 1924 leaves) :

$$\frac{1}{2\,a^3\,b^3\,(1+m)\,n^2\,(a+b\,x^n)^2}$$
$$x\,(e\,x)^m\,\Big(a^2\,A\,b^3\,c^2\,(1+m)\,n-a^3\,b^2\,B\,c^2\,(1+m)\,n-2\,a^3\,A\,b^2\,c\,d\,(1+m)\,n+2\,a^4\,b\,B\,c\,d\,(1+m)\,n+a^4\,A\,b\,d^2\,(1+m)\,n-a^5\,B\,d^2\,(1+m)\,n-$$
$$a\,A\,b^3\,c^2\,(1+m)\,(a+b\,x^n)+a^2\,b^2\,B\,c^2\,(1+m)\,(a+b\,x^n)+2\,a^2\,A\,b^2\,c\,d\,(1+m)\,(a+b\,x^n)-2\,a^3\,b\,B\,c\,d\,(1+m)\,(a+b\,x^n)-a^3\,A\,b\,d^2\,(1+m)\,(a+b\,x^n)+$$
$$a^4\,B\,d^2\,(1+m)\,(a+b\,x^n)-a\,A\,b^3\,c^2\,m\,(1+m)\,(a+b\,x^n)+a^2\,b^2\,B\,c^2\,m\,(1+m)\,(a+b\,x^n)+2\,a^2\,A\,b^2\,c\,d\,m\,(1+m)\,(a+b\,x^n)-$$
$$2\,a^3\,b\,B\,c\,d\,m\,(1+m)\,(a+b\,x^n)-a^3\,A\,b\,d^2\,m\,(1+m)\,(a+b\,x^n)+a^4\,B\,d^2\,m\,(1+m)\,(a+b\,x^n)+2\,a\,A\,b^3\,c^2\,(1+m)\,n\,(a+b\,x^n)-$$
$$4\,a^3\,b\,B\,c\,d\,(1+m)\,n\,(a+b\,x^n)-2\,a^3\,A\,b\,d^2\,(1+m)\,n\,(a+b\,x^n)+4\,a^4\,B\,d^2\,(1+m)\,n\,(a+b\,x^n)+2\,a^3\,B\,d^2\,n^2\,(a+b\,x^n)^2+$$
$$A\,b^3\,c^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-a\,b^2\,B\,c^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-$$
$$2\,a\,A\,b^2\,c\,d\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+2\,a^2\,b\,B\,c\,d\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$
$$a^2\,A\,b\,d^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-a^3\,B\,d^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$
$$2\,A\,b^3\,c^2\,m\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-2\,a\,b^2\,B\,c^2\,m\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-$$
$$4\,a\,A\,b^2\,c\,d\,m\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$
$$4\,a^2\,b\,B\,c\,d\,m\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$
$$2\,a^2\,A\,b\,d^2\,m\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-2\,a^3\,B\,d^2\,m\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$
$$A\,b^3\,c^2\,m^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-a\,b^2\,B\,c^2\,m^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-$$
$$2\,a\,A\,b^2\,c\,d\,m^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$
$$2\,a^2\,b\,B\,c\,d\,m^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$
$$a^2\,A\,b\,d^2\,m^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-a^3\,B\,d^2\,m^2\,(a+b\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-$$

$$3 A b^3 c^2 n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a b^2 B c^2 n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$2 a A b^2 c d n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$2 a^2 b B c d n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$a^2 A b d^2 n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - 3 a^3 B d^2 n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] -$$
$$3 A b^3 c^2 m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a b^2 B c^2 m n (a+b x^n)^2$$
$$\text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + 2 a A b^2 c d m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$2 a^2 b B c d m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] + a^2 A b d^2 m n (a+b x^n)^2$$
$$\text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - 3 a^3 B d^2 m n (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$2 A b^3 c^2 n^2 (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] - 2 a^3 B d^2 n^2 (a+b x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]\Bigg)\Bigg)$$

- **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^m (A+B x^n)}{(a+b x^n)^3 (c+d x^n)^2}\, dx$$

Optimal (type 5, 567 leaves, 7 steps):

$$\frac{d \left(a B c (b c (1+m) - a d (1+m-6 n)) + A \left(a b c d (1+m-6 n) - b^2 c^2 (1+m-2 n) - 2 a^2 d^2 n\right)\right) (e x)^{1+m}}{2 a^2 c (b c - a d)^3 e n^2 (c + d x^n)} +$$
$$\frac{(A b - a B) (e x)^{1+m}}{2 a (b c - a d) e n (a+b x^n)^2 (c+d x^n)} + \frac{(a B (b c (1+m) - a d (1+m-3 n)) + A b (a d (1+m-5 n) - b c (1+m-2 n))) (e x)^{1+m}}{2 a^2 (b c - a d)^2 e n^2 (a+b x^n) (c+d x^n)} +$$
$$\Big(b \left(a B \left(2 a b c d (1+m) (1+m-3 n) - b^2 c^2 (1+m) (1+m-n) - a^2 d^2 \left(1+m^2+m (2-5 n) - 5 n + 6 n^2\right)\right) +\right.$$
$$\left. A b \left(b^2 c^2 \left(1+m^2+m (2-3 n) - 3 n + 2 n^2\right) - 2 a b c d \left(1+m^2+m (2-5 n) - 5 n + 4 n^2\right) + a^2 d^2 \left(1+m^2+m (2-7 n) - 7 n + 12 n^2\right)\right)\right)$$
$$(e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right]\Big) \Big/ \left(2 a^3 (b c - a d)^4 e (1+m) n^2\right) + \frac{1}{c^2 (b c - a d)^4 e (1+m) n}$$
$$d^2 (b c (A d (1+m-4 n) - B c (1+m-3 n)) + a d (B c (1+m) - A d (1+m-n))) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right]$$

Result (type 5, 2176 leaves):

$$\frac{1}{2 a^3 c^2 (b c - a d)^4 (1+m) n^2 (a+b x^n)^2 (c+d x^n)}$$

$$x\,(e\,x)^m\,\Big(2\,a^3\,c\,d^2\,(b\,c-a\,d)\,(B\,c-A\,d)\,(1+m)\,n\,(a+b\,x^n)^2+a^2\,b\,(A\,b-a\,B)\,c^2\,(b\,c-a\,d)^2\,(1+m)\,n\,(c+d\,x^n)+$$
$$a\,b\,c^2\,(-b\,c+a\,d)\,(1+m)\,(a\,B\,(-b\,c\,(1+m)+a\,d\,(1+m-4\,n))+A\,b\,(-a\,d\,(1+m-6\,n)+b\,c\,(1+m-2\,n)))\,(a+b\,x^n)\,(c+d\,x^n)+$$
$$A\,b^4\,c^4\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-$$
$$a\,b^3\,B\,c^4\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-2\,a\,A\,b^3\,c^3\,d\,(a+b\,x^n)^2\,(c+d\,x^n)$$
$$\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+2\,a^2\,b^2\,B\,c^3\,d\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+$$
$$a^2\,A\,b^2\,c^2\,d^2\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-a^3\,b\,B\,c^2\,d^2\,(a+b\,x^n)^2\,(c+d\,x^n)$$
$$\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+2\,A\,b^4\,c^4\,m\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-$$
$$2\,a\,b^3\,B\,c^4\,m\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-4\,a\,A\,b^3\,c^3\,d\,m\,(a+b\,x^n)^2\,(c+d\,x^n)$$
$$\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+4\,a^2\,b^2\,B\,c^3\,d\,m\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+$$
$$2\,a^2\,A\,b^2\,c^2\,d^2\,m\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-2\,a^3\,b\,B\,c^2\,d^2\,m\,(a+b\,x^n)^2\,(c+d\,x^n)$$
$$\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+A\,b^4\,c^4\,m^2\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-$$
$$a\,b^3\,B\,c^4\,m^2\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-2\,a\,A\,b^3\,c^3\,d\,m^2\,(a+b\,x^n)^2\,(c+d\,x^n)$$
$$\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+2\,a^2\,b^2\,B\,c^3\,d\,m^2\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+$$
$$a^2\,A\,b^2\,c^2\,d^2\,m^2\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-a^3\,b\,B\,c^2\,d^2\,m^2\,(a+b\,x^n)^2\,(c+d\,x^n)$$
$$\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-3\,A\,b^4\,c^4\,n\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+$$
$$a\,b^3\,B\,c^4\,n\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+10\,a\,A\,b^3\,c^3\,d\,n\,(a+b\,x^n)^2\,(c+d\,x^n)$$
$$\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-6\,a^2\,b^2\,B\,c^3\,d\,n\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-$$
$$7\,a^2\,A\,b^2\,c^2\,d^2\,n\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+$$
$$5\,a^3\,b\,B\,c^2\,d^2\,n\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]-$$
$$3\,A\,b^4\,c^4\,m\,n\,(a+b\,x^n)^2\,(c+d\,x^n)\,\text{Hypergeometric2F1}\Big[1,\,\frac{1+m}{n},\,\frac{1+m+n}{n},\,-\frac{b\,x^n}{a}\Big]+$$

$$a b^3 B c^4 m n (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$10 a A b^3 c^3 d m n (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] -$$
$$6 a^2 b^2 B c^3 d m n (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] -$$
$$7 a^2 A b^2 c^2 d^2 m n (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$5 a^3 b B c^2 d^2 m n (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$2 A b^4 c^4 n^2 (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] -$$
$$8 a A b^3 c^3 d n^2 (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$12 a^2 A b^2 c^2 d^2 n^2 (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] -$$
$$6 a^3 b B c^2 d^2 n^2 (a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b x^n}{a}\right] +$$
$$2 a^3 d^2 n (b c (A d (1+m-4 n) - B c (1+m-3 n)) + a d (B c (1+m) + A d (-1-m+n)))$$
$$(a+b x^n)^2 (c+d x^n) \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right]\Bigg)\Bigg)$$

- **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^m (a+b x^n)^2 (A+B x^n)}{(c+d x^n)^3} \, dx$$

Optimal (type 5, 322 leaves, 4 steps):

$$\frac{b (a d (1+m) - b c (1+m+n)) (A d (1+m) - B c (1+m+2 n)) (e x)^{1+m}}{2 c^2 d^3 e (1+m) n^2} - \frac{(B c - A d) (e x)^{1+m} (a+b x^n)^2}{2 c d e n (c+d x^n)^2} -$$
$$\frac{(b c - a d) (e x)^{1+m} (a (B c (1+m) - A d (1+m-2 n)) - b (A d (1+m) - B c (1+m+2 n)) x^n)}{2 c^2 d^2 e n^2 (c+d x^n)} + \frac{1}{2 c^3 d^3 e (1+m) n^2}$$
$$(a d (B c (1+m) - A d (1+m-2 n)) (b c (1+m) - a d (1+m-n)) - b c (a d (1+m) - b c (1+m+n)) (A d (1+m) - B c (1+m+2 n)))$$
$$(e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right]$$

Result (type 5, 1924 leaves):

$$\frac{1}{2 c^3 d^3 (1+m) n^2 (c+d x^n)^2}$$

$$x\,(e\,x)^m\Big(-b^2 B c^5 (1+m)\,n + A b^2 c^4 d\,(1+m)\,n + 2\,a\,b\,B\,c^4 d\,(1+m)\,n - 2\,a\,A\,b\,c^3 d^2 (1+m)\,n - a^2 B c^3 d^2 (1+m)\,n + a^2 A c^2 d^3 (1+m)\,n +$$
$$b^2 B c^4 (1+m)(c+d x^n) - A b^2 c^3 d\,(1+m)(c+d x^n) - 2\,a\,b\,B\,c^3 d\,(1+m)(c+d x^n) + 2\,a\,A\,b\,c^2 d^2 (1+m)(c+d x^n) + a^2 B c^2 d^2 (1+m)(c+d x^n) -$$
$$a^2 A c\,d^3 (1+m)(c+d x^n) + b^2 B c^4 m\,(1+m)(c+d x^n) - A b^2 c^3 d\,m\,(1+m)(c+d x^n) - 2\,a\,b\,B\,c^3 d\,m\,(1+m)(c+d x^n) +$$
$$2\,a\,A\,b\,c^2 d^2 m\,(1+m)(c+d x^n) + a^2 B c^2 d^2 m\,(1+m)(c+d x^n) - a^2 A c\,d^3 m\,(1+m)(c+d x^n) + 4\,b^2 B c^4 (1+m)\,n\,(c+d x^n) -$$
$$2\,A b^2 c^3 d\,(1+m)\,n\,(c+d x^n) - 4\,a\,b\,B\,c^3 d\,(1+m)\,n\,(c+d x^n) + 2\,a^2 A c\,d^3 (1+m)\,n\,(c+d x^n) + 2\,b^2 B c^3 n^2 (c+d x^n)^2 -$$
$$b^2 B c^3 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + A b^2 c^2 d\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] +$$
$$2\,a\,b\,B\,c^2 d\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] - 2\,a\,A\,b\,c\,d^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$a^2 B c\,d^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + a^2 A d^3 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$2\,b^2 B c^3 m\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + 2\,A b^2 c^2 d\,m\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] +$$
$$4\,a\,b\,B\,c^2 d\,m\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$4\,a\,A\,b\,c\,d^2 m\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$2\,a^2 B c\,d^2 m\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + 2\,a^2 A d^3 m\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$b^2 B c^3 m^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + A b^2 c^2 d\,m^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] +$$
$$2\,a\,b\,B\,c^2 d\,m^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$2\,a\,A\,b\,c\,d^2 m^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$a^2 B c\,d^2 m^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + a^2 A d^3 m^2 (c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$3\,b^2 B c^3 n\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + A b^2 c^2 d\,n\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] +$$
$$2\,a\,b\,B\,c^2 d\,n\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] +$$
$$2\,a\,A\,b\,c\,d^2 n\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] +$$
$$a^2 B c\,d^2 n\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] - 3\,a^2 A d^3 n\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] -$$
$$3\,b^2 B c^3 m\,n\,(c+d x^n)^2\,\text{Hypergeometric2F1}\Big[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d x^n}{c}\Big] + A b^2 c^2 d\,m\,n\,(c+d x^n)^2$$

$$\text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + 2 a b B c^2 d m n (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$2 a A b c d^2 m n (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + a^2 B c d^2 m n (c + d x^n)^2$$
$$\text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] - 3 a^2 A d^3 m n (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$
$$2 b^2 B c^3 n^2 (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + 2 a^2 A d^3 n^2 (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right]\Bigg)$$

- **Problem 37: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^m (a + b x^n) (A + B x^n)}{(c + d x^n)^3} \, dx$$

Optimal (type 5, 228 leaves, 3 steps):

$$-\frac{(b c - a d) (e x)^{1+m} (A + B x^n)}{2 c d e n (c + d x^n)^2} - \frac{(a d (A d (1 + m - 2 n) - B c (1 + m - n)) - b c (A d (1 + m) - B c (1 + m + n))) (e x)^{1+m}}{2 c^2 d^2 e n^2 (c + d x^n)} - \frac{1}{2 c^3 d^2 e (1 + m) n^2}$$
$$(A d (b c (1 + m) - a d (1 + m - 2 n)) (1 + m - n) + B c (1 + m) (a d (1 + m - n) - b c (1 + m + n))) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right]$$

Result (type 5, 1153 leaves):

$$\frac{1}{2 c^3 d^2 (1+m) n^2 (c+d x^n)^2} x (e x)^m \Big(b B c^4 (1+m) n - A b c^3 d (1+m) n - a B c^3 d (1+m) n + a A c^2 d^2 (1+m) n - b B c^3 (1+m) (c+d x^n) + A b c^2 d (1+m) (c+d x^n) +$$
$$a B c^2 d (1+m) (c+d x^n) - a A c d^2 (1+m) (c+d x^n) - b B c^3 m (1+m) (c+d x^n) + A b c^2 d m (1+m) (c+d x^n) +$$
$$a B c^2 d m (1+m) (c+d x^n) - a A c d^2 m (1+m) (c+d x^n) - 2 b B c^3 (1+m) n (c+d x^n) + 2 a A c d^2 (1+m) n (c+d x^n) +$$
$$b B c^2 (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] - A b c d (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$
$$a B c d (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + a A d^2 (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$2 b B c^2 m (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] - 2 A b c d m (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$
$$2 a B c d m (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$2 a A d^2 m (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + b B c^2 m^2 (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$
$$A b c d m^2 (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] - a B c d m^2 (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$a A d^2 m^2 (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + b B c^2 n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$A b c d n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + a B c d n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$
$$3 a A d^2 n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + b B c^2 m n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$A b c d m n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + a B c d m n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$
$$3 a A d^2 m n (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] + 2 a A d^2 n^2 (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right]\Big)$$

- **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^m (A+B x^n)}{(a+b x^n)^2 (c+d x^n)^3} \, dx$$

Optimal (type 5, 482 leaves, 7 steps) :

$$\frac{d\,(2\,A\,b\,c-3\,a\,B\,c+a\,A\,d)\,(e\,x)^{1+m}}{2\,a\,c\,(b\,c-a\,d)^2\,e\,n\,(c+d\,x^n)^2}+\frac{(A\,b-a\,B)\,(e\,x)^{1+m}}{a\,(b\,c-a\,d)\,e\,n\,(a+b\,x^n)\,(c+d\,x^n)^2}-$$

$$\frac{d\left(a^2\,d\,(B\,c\,(1+m)-A\,d\,(1+m-2\,n))-a\,b\,c\,(B\,c-A\,d)\,(1+m-6\,n)-2\,A\,b^2\,c^2\,n\right)\,(e\,x)^{1+m}}{2\,a\,c^2\,(b\,c-a\,d)^3\,e\,n^2\,(c+d\,x^n)}+\frac{1}{a^2\,(b\,c-a\,d)^4\,e\,(1+m)\,n}$$

$$b^2\,(a\,B\,(b\,c\,(1+m)-a\,d\,(1+m-3\,n))+A\,b\,(a\,d\,(1+m-4\,n)-b\,c\,(1+m-n)))\,(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]+$$

$$\Big(d\left(b^2\,c^2\,(A\,d\,(1+m-4\,n)-B\,c\,(1+m-2\,n))\,(1+m-3\,n)-\right.$$
$$a^2\,d^2\,(B\,c\,(1+m)-A\,d\,(1+m-2\,n))\,(1+m-n)+2\,a\,b\,c\,d\left(B\,c\,(1+m)\,(1+m-3\,n)-A\,d\left(1+m^2+m\,(2-5\,n)-5\,n+4\,n^2\right)\right)\Big)$$
$$(e\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]\Big)\Big/\left(2\,c^3\,(b\,c-a\,d)^4\,e\,(1+m)\,n^2\right)$$

Result (type 5, 2178 leaves) :

$$\frac{1}{2\,a^2\,c^3\,(b\,c-a\,d)^4\,(1+m)\,n^2\,(a+b\,x^n)\,(c+d\,x^n)^2}\,x\,(e\,x)^m\,\Big(-a^2\,c^2\,d\,(b\,c-a\,d)^2\,(B\,c-A\,d)\,(1+m)\,n\,(a+b\,x^n)+$$
$$a^2\,c\,d\,(-b\,c+a\,d)\,(1+m)\,(b\,c\,(A\,d\,(1+m-6\,n)-B\,c\,(1+m-4\,n))+a\,d\,(B\,c\,(1+m)-A\,d\,(1+m-2\,n)))\,(a+b\,x^n)\,(c+d\,x^n)+$$
$$2\,a\,b^2\,(-A\,b+a\,B)\,c^3\,(-b\,c+a\,d)\,(1+m)\,n\,(c+d\,x^n)^2+2\,b^2\,c^3\,(a\,B\,(b\,c\,(1+m)-a\,d\,(1+m-3\,n))+A\,b\,(a\,d\,(1+m-4\,n)-b\,c\,(1+m-n)))$$
$$n\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{b\,x^n}{a}\right]-$$
$$a^2\,b^2\,B\,c^3\,d\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]+a^2\,A\,b^2\,c^2\,d^2\,(a+b\,x^n)\,(c+d\,x^n)^2$$
$$\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]+2\,a^3\,b\,B\,c^2\,d^2\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]-$$
$$2\,a^3\,A\,b\,c\,d^3\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]-a^4\,B\,c\,d^3\,(a+b\,x^n)\,(c+d\,x^n)^2$$
$$\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]+a^4\,A\,d^4\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]-$$
$$2\,a^2\,b^2\,B\,c^3\,d\,m\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]+$$
$$2\,a^2\,A\,b^2\,c^2\,d^2\,m\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]+$$
$$4\,a^3\,b\,B\,c^2\,d^2\,m\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]-$$
$$4\,a^3\,A\,b\,c\,d^3\,m\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]-$$
$$2\,a^4\,B\,c\,d^3\,m\,(a+b\,x^n)\,(c+d\,x^n)^2\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{d\,x^n}{c}\right]+$$

$$2 a^4 A d^4 m (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$a^2 b^2 B c^3 d m^2 (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$a^2 A b^2 c^2 d^2 m^2 (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$2 a^3 b B c^2 d^2 m^2 (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$2 a^3 A b c d^3 m^2 (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$a^4 B c d^3 m^2 (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$a^4 A d^4 m^2 (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$5 a^2 b^2 B c^3 d n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$7 a^2 A b^2 c^2 d^2 n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$6 a^3 b B c^2 d^2 n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$10 a^3 A b c d^3 n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$a^4 B c d^3 n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$3 a^4 A d^4 n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$5 a^2 b^2 B c^3 d m n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$7 a^2 A b^2 c^2 d^2 m n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$6 a^3 b B c^2 d^2 m n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$10 a^3 A b c d^3 m n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$

$$a^4 B c d^3 m n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$3 a^4 A d^4 m n (a + b x^n) (c + d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$

$$6 a^2 b^2 B c^3 d n^2 (a+b x^n) (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$12 a^2 A b^2 c^2 d^2 n^2 (a+b x^n) (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] -$$
$$8 a^3 A b c d^3 n^2 (a+b x^n) (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right] +$$
$$2 a^4 A d^4 n^2 (a+b x^n) (c+d x^n)^2 \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{d x^n}{c}\right]\Bigg)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int (e x)^m (a+b x^n)^p (A+B x^n) (c+d x^n)^q \, dx$$

Optimal (type 6, 211 leaves, 7 steps):

$$\frac{A (e x)^{1+m} (a+b x^n)^p \left(1+\frac{b x^n}{a}\right)^{-p} (c+d x^n)^q \left(1+\frac{d x^n}{c}\right)^{-q} \text{AppellF1}\left[\frac{1+m}{n}, -p, -q, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]}{e (1+m)} +$$
$$\frac{B x^{1+n} (e x)^m (a+b x^n)^p \left(1+\frac{b x^n}{a}\right)^{-p} (c+d x^n)^q \left(1+\frac{d x^n}{c}\right)^{-q} \text{AppellF1}\left[\frac{1+m+n}{n}, -p, -q, \frac{1+m+2 n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]}{1+m+n}$$

Result (type 6, 458 leaves):

$$\frac{1}{1+m+n} a c x (e x)^m (a+b x^n)^p (c+d x^n)^q \left(\left(A (1+m+n)^2 \text{AppellF1}\left[\frac{1+m}{n}, -p, -q, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right) \Big/\right.$$
$$\left((1+m) \left(a c (1+m+n) \text{AppellF1}\left[\frac{1+m}{n}, -p, -q, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] + n x^n\right.\right.$$
$$\left.\left.\left(b c p \text{AppellF1}\left[\frac{1+m+n}{n}, 1-p, -q, \frac{1+m+2 n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] + a d q \text{AppellF1}\left[\frac{1+m+n}{n}, -p, 1-q, \frac{1+m+2 n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right)\right)\right) +$$
$$\left(B (1+m+2 n) x^n \text{AppellF1}\left[\frac{1+m+n}{n}, -p, -q, \frac{1+m+2 n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right) \Big/$$
$$\left(a c (1+m+2 n) \text{AppellF1}\left[\frac{1+m+n}{n}, -p, -q, \frac{1+m+2 n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] +\right.$$
$$\left.\left. n x^n \left(b c p \text{AppellF1}\left[\frac{1+m+2 n}{n}, 1-p, -q, \frac{1+m+3 n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] + a d q \text{AppellF1}\left[\frac{1+m+2 n}{n}, -p, 1-q, \frac{1+m+3 n}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right)\right)\right)$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^m (a+b x^n)^p (A+B x^n)}{c+d x^n} \, dx$$

Optimal (type 6, 164 leaves, 6 steps):

$$-\frac{(B c-A d)\,(e x)^{1+m}\,(a+b x^n)^p\,\left(1+\frac{b x^n}{a}\right)^{-p}\,\text{AppellF1}\left[\frac{1+m}{n},-p,1,\frac{1+m+n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]}{c\,d\,e\,(1+m)}+\frac{B\,(e x)^{1+m}\,(a+b x^n)^p\,\left(1+\frac{b x^n}{a}\right)^{-p}\,\text{Hypergeometric2F1}\left[\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{b x^n}{a}\right]}{d\,e\,(1+m)}$$

Result (type 6, 438 leaves) :

$$\frac{1}{(1+m+n)\,(c+d x^n)}a\,c\,x\,(e x)^m\,(a+b x^n)^p\left(\left(A\,(1+m+n)^2\,\text{AppellF1}\left[\frac{1+m}{n},-p,1,\frac{1+m+n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right)\Big/\left((1+m)\left(a\,c\,(1+m+n)\,\text{AppellF1}\left[\frac{1+m}{n},-p,1,\frac{1+m+n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]+n\,x^n\left(b\,c\,p\,\text{AppellF1}\left[\frac{1+m+n}{n},1-p,1,\frac{1+m+2n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]-a\,d\,\text{AppellF1}\left[\frac{1+m+n}{n},-p,2,\frac{1+m+2n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right)\right)\right)+\left(B\,(1+m+2n)\,x^n\,\text{AppellF1}\left[\frac{1+m+n}{n},-p,1,\frac{1+m+2n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right)\Big/\left(a\,c\,(1+m+2n)\,\text{AppellF1}\left[\frac{1+m+n}{n},-p,1,\frac{1+m+2n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]+n\,x^n\left(b\,c\,p\,\text{AppellF1}\left[\frac{1+m+2n}{n},1-p,1,\frac{1+m+3n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]-a\,d\,\text{AppellF1}\left[\frac{1+m+2n}{n},-p,2,\frac{1+m+3n}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right)\right)\right)$$

Test results for the 594 problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

- **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \frac{a^{1/3}\,(-b)^{1/3}\,B-(-b)^{2/3}\,B\,x}{a+b\,x^3}\,dx$$

Optimal (type 3, 41 leaves, 3 steps) :

$$\frac{2\,B\,\text{ArcTan}\left[\frac{a^{1/3}+2\,(-b)^{1/3}\,x}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{1/3}}$$

Result (type 3, 129 leaves) :

$$\frac{1}{6\,a^{1/3}\,b^{2/3}}(-b)^{1/3}\,B\left(2\sqrt{3}\,\left((-b)^{1/3}-b^{1/3}\right)\,\text{ArcTan}\left[\frac{1-\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]+\left((-b)^{1/3}+b^{1/3}\right)\,\left(2\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]-\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]\right)\right)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{(-a)^{2/3} C + 2 C x^2}{a - 8 x^3} \, dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{C \operatorname{ArcTan}\left[\frac{1-\frac{4 x}{(-a)^{1/3}}}{\sqrt{3}}\right]}{2 \sqrt{3}} - \frac{1}{4} C \operatorname{Log}\left[(-a)^{1/3} + 2 x\right]$$

Result (type 3, 106 leaves):

$$\frac{1}{12 a^{2/3}} C \left(2 \sqrt{3} (-a)^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{4 x}{a^{1/3}}}{\sqrt{3}}\right] - 2 (-a)^{2/3} \operatorname{Log}\left[a^{1/3} - 2 x\right] + (-a)^{2/3} \operatorname{Log}\left[a^{2/3} + 2 a^{1/3} x + 4 x^2\right] - a^{2/3} \operatorname{Log}\left[-a + 8 x^3\right]\right)$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{2 \left(\frac{a}{b}\right)^{2/3} C + C x^2}{a + b x^3} \, dx$$

Optimal (type 3, 50 leaves, 4 steps):

$$-\frac{2 C \operatorname{ArcTan}\left[\frac{1-\frac{2 x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b} + \frac{C \operatorname{Log}\left[\left(\frac{a}{b}\right)^{1/3} + x\right]}{b}$$

Result (type 3, 146 leaves):

$$\frac{1}{3 a^{2/3} b} C \left(-2 \sqrt{3} \left(\frac{a}{b}\right)^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{1-\frac{2 b^{1/3} x}{a^{1/3}}}{\sqrt{3}}\right] + 2 \left(\frac{a}{b}\right)^{2/3} b^{2/3} \operatorname{Log}\left[a^{1/3} + b^{1/3} x\right] - \left(\frac{a}{b}\right)^{2/3} b^{2/3} \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2\right] + a^{2/3} \operatorname{Log}\left[a + b x^3\right]\right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{2 \left(-\frac{a}{b}\right)^{2/3} C + C x^2}{a - b x^3} \, dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{2 C \operatorname{ArcTan}\left[\frac{1-\frac{2 x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b} - \frac{C \operatorname{Log}\left[\left(-\frac{a}{b}\right)^{1/3} + x\right]}{b}$$

Result (type 3, 150 leaves):

$$\frac{1}{3 a^{2/3} b}C \left(2 \sqrt{3} \left(-\frac{a}{b}\right)^{2/3} b^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{a^{1/3}}}{\sqrt{3}}\right] - 2 \left(-\frac{a}{b}\right)^{2/3} b^{2/3} \text{Log}\left[a^{1/3} - b^{1/3} x\right] + \left(-\frac{a}{b}\right)^{2/3} b^{2/3} \text{Log}\left[a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2\right] - a^{2/3} \text{Log}\left[a - b x^3\right]\right)$$

- **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \frac{2 \left(-\frac{a}{b}\right)^{2/3} C + C x^2}{a + b x^3} \, dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$-\frac{2 C \text{ArcTan}\left[\frac{1+\frac{2 x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b} + \frac{C \text{Log}\left[\left(-\frac{a}{b}\right)^{1/3} - x\right]}{b}$$

Result (type 3, 149 leaves):

$$\frac{1}{3 a^{2/3} b}C \left(-2 \sqrt{3} \left(-\frac{a}{b}\right)^{2/3} b^{2/3} \text{ArcTan}\left[\frac{1-\frac{2 b^{1/3} x}{a^{1/3}}}{\sqrt{3}}\right] + 2 \left(-\frac{a}{b}\right)^{2/3} b^{2/3} \text{Log}\left[a^{1/3} + b^{1/3} x\right] - \left(-\frac{a}{b}\right)^{2/3} b^{2/3} \text{Log}\left[a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2\right] + a^{2/3} \text{Log}\left[a + b x^3\right]\right)$$

- **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \frac{2 \left(\frac{a}{b}\right)^{2/3} C + C x^2}{a - b x^3} \, dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{2 C \text{ArcTan}\left[\frac{1+\frac{2 x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b} - \frac{C \text{Log}\left[\left(\frac{a}{b}\right)^{1/3} - x\right]}{b}$$

Result (type 3, 147 leaves):

$$\frac{1}{3 a^{2/3} b}C \left(2 \sqrt{3} \left(\frac{a}{b}\right)^{2/3} b^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{a^{1/3}}}{\sqrt{3}}\right] - 2 \left(\frac{a}{b}\right)^{2/3} b^{2/3} \text{Log}\left[a^{1/3} - b^{1/3} x\right] + \left(\frac{a}{b}\right)^{2/3} b^{2/3} \text{Log}\left[a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2\right] - a^{2/3} \text{Log}\left[a - b x^3\right]\right)$$

- **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \frac{a^{1/3} (-b)^{1/3} B - 2 a^{2/3} C - (-b)^{2/3} B x - (-b)^{2/3} C x^2}{a + b x^3} \, dx$$

Optimal (type 3, 88 leaves, 4 steps):

$$\frac{2 \left(b B + a^{1/3} (-b)^{2/3} C\right) \text{ArcTan}\left[\frac{a^{1/3}+2 (-b)^{1/3} x}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3} b} + \frac{C \text{Log}\left[a^{1/3} - (-b)^{1/3} x\right]}{(-b)^{1/3}}$$

Result (type 3, 238 leaves):

$$\frac{1}{6\, a^{1/3}\, b}\left(2\sqrt{3}\, b^{1/3}\left(\left((-b)^{2/3}-\left(-b^2\right)^{1/3}\right)B+2\, a^{1/3}\, b^{1/3}\, C\right)\text{ArcTan}\left[\frac{1-\frac{2\, b^{1/3}\, x}{a^{1/3}}}{\sqrt{3}}\right]+1\Big/\left(-b^2\right)^{1/3}\left(-2\, b\left(\left(-(-b)^{2/3}+b^{2/3}\right)B+2\, a^{1/3}\,(-b)^{1/3}\, C\right)\text{Log}\left[a^{1/3}+b^{1/3}\, x\right]+\left((-b)^{5/3}\, B+b^{5/3}\, B+2\, a^{1/3}\,(-b)^{1/3}\, b\, C\right)\text{Log}\left[a^{2/3}-a^{1/3}\, b^{1/3}\, x+b^{2/3}\, x^2\right]-2\, a^{1/3}\,(-b)^{2/3}\left(-b^2\right)^{1/3}\, C\,\text{Log}\left[a+b\, x^3\right]\right)\right)$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{a}{b}\right)^{1/3} B+2\left(\frac{a}{b}\right)^{2/3} C+B\, x+C\, x^2}{a+b\, x^3}\, dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B+\left(\frac{a}{b}\right)^{1/3} C\right)\text{ArcTan}\left[\frac{1-\frac{2x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a}+\frac{C\,\text{Log}\left[\left(\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 247 leaves):

$$\frac{1}{6\, a\, b}\left(2\sqrt{3}\, a^{1/3}\, b^{1/3}\left(a^{1/3}\, B+\left(\frac{a}{b}\right)^{1/3} b^{1/3}\left(B+2\left(\frac{a}{b}\right)^{1/3} C\right)\right)\text{ArcTan}\left[\frac{-a^{1/3}+2\, b^{1/3}\, x}{\sqrt{3}\, a^{1/3}}\right]+2\, b^{1/3}\left(-a^{2/3}\, B+a^{1/3}\left(\frac{a}{b}\right)^{1/3} b^{1/3}\left(B+2\left(\frac{a}{b}\right)^{1/3} C\right)\right)\text{Log}\left[a^{1/3}+b^{1/3}\, x\right]+b^{1/3}\left(a^{2/3}\, B-a^{1/3}\left(\frac{a}{b}\right)^{1/3} b^{1/3}\left(B+2\left(\frac{a}{b}\right)^{1/3} C\right)\right)\text{Log}\left[a^{2/3}-a^{1/3}\, b^{1/3}\, x+b^{2/3}\, x^2\right]+2\, a\, C\,\text{Log}\left[a+b\, x^3\right]\right)$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(-\frac{a}{b}\right)^{1/3} B+2\left(-\frac{a}{b}\right)^{2/3} C+B\, x+C\, x^2}{a-b\, x^3}\, dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$\frac{2\left(B+\left(-\frac{a}{b}\right)^{1/3} C\right)\text{ArcTan}\left[\frac{1-\frac{2x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\left(-\frac{a}{b}\right)^{1/3} b}-\frac{C\,\text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 288 leaves):

$$-\frac{\left(a^{2/3} B-a^{1/3} \left(-\frac{a}{b}\right)^{1/3} b^{1/3} B-2 a^{1/3} \left(-\frac{a}{b}\right)^{2/3} b^{1/3} C\right) \text{ArcTan}\left[\frac{a^{1/3}+2 b^{1/3} x}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a b^{2/3}}-\frac{\left(a^{2/3} B+a^{1/3} \left(-\frac{a}{b}\right)^{1/3} b^{1/3} B+2 a^{1/3} \left(-\frac{a}{b}\right)^{2/3} b^{1/3} C\right) \text{Log}\left[a^{1/3}-b^{1/3} x\right]}{3 a b^{2/3}}-$$

$$\frac{\left(-a^{2/3} B-a^{1/3} \left(-\frac{a}{b}\right)^{1/3} b^{1/3} B-2 a^{1/3} \left(-\frac{a}{b}\right)^{2/3} b^{1/3} C\right) \text{Log}\left[a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2\right]}{6 a b^{2/3}}-\frac{C \text{Log}\left[a-b x^3\right]}{3 b}$$

- **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \frac{-\left(-\frac{a}{b}\right)^{1/3} B+2 \left(-\frac{a}{b}\right)^{2/3} C+B x+C x^2}{a+b x^3} \, dx$$

Optimal (type 3, 78 leaves, 4 steps):

$$\frac{2 \left(B-\left(-\frac{a}{b}\right)^{1/3} C\right) \text{ArcTan}\left[\frac{1+\frac{2 x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} \left(-\frac{a}{b}\right)^{1/3} b}+\frac{C \text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 253 leaves):

$$\frac{1}{6 a b}\left(2 \sqrt{3} a^{1/3} b^{1/3} \left(a^{1/3} B+\left(-\frac{a}{b}\right)^{1/3} b^{1/3} \left(-B+2 \left(-\frac{a}{b}\right)^{1/3} C\right)\right) \text{ArcTan}\left[\frac{-a^{1/3}+2 b^{1/3} x}{\sqrt{3} a^{1/3}}\right]-\right.$$
$$2 b^{1/3} \left(a^{2/3} B+a^{1/3} \left(-\frac{a}{b}\right)^{1/3} b^{1/3} \left(B-2 \left(-\frac{a}{b}\right)^{1/3} C\right)\right) \text{Log}\left[a^{1/3}+b^{1/3} x\right]+$$
$$\left.b^{1/3} \left(a^{2/3} B+a^{1/3} \left(-\frac{a}{b}\right)^{1/3} b^{1/3} \left(B-2 \left(-\frac{a}{b}\right)^{1/3} C\right)\right) \text{Log}\left[a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2\right]+2 a C \text{Log}\left[a+b x^3\right]\right)$$

- **Problem 47: Result more than twice size of optimal antiderivative.**

$$\int \frac{-\left(\frac{a}{b}\right)^{1/3} B+2 \left(\frac{a}{b}\right)^{2/3} C+B x+C x^2}{a-b x^3} \, dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$-\frac{2 \left(\frac{a}{b}\right)^{2/3} \left(B-\left(\frac{a}{b}\right)^{1/3} C\right) \text{ArcTan}\left[\frac{1+\frac{2 x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a}-\frac{C \text{Log}\left[\left(\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 244 leaves):

$$\frac{1}{6ab}\left(-2\sqrt{3}\,a^{1/3}b^{1/3}\left(a^{1/3}B+\left(\frac{a}{b}\right)^{1/3}b^{1/3}\left(B-2\left(\frac{a}{b}\right)^{1/3}C\right)\right)\text{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{a^{1/3}}}{\sqrt{3}}\right]-2b^{1/3}\left(a^{2/3}B+a^{1/3}\left(\frac{a}{b}\right)^{1/3}b^{1/3}\left(-B+2\left(\frac{a}{b}\right)^{1/3}C\right)\right)\text{Log}\left[a^{1/3}-b^{1/3}x\right]+\right.$$
$$\left.b^{1/3}\left(a^{2/3}B+a^{1/3}\left(\frac{a}{b}\right)^{1/3}b^{1/3}\left(-B+2\left(\frac{a}{b}\right)^{1/3}C\right)\right)\text{Log}\left[a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2\right]-2aC\,\text{Log}\left[a-bx^3\right]\right)$$

- **Problem 59: Result unnecessarily involves imaginary or complex numbers.**

$$\int\left(a+bx^3\right)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)dx$$

Optimal (type 4, 585 leaves, 7 steps):

$$\frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+\frac{54a^2\left(1729cx+935dx^2\right)\sqrt{a+bx^3}}{323323}+\frac{30a\left(247cx+187dx^2\right)\left(a+bx^3\right)^{3/2}}{46189}+\frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2}-$$
$$\left(405\times3^{1/4}\sqrt{2-\sqrt{3}}\,a^{10/3}d\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(1729b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)+$$
$$\left(54\times3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(1729b^{1/3}c-935\left(1-\sqrt{3}\right)a^{1/3}d\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(323323b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 349 leaves):

$$\frac{1}{323\,323\,(-b)^{2/3}\sqrt{a+b\,x^3}}\Bigg(2\,(-b)^{2/3}\,x\,\left(a+b\,x^3\right)\,\left(1001\,b^2\,x^6\,(19\,c+17\,d\,x)+7\,a\,b\,x^3\,(9139\,c+7667\,d\,x)+a^2\,(91\,637\,c+61\,897\,d\,x)\right)-$$

$$151\,470\,(-1)^{2/3}\,3^{1/4}\,a^{11/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$54\,i\,3^{3/4}\,a^{10/3}\,\left(1729\,(-b)^{1/3}\,c+935\,a^{1/3}\,d\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\Bigg)$$

- **Problem 60: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{a+b\,x^3}\,\left(a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4\right)\,dx$$

Optimal (type 4, 556 leaves, 6 steps) :

$$\frac{54\,a^2\,d\,\sqrt{a+b\,x^3}}{91\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{18\,a\,\left(91\,c\,x+55\,d\,x^2\right)\sqrt{a+b\,x^3}}{5005}+\frac{2}{143}\left(13\,c\,x+11\,d\,x^2\right)\left(a+b\,x^3\right)^{3/2}-$$

$$\left(27\times3^{1/4}\sqrt{2-\sqrt{3}}\,a^{7/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(91\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\Bigg(18\times3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\,\left(91\,b^{1/3}\,c-55\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)$$

$$\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\Bigg)\Bigg/\left(5005\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 329 leaves) :

$$\frac{1}{5005\,(-b)^{2/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,x\,\left(a+b\,x^3\right)\,\left(1274\,a\,c+880\,a\,d\,x+455\,b\,c\,x^3+385\,b\,d\,x^4\right)-\right.$$

$$2970\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$18\,i\,3^{3/4}\,a^{7/3}\,\left(91\,(-b)^{1/3}\,c+55\,a^{1/3}\,d\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 61: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 525 leaves, 5 steps):

$$\frac{6\,a\,d\,\sqrt{a+b\,x^3}}{7\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{2}{35}\left(7\,c\,x+5\,d\,x^2\right)\sqrt{a+b\,x^3}-$$

$$\left(3\times3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(7\,b^{2/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(2\times3^{3/4}\,\sqrt{2+\sqrt{3}}\,a\,\left(7\,b^{1/3}\,c-5\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/\left(35\,b^{2/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 313 leaves):

$$\frac{1}{35\,(-b)^{2/3}\sqrt{a+b\,x^3}}\Bigg(2\,(-b)^{2/3}\,x\,(7\,c+5\,d\,x)\,\left(a+b\,x^3\right)-$$

$$30\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+2\,i\,3^{3/4}$$

$$a^{4/3}\left(7\,(-b)^{1/3}\,c+5\,a^{1/3}\,d\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\Bigg)$$

- **Problem 62: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 490 leaves, 4 steps) :

$$\frac{2\,d\,\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(2\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,c-\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 221 leaves) :

$$-\frac{1}{3^{1/4}\,(-b)^{2/3}\sqrt{a+b\,x^3}}\,2\,a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\left((-1)^{2/3}\sqrt{3}\,a^{1/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-\right.$$

$$\left.i\left((-b)^{1/3}\,c+a^{1/3}\,d\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 63: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 522 leaves, 5 steps) :

$$\frac{2\,x\,(c+d\,x)}{3\,a\sqrt{a+b\,x^3}}-\frac{2\,d\sqrt{a+b\,x^3}}{3\,a\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{\sqrt{2-\sqrt{3}}\,d\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]}{3^{3/4}\,a^{2/3}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}+$$

$$\left(2\sqrt{2+\sqrt{3}}\left(b^{1/3}\,c+\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3\times3^{1/4}\,a\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 305 leaves) :

$$\frac{1}{9\,a\,(-b)^{2/3}\sqrt{a+b\,x^3}}\left(6\,(-b)^{2/3}\,x\,(c+d\,x)+\right.$$

$$6\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+2\,i\,3^{3/4}$$

$$\left.a^{1/3}\,\left((-b)^{1/3}\,c-a^{1/3}\,d\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 64: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\left(a+b\,x^3\right)^{7/2}}\,dx$$

Optimal (type 4, 554 leaves, 6 steps) :

$$\frac{2\,x\,(c+d\,x)}{9\,a\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,x\,(7\,c+5\,d\,x)}{27\,a^2\,\sqrt{a+b\,x^3}}-\frac{10\,d\,\sqrt{a+b\,x^3}}{27\,a^2\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\frac{5\,\sqrt{2-\sqrt{3}}\;d\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{9\times 3^{3/4}\,a^{5/3}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}}+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\,\left(7\,b^{1/3}\,c+5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(27\times 3^{1/4}\,a^2\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 267 leaves) :

$$\frac{1}{81\,a^2\,(-b)^{2/3}\,\left(a+b\,x^3\right)^{3/2}}\,2\left(3\,(-b)^{2/3}\,\left(2\,a\,x\,(5\,c+4\,d\,x)+b\,x^4\,(7\,c+5\,d\,x)\right)+3^{3/4}\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)\right.$$

$$\left(5\,(-1)^{2/3}\,\sqrt{3}\,a^{2/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left.i\,a^{1/3}\,\left(7\,(-b)^{1/3}\,c-5\,a^{1/3}\,d\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 65: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\left(a+b\,x^3\right)^{9/2}}\,dx$$

Optimal (type 4, 581 leaves, 7 steps) :

$$\frac{2x(c+dx)}{15a(a+bx^3)^{5/2}}+\frac{2x(13c+11dx)}{135a^2(a+bx^3)^{3/2}}+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\frac{11\sqrt{2-\sqrt{3}}\,d\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]}{27\times 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}}+$$

$$\left(2\sqrt{2+\sqrt{3}}\left(91b^{1/3}c+55\left(1-\sqrt{3}\right)a^{1/3}d\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(405\times 3^{1/4}a^3b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 287 leaves) :

$$\frac{1}{1215a^3(-b)^{2/3}\left(a+bx^3\right)^{5/2}}$$

$$2\left(3(-b)^{2/3}\left(13abx^4(17c+11dx)+b^2x^7(91c+55dx)+a^2x(157c+115dx)\right)+3^{3/4}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left(a+bx^3\right)^2\left(55(-1)^{2/3}\sqrt{3}\,a^{2/3}d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left.i\,a^{1/3}\left(91(-b)^{1/3}c-55a^{1/3}d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

- **Problem 66: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{c + d x + e x^2 + f x^3 + g x^4}{\sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 590 leaves, 7 steps):

$$\frac{2 e \sqrt{a + b x^3}}{3 b} + \frac{2 f x \sqrt{a + b x^3}}{5 b} + \frac{2 g x^2 \sqrt{a + b x^3}}{7 b} + \frac{2 (7 b d - 4 a g) \sqrt{a + b x^3}}{7 b^{5/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} -$$

$$\left(3^{1/4} \sqrt{2 - \sqrt{3}} \, a^{1/3} (7 b d - 4 a g) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(7 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) +$$

$$\left(2 \sqrt{2 + \sqrt{3}} \left(7 b^{1/3} (5 b c - 2 a f) - 5 \left(1 - \sqrt{3}\right) a^{1/3} (7 b d - 4 a g)\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(35 \times 3^{1/4} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 357 leaves):

$$-\frac{1}{105\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\,(35\,e+3\,x\,(7\,f+5\,g\,x))-30\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,(7\,b\,d-4\,a\,g)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+2\,i\,3^{3/4}\,a^{1/3}\,\left(35\,b\,\left((-b)^{1/3}\,c+a^{1/3}\,d\right)-2\,a\,\left(7\,(-b)^{1/3}\,f+10\,a^{1/3}\,g\right)\right)$$

$$\left.\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 67: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2+f\,x^3+g\,x^4}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 594 leaves, 6 steps) :

$$\frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2e\sqrt{a+bx^3}}{3ab}-\frac{2(bd-4ag)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\left(\sqrt{2-\sqrt{3}}\,(bd-4ag)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)+$$

$$\left(2\sqrt{2+\sqrt{3}}\left(b^{1/3}(bc+2af)+\left(1-\sqrt{3}\right)a^{1/3}(bd-4ag)\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(3\times 3^{1/4}ab^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 354 leaves):

$$-\frac{1}{9a(-b)^{5/3}\sqrt{a+bx^3}}$$

$$\left(6(-b)^{2/3}(bx(c+dx)-a(e+x(f+gx)))+6(-1)^{2/3}3^{1/4}a^{2/3}(bd-4ag)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+2i\,3^{3/4}a^{1/3}\left((-b)^{1/3}bc-a^{1/3}bd+2a(-b)^{1/3}f+4a^{4/3}g\right)$$

$$\left.\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c + d x + e x^2 + f x^3 + g x^4}{\left(a + b x^3\right)^{5/2}} \, dx$$

Optimal (type 4, 628 leaves, 5 steps):

$$\frac{2 x \left(b c - a f + (b d - a g) x + b e x^2\right)}{9 a b \left(a + b x^3\right)^{3/2}} - \frac{2 (5 b d + 4 a g) \sqrt{a + b x^3}}{27 a^2 b^{5/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} - \frac{2 (3 a e - x (7 b c + 2 a f + (5 b d + 4 a g) x))}{27 a^2 b \sqrt{a + b x^3}} +$$

$$\left(\sqrt{2 - \sqrt{3}} (5 b d + 4 a g) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(9 \times 3^{3/4} a^{5/3} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) +$$

$$\left(2 \sqrt{2 + \sqrt{3}} \left(b^{1/3} (7 b c + 2 a f) + \left(1 - \sqrt{3}\right) a^{1/3} (5 b d + 4 a g)\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(27 \times 3^{1/4} a^2 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 329 leaves):

$$-\frac{1}{81\,a^2\,(-b)^{5/3}\,\left(a+b\,x^3\right)^{3/2}}\,2\left(-3\,(-b)^{2/3}\,\left(-x\,(7\,b\,c+2\,a\,f+5\,b\,d\,x+4\,a\,g\,x)\,\left(a+b\,x^3\right)+3\,a\,(-b\,x\,(c+d\,x)+a\,(e+x\,(f+g\,x)))\right)+\mathrm{i}\,3^{3/4}\,a^{1/3}\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)\left((-1)^{1/6}\,\sqrt{3}\,a^{1/3}\,(5\,b\,d+4\,a\,g)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left.\left(7\,(-b)^{1/3}\,b\,c-5\,a^{1/3}\,b\,d+2\,a\,(-b)^{1/3}\,f-4\,a^{4/3}\,g\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

- **Problem 69: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2+f\,x^3+g\,x^4}{\left(a+b\,x^3\right)^{7/2}}\,dx$$

Optimal (type 4, 676 leaves, 6 steps) :

$$\frac{2\,x\,\left(b\,c-a\,f+(b\,d-a\,g)\,x+b\,e\,x^2\right)}{15\,a\,b\,\left(a+b\,x^3\right)^{5/2}}+\frac{2\,x\,(7\,(13\,b\,c+2\,a\,f)+5\,(11\,b\,d+4\,a\,g)\,x)}{405\,a^3\,b\,\sqrt{a+b\,x^3}}-$$

$$\frac{2\,(11\,b\,d+4\,a\,g)\,\sqrt{a+b\,x^3}}{81\,a^3\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,(9\,a\,e-x\,(13\,b\,c+2\,a\,f+(11\,b\,d+4\,a\,g)\,x))}{135\,a^2\,b\,\left(a+b\,x^3\right)^{3/2}}+$$

$$\left(\sqrt{2-\sqrt{3}}\,(11\,b\,d+4\,a\,g)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(27\times 3^{3/4}\,a^{8/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\,\left(7\,b^{1/3}\,(13\,b\,c+2\,a\,f)+5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,(11\,b\,d+4\,a\,g)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(405\times 3^{1/4}\,a^3\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 366 leaves) :

$$-\frac{1}{1215\,a^3\,(-b)^{5/3}\,\left(a+b\,x^3\right)^{5/2}}$$

$$2\left(-3\,(-b)^{2/3}\,\left(-3\,a\,x\,(13\,b\,c+2\,a\,f+11\,b\,d\,x+4\,a\,g\,x)\,\left(a+b\,x^3\right)-x\,(91\,b\,c+14\,a\,f+55\,b\,d\,x+20\,a\,g\,x)\,\left(a+b\,x^3\right)^2+27\,a^2\right.\right.$$

$$\left.(-b\,x\,(c+d\,x)+a\,(e+x\,(f+g\,x)))\right)+\mathrm{i}\,3^{3/4}\,a^{1/3}\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(a+b\,x^3\right)^2\,\left(5\,(-1)^{1/6}\,\sqrt{3}\,a^{1/3}\,(11\,b\,d+4\,a\,g)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left.\left(91\,(-b)^{1/3}\,b\,c-55\,a^{1/3}\,b\,d+14\,a\,(-b)^{1/3}\,f-20\,a^{4/3}\,g\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 230 leaves, 3 steps) :

$$\frac{2\,\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{3^{1/4}\,\sqrt{2-\sqrt{3}}\,(1+x)\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],\,-7-4\,\sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}+$$

$$\frac{4\times 3^{1/4}\,\sqrt{2+\sqrt{3}}\,(1+x)\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],\,-7-4\,\sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Result (type 4, 127 leaves) :

$$\frac{1}{\sqrt{1+x^3}} 3^{1/4} \sqrt{-(-1)^{1/6} \left((-1)^{2/3}+x\right)} \sqrt{1+(-1)^{1/3} x+(-1)^{2/3} x^2} \left(-2 \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{1/6} \left((2-i)+\sqrt{3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}} \, dx$$

Optimal (type 4, 257 leaves, 3 steps) :

$$-\frac{2 \sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{3^{1/4} \sqrt{2-\sqrt{3}} \,(1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}}-$$

$$\frac{4\times 3^{1/4} \sqrt{2+\sqrt{3}} \,(1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}}$$

Result (type 4, 112 leaves) :

$$\frac{1}{\sqrt{1-x^3}} 2\times 3^{1/4} \sqrt{(-1)^{5/6} (-1+x)} \sqrt{1+x+x^2} \left((-1)^{2/3} \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right]+i \,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} \, dx$$

Optimal (type 4, 144 leaves, 1 step) :

$$\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Result (type 4, 110 leaves):

$$\frac{1}{\sqrt{-1+x^3}}2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}$$
$$\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 82: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 135 leaves, 1 step):

$$-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 147 leaves):

$$\frac{1}{\sqrt{-1-x^3}}(1-i)\,(-1)^{1/6}\,3^{1/4}\sqrt{-(-1)^{5/6}+i\,x}\,\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}$$
$$\left((1+i)\,(-1)^{1/6}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]-\left(1+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 83: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 468 leaves, 3 steps):

$$\frac{2\sqrt{a+b x^3}}{b^{1/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)+$$

$$\left(4\times 3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 225 leaves) :

$$\frac{1}{3^{3/4}(-b)^{2/3}\sqrt{a+b x^3}}$$

$$2\,i\,a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\left(-3(-1)^{1/6}b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left(\left(3+\sqrt{3}\right)(-b)^{1/3}+\sqrt{3}\,b^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 84: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\sqrt{a-b x^3}}\,dx$$

Optimal (type 4, 481 leaves, 3 steps) :

$$-\frac{2\sqrt{a-b\,x^3}}{b^{1/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{a-b\,x^3}\right)-$$

$$\left(4\times 3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{a-b\,x^3}\right)$$

Result (type 4, 182 leaves) :

$$\frac{1}{b^{1/3}\sqrt{a-b\,x^3}}2\times 3^{1/4}\,a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{b^{1/3}\,x}{a^{1/3}}+\frac{b^{2/3}\,x^2}{a^{2/3}}}$$

$$\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 85: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\sqrt{-a+b\,x^3}}\,dx$$

Optimal (type 4, 271 leaves, 1 step) :

$$\frac{2\sqrt{-a+b x^3}}{b^{1/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)$$

Result (type 4, 257 leaves) :

$$\frac{1}{3^{3/4}(-b)^{2/3}\sqrt{-a+b x^3}}2(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}(-b)^{1/3}x\right)}{a}}$$

$$\sqrt{1+\frac{(-b)^{1/3}x\left((-a)^{1/3}+(-b)^{1/3}x\right)}{(-a)^{2/3}}}\left(3(-1)^{2/3}(-a)^{1/3}b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.i\left(\left(3+\sqrt{3}\right)a^{1/3}(-b)^{1/3}-\sqrt{3}\,(-a)^{1/3}b^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 86: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\sqrt{-a-b x^3}}\,dx$$

Optimal (type 4, 266 leaves, 1 step) :

$$-\frac{2\sqrt{-a-b\,x^3}}{b^{1/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{-a-b\,x^3}\right)$$

Result (type 4, 227 leaves) :

$$\frac{1}{3^{3/4}\,b^{1/3}\sqrt{-a-b\,x^3}}2\,i\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}\,b^{1/3}\,x\right)}{a}}$$

$$\sqrt{1+\frac{b^{1/3}\,x\left((-a)^{1/3}+b^{1/3}\,x\right)}{(-a)^{2/3}}}\left(-3\,(-1)^{1/6}\,(-a)^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left(\sqrt{3}\,(-a)^{1/3}+\left(3+\sqrt{3}\right)a^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 87: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 520 leaves, 3 steps) :

$$\frac{2\left(\frac{b}{a}\right)^{1/3}\sqrt{a+b x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(\frac{b}{a}\right)^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+b x^3}\right)+\left(2\sqrt{2+\sqrt{3}}\;\left(\left(1+\sqrt{3}\right)b^{1/3}-\left(1-\sqrt{3}\right)a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(3^{1/4}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+b x^3}\right)$$

Result (type 4, 243 leaves) :

$$\frac{1}{3^{3/4}\left(-b\right)^{2/3}\sqrt{a+b x^3}}$$

$$2\,i\,a^{1/3}\sqrt{\frac{\left(-1\right)^{5/6}\left(-a^{1/3}+\left(-b\right)^{1/3}x\right)}{a^{1/3}}}\;\sqrt{1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}x^2}{a^{2/3}}}\left(-3\left(-1\right)^{1/6}a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\left(-b\right)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]+\right.$$

$$\left.\left(\left(3+\sqrt{3}\right)\left(-b\right)^{1/3}+\sqrt{3}\;a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\left(-b\right)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]\right)$$

- **Problem 88: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{a-b x^3}}\,dx$$

Optimal (type 4, 533 leaves, 3 steps) :

$$-\frac{2\left(\frac{b}{a}\right)^{1/3}\sqrt{a-b x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(\frac{b}{a}\right)^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b x^3}\right)-\left(2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)b^{1/3}-\left(1-\sqrt{3}\right)a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\left(a^{1/3}-b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(3^{1/4}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b x^3}\right)$$

Result (type 4, 232 leaves) :

$$\frac{1}{3^{3/4}b^{2/3}\sqrt{a-b x^3}}$$

$$2a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{b^{1/3}x}{a^{1/3}}+\frac{b^{2/3}x^2}{a^{2/3}}}\left(3(-1)^{2/3}a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.i\left(\left(3+\sqrt{3}\right)b^{1/3}-\sqrt{3}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

■ Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{-a+b x^3}}\,dx$$

Optimal (type 4, 256 leaves, 1 step) :

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+b x^3}}{b\left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x\right)}-\frac{3^{1/4}\sqrt{2+\sqrt{3}}\left(1-\left(\frac{b}{a}\right)^{1/3}x\right)\sqrt{\frac{1+\left(\frac{b}{a}\right)^{1/3}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}\right],-7+4\sqrt{3}\right]}{\left(\frac{b}{a}\right)^{1/3}\sqrt{-\frac{1-\left(\frac{b}{a}\right)^{1/3}x}{\left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\sqrt{-a+b x^3}}$$

Result (type 4, 267 leaves) :

$$\frac{1}{3^{3/4}(-b)^{2/3}\sqrt{-a+b x^3}}2(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}(-b)^{1/3}x\right)}{a}}$$

$$\sqrt{1+\frac{(-b)^{1/3}x\left((-a)^{1/3}+(-b)^{1/3}x\right)}{(-a)^{2/3}}}\left(3(-1)^{2/3}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.i\left(3(-b)^{1/3}+\sqrt{3}(-b)^{1/3}-\sqrt{3}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

■ Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{-a-b x^3}}\,dx$$

Optimal (type 4, 251 leaves, 1 step) :

$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-b x^3}}{b\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\left(1+\left(\frac{b}{a}\right)^{1/3}x\right)\sqrt{\frac{1-\left(\frac{b}{a}\right)^{1/3}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}\right],-7+4\sqrt{3}\right]}{\left(\frac{b}{a}\right)^{1/3}\sqrt{-\frac{1+\left(\frac{b}{a}\right)^{1/3}x}{\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\sqrt{-a-b x^3}}$$

Result (type 4, 245 leaves) :

$$\frac{1}{3^{3/4}\, b^{2/3}\sqrt{-a-b\,x^3}} 2\, i\, (-a)^{1/3} \sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}\, b^{1/3}\, x\right)}{a}}$$

$$\sqrt{1+\frac{b^{1/3}\, x\left((-a)^{1/3}+b^{1/3}\, x\right)}{(-a)^{2/3}}} \left(-3\, (-1)^{1/6}\, (-a)^{1/3} \left(\frac{b}{a}\right)^{1/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, b^{1/3}\, x}{(-a)^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] +\right.$$

$$\left.\left(\left(3+\sqrt{3}\right) b^{1/3}+\sqrt{3}\, (-a)^{1/3} \left(\frac{b}{a}\right)^{1/3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, b^{1/3}\, x}{(-a)^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 91: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\, dx$$

Optimal (type 4, 127 leaves, 1 step):

$$\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{3^{1/4}\sqrt{2-\sqrt{3}}\, (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\, \sqrt{1+x^3}}$$

Result (type 4, 127 leaves):

$$\frac{1}{\sqrt{1+x^3}} 3^{1/4} \sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\, \sqrt{1+(-1)^{1/3}\, x+(-1)^{2/3}\, x^2}$$

$$\left(-2\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}\, (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{1/6}\left((-2-i)+\sqrt{3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}\, (1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 92: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\, dx$$

Optimal (type 4, 142 leaves, 1 step):

$$-\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Result (type 4, 112 leaves):

$$\frac{1}{\sqrt{1-x^3}}2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]-i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 93: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 264 leaves, 3 steps):

$$\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}+$$

$$\frac{4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Result (type 4, 110 leaves):

$$\frac{1}{\sqrt{-1+x^3}}2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]-i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 247 leaves, 3 steps):

$$-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}-$$
$$\frac{4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 147 leaves):

$$\frac{1}{\sqrt{-1-x^3}}(1+i)\,(-1)^{1/6}\,3^{1/4}\sqrt{-(-1)^{5/6}+i\,x}\,\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}$$
$$\left((1-i)\,(-1)^{1/6}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]-\left(-1+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 126 leaves, 1 step):

$$-\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}+\frac{3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Result (type 4, 129 leaves):

$$\frac{1}{\sqrt{1+x^3}} 3^{1/4} \sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)} \sqrt{1+(-1)^{1/3} x+(-1)^{2/3} x^2}$$

$$\left(2 \,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{1/6}\left((2+i)-\sqrt{3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 96: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}} \, dx$$

Optimal (type 4, 143 leaves, 1 step):

$$\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Result (type 4, 112 leaves):

$$-\frac{1}{\sqrt{1-x^3}} 2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}$$

$$\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right]-i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 97: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}} \, dx$$

Optimal (type 4, 263 leaves, 3 steps):

$$-\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}-$$

$$\frac{4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Result (type 4, 110 leaves) :

$$-\frac{1}{\sqrt{-1+x^3}}2\times 3^{1/4}\sqrt{(-1)^{5/6}\,(-1+x)}\,\sqrt{1+x+x^2}$$

$$\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]-\mathrm{i}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

■ Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 248 leaves, 3 steps) :

$$\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}-\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}+$$

$$\frac{4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Result (type 4, 146 leaves) :

$$\frac{1}{\sqrt{-1-x^3}}(1+i)(-1)^{1/6}3^{1/4}\sqrt{-(-1)^{5/6}+i x}\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}$$
$$\left((-1+i)(-1)^{1/6}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]+\left(-1+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 99: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 256 leaves, 1 step):

$$\frac{2\sqrt{a+b x^3}}{b^{1/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$
$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 225 leaves):

$$\frac{1}{3^{3/4}(-b)^{2/3}\sqrt{a+b x^3}}$$
$$2\,i\,a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\left(-3(-1)^{1/6}b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$
$$\left.\left(\left(-3+\sqrt{3}\right)(-b)^{1/3}+\sqrt{3}\,b^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\sqrt{a-b x^3}} \, dx$$

Optimal (type 4, 263 leaves, 1 step):

$$-\frac{2 \sqrt{a-b x^3}}{b^{1/3} \left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)}+$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}} \, a^{1/3} \left(a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}-b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}} \sqrt{a-b x^3}\right)$$

Result (type 4, 182 leaves):

$$\frac{1}{b^{1/3} \sqrt{a-b x^3}} 2\times 3^{1/4} a^{2/3} \sqrt{\frac{(-1)^{5/6} \left(-a^{1/3}+b^{1/3} x\right)}{a^{1/3}}} \sqrt{1+\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}}$$

$$\left((-1)^{2/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]-i \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\sqrt{-a+b x^3}} \, dx$$

Optimal (type 4, 497 leaves, 3 steps):

$$\frac{2\sqrt{-a+b x^3}}{b^{1/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)+$$

$$\left(4\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)$$

Result (type 4, 257 leaves):

$$\frac{1}{3^{3/4}(-b)^{2/3}\sqrt{-a+b x^3}}2(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}(-b)^{1/3}x\right)}{a}}$$

$$\sqrt{1+\frac{(-b)^{1/3}x\left((-a)^{1/3}+(-b)^{1/3}x\right)}{(-a)^{2/3}}}\left(3(-1)^{2/3}(-a)^{1/3}b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.i\left(\left(-3+\sqrt{3}\right)a^{1/3}(-b)^{1/3}-\sqrt{3}\,(-a)^{1/3}b^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 102: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\sqrt{-a-b x^3}}\,dx$$

Optimal (type 4, 488 leaves, 3 steps):

$$-\frac{2\sqrt{-a-b\,x^3}}{b^{1/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{-a-b\,x^3}\right)-$$

$$\left(4\times3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{-a-b\,x^3}\right)$$

Result (type 4, 227 leaves):

$$\frac{1}{3^{3/4}\,b^{1/3}\sqrt{-a-b\,x^3}}2\,i\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}\,b^{1/3}\,x\right)}{a}}$$

$$\sqrt{1+\frac{b^{1/3}\,x\left((-a)^{1/3}+b^{1/3}\,x\right)}{(-a)^{2/3}}}\left(-3\,(-1)^{1/6}\,(-a)^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left(\sqrt{3}\,(-a)^{1/3}+\left(-3+\sqrt{3}\right)a^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 103: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x}{\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 241 leaves, 1 step):

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+b x^3}}{b\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x\right)}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\left(1+\left(\frac{b}{a}\right)^{1/3} x\right)\sqrt{\frac{1-\left(\frac{b}{a}\right)^{1/3} x+\left(\frac{b}{a}\right)^{2/3} x^2}{\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x}{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x}\right],-7-4\sqrt{3}\right]}{\left(\frac{b}{a}\right)^{1/3}\sqrt{\frac{1+\left(\frac{b}{a}\right)^{1/3} x}{\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x\right)^2}}\ \sqrt{a+b x^3}}$$

Result (type 4, 243 leaves):

$$\frac{1}{3^{3/4}(-b)^{2/3}\sqrt{a+b x^3}}$$

$$2\,i\,a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}\ \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \left(-3(-1)^{1/6} a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left(\left(-3+\sqrt{3}\right)(-b)^{1/3}+\sqrt{3}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 104: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x}{\sqrt{a-b x^3}}\,dx$$

Optimal (type 4, 248 leaves, 1 step):

$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-b x^3}}{b\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x\right)}+\frac{3^{1/4}\sqrt{2-\sqrt{3}}\left(1-\left(\frac{b}{a}\right)^{1/3} x\right)\sqrt{\frac{1+\left(\frac{b}{a}\right)^{1/3} x+\left(\frac{b}{a}\right)^{2/3} x^2}{\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x}{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x}\right],-7-4\sqrt{3}\right]}{\left(\frac{b}{a}\right)^{1/3}\sqrt{\frac{1-\left(\frac{b}{a}\right)^{1/3} x}{\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x\right)^2}}\ \sqrt{a-b x^3}}$$

Result (type 4, 232 leaves) :

$$\frac{1}{3^{3/4} b^{2/3}\sqrt{a-b x^3}}$$

$$2 a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3} x\right)}{a^{1/3}}}\ \sqrt{1+\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}}\left(3(-1)^{2/3} a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.i\left(\left(-3+\sqrt{3}\right) b^{1/3}-\sqrt{3}\ a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 105: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x}{\sqrt{-a+b x^3}}\,dx$$

Optimal (type 4, 549 leaves, 3 steps) :

$$\frac{2\left(\frac{b}{a}\right)^{1/3}\sqrt{-a+b x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(\frac{b}{a}\right)^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)-\left(2\sqrt{2-\sqrt{3}}\;\left(\left(1-\sqrt{3}\right)b^{1/3}-\left(1+\sqrt{3}\right)a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(3^{1/4}b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)$$

Result (type 4, 267 leaves):

$$\frac{1}{3^{3/4}(-b)^{2/3}\sqrt{-a+b x^3}}2(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}(-b)^{1/3}x\right)}{a}}$$

$$\sqrt{1+\frac{(-b)^{1/3}x\left((-a)^{1/3}+(-b)^{1/3}x\right)}{(-a)^{2/3}}}\left(3(-1)^{2/3}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.i\left(-3(-b)^{1/3}+\sqrt{3}(-b)^{1/3}-\sqrt{3}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 106: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{-a-b x^3}}\,dx$$

Optimal (type 4, 540 leaves, 3 steps):

$$-\frac{2\left(\frac{b}{a}\right)^{1/3}\sqrt{-a-b x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{-a-b x^3}\right)+\left(2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)b^{1/3}-\left(1+\sqrt{3}\right)a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(3^{1/4}b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{-a-b x^3}\right)$$

Result (type 4, 245 leaves) :

$$\frac{1}{3^{3/4}b^{2/3}\sqrt{-a-b x^3}}2\,i\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}b^{1/3}x\right)}{a}}$$

$$\sqrt{1+\frac{b^{1/3}x\left((-a)^{1/3}+b^{1/3}x\right)}{(-a)^{2/3}}}\left(-3\,(-1)^{1/6}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left(\left(-3+\sqrt{3}\right)b^{1/3}+\sqrt{3}\,(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

■ Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d x}{\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 490 leaves, 3 steps) :

$$\frac{2\,d\,\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(2\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,c-\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 221 leaves) :

$$-\frac{1}{3^{1/4}\,(-b)^{2/3}\sqrt{a+b\,x^3}}$$

$$2\,a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\left((-1)^{2/3}\sqrt{3}\;a^{1/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-\right.$$

$$\left.i\left((-b)^{1/3}\,c+a^{1/3}\,d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 108: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x}{\sqrt{a-b\,x^3}}\,dx$$

Optimal (type 4, 503 leaves, 3 steps) :

$$\frac{2 d \sqrt{a-b x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\, a^{1/3} d\left(a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}}\sqrt{a-b x^3}\right)-$$

$$\left(2\sqrt{2+\sqrt{3}}\left(b^{1/3} c+\left(1-\sqrt{3}\right) a^{1/3} d\right)\left(a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4} b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}}\sqrt{a-b x^3}\right)$$

Result (type 4, 208 leaves) :

$$-\frac{1}{3^{1/4} b^{2/3}\sqrt{a-b x^3}} 2\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3} x\right)}{a^{1/3}}}\sqrt{1+\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}}\left((-1)^{2/3}\sqrt{3}\, a^{2/3} d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]-\right.$$

$$\left.i\, a^{1/3}\left(b^{1/3} c+a^{1/3} d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 109: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{c+d x}{\sqrt{-a+b x^3}}\, dx$$

Optimal (type 4, 515 leaves, 3 steps) :

$$-\frac{2\,d\,\sqrt{-a+b\,x^3}}{b^{2/3}\,\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)}+$$

$$\left(3^{1/4}\,\sqrt{2+\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{-a+b\,x^3}\right)-$$

$$\left(2\,\sqrt{2-\sqrt{3}}\;\left(b^{1/3}\,c+\left(1+\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{-a+b\,x^3}\right)$$

Result (type 4, 236 leaves) :

$$-\frac{1}{3^{1/4}\,(-b)^{2/3}\,\sqrt{-a+b\,x^3}}2\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\,\left(a+(-a)^{2/3}\,(-b)^{1/3}\,x\right)}{a}}$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x\,\left((-a)^{1/3}+(-b)^{1/3}\,x\right)}{(-a)^{2/3}}}\left((-1)^{2/3}\,\sqrt{3}\,(-a)^{1/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-\right.$$

$$\left.i\,\left((-b)^{1/3}\,c+(-a)^{1/3}\,d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

■ **Problem 110: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x}{\sqrt{-a-b\,x^3}}\,dx$$

Optimal (type 4, 508 leaves, 3 steps):

$$-\frac{2\,d\,\sqrt{-a-b\,x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(3^{1/4}\,\sqrt{2+\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7+4\,\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\,\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{-a-b\,x^3}\right)+$$

$$\left(2\,\sqrt{2-\sqrt{3}}\;\left(b^{1/3}\,c-\left(1+\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7+4\,\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\,\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{-a-b\,x^3}\right)$$

Result (type 4, 223 leaves):

$$-\frac{1}{3^{1/4}\,b^{2/3}\,\sqrt{-a-b\,x^3}}\,2\,(-a)^{1/3}\,\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}\,b^{1/3}\,x\right)}{a}}$$

$$\sqrt{1+\frac{b^{1/3}\,x\,\left((-a)^{1/3}+b^{1/3}\,x\right)}{(-a)^{2/3}}}\;\left((-1)^{2/3}\,\sqrt{3}\;(-a)^{1/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-\right.$$

$$\left.i\,\left(b^{1/3}\,c+(-a)^{1/3}\,d\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 111: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{c + d x}{\sqrt{1 + x^3}} \, dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$\frac{2 d \sqrt{1 + x^3}}{1 + \sqrt{3} + x} - \frac{3^{1/4} \sqrt{2 - \sqrt{3}} \, d \, (1 + x) \sqrt{\frac{1 - x + x^2}{\left(1 + \sqrt{3} + x\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}\right]}{\sqrt{\frac{1 + x}{\left(1 + \sqrt{3} + x\right)^2}} \sqrt{1 + x^3}} +$$

$$\frac{2 \sqrt{2 + \sqrt{3}} \left(c - \left(1 - \sqrt{3}\right) d\right) (1 + x) \sqrt{\frac{1 - x + x^2}{\left(1 + \sqrt{3} + x\right)^2}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1 + x}{\left(1 + \sqrt{3} + x\right)^2}} \sqrt{1 + x^3}}$$

Result (type 4, 136 leaves):

$$-\frac{1}{3^{3/4} \sqrt{1 + x^3}} 2 \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + x\right)} \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2}$$

$$\left(3 d \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1 + x)}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{1/6} \sqrt{3} \left(-c + (-1)^{2/3} d\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1 + x)}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

- **Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{c + d x}{\sqrt{1 - x^3}} \, dx$$

Optimal (type 4, 271 leaves, 3 steps):

$$\frac{2\,d\,\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{3^{1/4}\sqrt{2-\sqrt{3}}\;d\;(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],\,-7-4\sqrt{3}\right]}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\;\sqrt{1-x^3}}-$$

$$\frac{2\sqrt{2+\sqrt{3}}\;\left(c+d-\sqrt{3}\;d\right)\;(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],\,-7-4\sqrt{3}\right]}{3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\;\sqrt{1-x^3}}$$

Result (type 4, 121 leaves) :

$$\frac{1}{3^{3/4}\sqrt{1-x^3}}2\,i\sqrt{(-1)^{5/6}\,(-1+x)}\;\sqrt{1+x+x^2}$$

$$\left(-3\,(-1)^{1/6}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\sqrt{3}\;(c+d)\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 275 leaves, 3 steps) :

$$-\frac{2\,d\,\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\;d\;(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],\,-7+4\sqrt{3}\right]}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\;\sqrt{-1+x^3}}-$$

$$\frac{2\sqrt{2-\sqrt{3}}\;\left(c+d+\sqrt{3}\;d\right)\;(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],\,-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\;\sqrt{-1+x^3}}$$

Result (type 4, 119 leaves) :

$$\frac{1}{3^{3/4}\sqrt{-1+x^3}}2\,i\sqrt{(-1)^{5/6}(-1+x)}\sqrt{1+x+x^2}$$
$$\left(-3(-1)^{1/6}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+\sqrt{3}\,(c+d)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 114: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 261 leaves, 3 steps):

$$-\frac{2\,d\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{3^{1/4}\sqrt{2+\sqrt{3}}\,d\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+$$
$$\frac{2\sqrt{2-\sqrt{3}}\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]}{3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Result (type 4, 152 leaves):

$$\frac{1}{3^{3/4}\sqrt{-1-x^3}}2\,(-1)^{1/6}\sqrt{-(-1)^{5/6}+i\,x}\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\left(3\,(-1)^{1/6}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$
$$\left.\sqrt{3}\left((-1)^{2/3}c-d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 124: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x}{1+x^4}\,dx$$

Optimal (type 3, 98 leaves, 13 steps):

$$\frac{1}{2}d\,\text{ArcTan}\left[x^2\right]-\frac{c\,\text{ArcTan}\left[1-\sqrt{2}\,x\right]}{2\sqrt{2}}+\frac{c\,\text{ArcTan}\left[1+\sqrt{2}\,x\right]}{2\sqrt{2}}-\frac{c\,\text{Log}\left[1-\sqrt{2}\,x+x^2\right]}{4\sqrt{2}}+\frac{c\,\text{Log}\left[1+\sqrt{2}\,x+x^2\right]}{4\sqrt{2}}$$

Result (type 3, 99 leaves) :

$$\frac{1}{4}\left(-\left((-1)^{1/4}c+i\,d\right)\text{Log}\left[(-1)^{1/4}-x\right]+\left(-(-1)^{3/4}c+i\,d\right)\text{Log}\left[(-1)^{3/4}-x\right]+\left((-1)^{1/4}c-i\,d\right)\text{Log}\left[(-1)^{1/4}+x\right]+\left((-1)^{3/4}c+i\,d\right)\text{Log}\left[(-1)^{3/4}+x\right]\right)$$

- **Problem 161: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{b\,x+d\,x^3}{2+3\,x^4}\,dx$$

Optimal (type 3, 36 leaves, 5 steps) :

$$\frac{b\,\text{ArcTan}\left[\sqrt{\frac{3}{2}}\,x^2\right]}{2\,\sqrt{6}}+\frac{1}{12}\,d\,\text{Log}\left[2+3\,x^4\right]$$

Result (type 3, 65 leaves) :

$$\frac{1}{24}\left(i\,\sqrt{6}\,b+2\,d\right)\text{Log}\left[\sqrt{6}-3\,i\,x^2\right]+\frac{1}{24}\left(-i\,\sqrt{6}\,b+2\,d\right)\text{Log}\left[\sqrt{6}+3\,i\,x^2\right]$$

- **Problem 210: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 121 leaves, 6 steps) :

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\,\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,b^{1/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 107 leaves) :

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\,\sqrt{b}}-\frac{i\,c\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\,x^4}}$$

- **Problem 211: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x}{\sqrt{a-b\,x^4}}\,dx$$

Optimal (type 4, 87 leaves, 7 steps) :

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a-b\,x^4}}\right]}{2\sqrt{b}}+\frac{a^{1/4}\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{b^{1/4}\,\sqrt{a-b\,x^4}}$$

Result (type 4, 106 leaves) :

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a-b\,x^4}}\right]}{2\sqrt{b}}-\frac{i\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{a-b\,x^4}}$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-a+b\,x^4}}\,dx$$

Optimal (type 4, 89 leaves, 7 steps) :

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a+b\,x^4}}\right]}{2\sqrt{b}}+\frac{a^{1/4}\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{b^{1/4}\,\sqrt{-a+b\,x^4}}$$

Result (type 4, 108 leaves) :

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a+b\,x^4}}\right]}{2\sqrt{b}}-\frac{i\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-a+b\,x^4}}$$

Problem 213: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-a-b\,x^4}}\,dx$$

Optimal (type 4, 127 leaves, 6 steps) :

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a-b\,x^4}}\right]}{2\sqrt{b}}+\frac{c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,b^{1/4}\,\sqrt{-a-b\,x^4}}$$

Result (type 4, 113 leaves) :

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a-b\,x^4}}\right]}{2\,\sqrt{b}}-\frac{i\,c\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]}{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;\sqrt{-a-b\,x^4}}$$

- **Problem 214: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 257 leaves, 8 steps):

$$\frac{e\,x\,\sqrt{a+b\,x^4}}{\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\,\sqrt{b}}-\frac{a^{1/4}\,e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{a^{1/4}\,\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 201 leaves):

$$\frac{1}{2\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;\sqrt{b}\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;d\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+\right.$$

$$\left.2\,\sqrt{a}\,e\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]-2\,\left(i\,\sqrt{b}\,c+\sqrt{a}\,e\right)\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

- **Problem 220: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2+f\,x^3+g\,x^4+h\,x^5+i\,x^6}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 385 leaves, 12 steps):

$$\frac{f\sqrt{a+b x^4}}{2 b}+\frac{g x\sqrt{a+b x^4}}{3 b}+\frac{h x^2\sqrt{a+b x^4}}{4 b}+\frac{i x^3\sqrt{a+b x^4}}{5 b}+\frac{(5 b e-3 a i) x\sqrt{a+b x^4}}{5 b^{3/2}\left(\sqrt{a}+\sqrt{b} x^2\right)}+$$

$$\frac{(2 b d-a h)\,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{4 b^{3/2}}-\frac{a^{1/4}(5 b e-3 a i)\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{5 b^{7/4}\sqrt{a+b x^4}}+$$

$$\frac{a^{1/4}\left(15 b e+\frac{5\sqrt{b}(3 b c-a g)}{\sqrt{a}}-9 a i\right)\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{30 b^{7/4}\sqrt{a+b x^4}}$$

Result (type 4, 275 leaves):

$$\frac{1}{60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\sqrt{a+b x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(a+b x^4\right)(30 f+x(20 g+3 x(5 h+4 i x)))+15(2 b d-a h)\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)-\right.$$

$$12\sqrt{a}(-5 b e+3 a i)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$\left.4\left(-15 i b^{3/2} c-15\sqrt{a} b e+5 i a\sqrt{b} g+9 a^{3/2} i\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 221: Result is not expressed in closed-form.**

$$\int\frac{1+x}{1+x^5}\,dx$$

Optimal (type 3, 109 leaves, 3 steps):

$$-\frac{1}{5}(-1)^{1/5}\left(1+(-1)^{1/5}\right)\text{Log}\left[(-1)^{1/5}-x\right]+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\text{Log}\left[-(-1)^{4/5}-x\right]+$$

$$\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\text{Log}\left[(-1)^{2/5}+x\right]-\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\text{Log}\left[-(-1)^{3/5}+x\right]$$

Result (type 7, 51 leaves):

$$\text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\,\&,\ \frac{\text{Log}[x-\#1]}{-1+2\#1-3\#1^2+4\#1^3}\,\&\right]$$

Problem 222: Result is not expressed in closed-form.

$$\int \frac{1-x}{1-x^5}\,dx$$

Optimal (type 3, 109 leaves, 3 steps):

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\text{Log}\left[(-1)^{2/5}-x\right]+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\text{Log}\left[-(-1)^{3/5}-x\right]+\frac{1}{5}(-1)^{1/5}\left(1+(-1)^{1/5}\right)\text{Log}\left[(-1)^{1/5}+x\right]-\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\text{Log}\left[-(-1)^{4/5}+x\right]$$

Result (type 7, 47 leaves):

$$\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\,\&,\ \frac{\text{Log}[x-\#1]}{1+2\,\#1+3\,\#1^2+4\,\#1^3}\,\&\right]$$

Problem 369: Result more than twice size of optimal antiderivative.

$$\int \frac{x\left(-2\left(\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a+b\,x^3}\,dx$$

Optimal (type 3, 50 leaves, 4 steps):

$$\frac{2\,C\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}+\frac{C\,\text{Log}\left[\left(\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 146 leaves):

$$\frac{1}{3\,a^{1/3}\,b}C\left(2\sqrt{3}\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]+2\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]-\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a+b\,x^3\right]\right)$$

Problem 370: Result more than twice size of optimal antiderivative.

$$\int \frac{x\left(-2\left(-\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a-b\,x^3}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{2\,C\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}-\frac{C\,\text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 149 leaves):

$$-\frac{1}{3\,a^{1/3}\,b}\,C\left(-2\sqrt{3}\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]-2\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}-b^{1/3}\,x\right]+\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a-b\,x^3\right]\right)$$

- **Problem 371: Result more than twice size of optimal antiderivative.**

$$\int\frac{x\left(2\left(-\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a+b\,x^3}\,dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{2\,C\,\text{ArcTan}\left[\frac{1+\frac{2\,x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}+\frac{C\,\text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 148 leaves):

$$\frac{1}{3\,a^{1/3}\,b}C\left(-2\sqrt{3}\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]-2\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]+\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a+b\,x^3\right]\right)$$

- **Problem 372: Result more than twice size of optimal antiderivative.**

$$\int\frac{x\left(2\left(\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a-b\,x^3}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{2\,C\,\text{ArcTan}\left[\frac{1+\frac{2\,x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}-\frac{C\,\text{Log}\left[\left(\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 147 leaves):

$$-\frac{1}{3\,a^{1/3}\,b}C\left(2\sqrt{3}\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]+2\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}-b^{1/3}\,x\right]-\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a-b\,x^3\right]\right)$$

- **Problem 430: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3\left(c+d\,x+e\,x^2\right)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 583 leaves, 10 steps):

$$-\frac{4\,a\,e\,\sqrt{a+b\,x^3}}{9\,b^2}+\frac{2\,c\,x\,\sqrt{a+b\,x^3}}{5\,b}+\frac{2\,d\,x^2\,\sqrt{a+b\,x^3}}{7\,b}+\frac{2\,e\,x^3\,\sqrt{a+b\,x^3}}{9\,b}-\frac{8\,a\,d\,\sqrt{a+b\,x^3}}{7\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(4\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(7\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(4\,\sqrt{2+\sqrt{3}}\,a\,\left(7\,b^{1/3}\,c-10\,\left(1-\sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(35\times 3^{1/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 329 leaves) :

$$\frac{1}{315\,(-b)^{8/3}\,\sqrt{a+b\,x^3}}$$

$$\left(2\,(-b)^{2/3}\,\left(a+b\,x^3\right)\,\left(-70\,a\,e+b\,x\,\left(63\,c+5\,x\,\left(9\,d+7\,e\,x\right)\right)\right)+360\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,b\,d\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-12\,i\,3^{3/4}\,a^{4/3}\,b\,\left(7\,(-b)^{1/3}\,c+10\,a^{1/3}\,d\right)$$

$$\left.\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

■ Problem 431: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(c + d x + e x^2\right)}{\sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 560 leaves, 8 steps) :

$$\frac{2 c \sqrt{a + b x^3}}{3 b} + \frac{2 d x \sqrt{a + b x^3}}{5 b} + \frac{2 e x^2 \sqrt{a + b x^3}}{7 b} - \frac{8 a e \sqrt{a + b x^3}}{7 b^{5/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} +$$

$$\left(4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} \; a^{4/3} e \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(7 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) -$$

$$\left(4 \sqrt{2 + \sqrt{3}} \; a \left(7 b^{1/3} d - 10 \left(1 - \sqrt{3}\right) a^{1/3} e\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(35 \times 3^{1/4} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 319 leaves) :

$$-\frac{1}{105 (-b)^{5/3} \sqrt{a+b x^3}}\left(2 (-b)^{2/3} \left(a+b x^3\right) (35 c+3 x (7 d+5 e x))+120 (-1)^{2/3} 3^{1/4} a^{5/3} e \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$
$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]-4 i 3^{3/4} a^{4/3} \left(7 (-b)^{1/3} d+10 a^{1/3} e\right)$$
$$\left.\sqrt{\frac{(-1)^{5/6} \left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 432: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x \left(c+d x+e x^2\right)}{\sqrt{a+b x^3}}\, dx$$

Optimal (type 4, 537 leaves, 6 steps) :

$$\frac{2 d \sqrt{a+b x^3}}{3 b}+\frac{2 e x \sqrt{a+b x^3}}{5 b}+\frac{2 c \sqrt{a+b x^3}}{b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$
$$\left(3^{1/4} \sqrt{2-\sqrt{3}}\ a^{1/3} c \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$
$$\left(b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\left(2 \sqrt{2+\sqrt{3}}\ a^{1/3} \left(5 \left(1-\sqrt{3}\right) b^{2/3} c+2 a^{2/3} e\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(5\times 3^{1/4} b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 314 leaves) :

$$\frac{1}{15\,(-b)^{5/3}\,\sqrt{a+b\,x^3}}\left(-2\,(-b)^{2/3}\,(5\,d+3\,e\,x)\,\left(a+b\,x^3\right)+30\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,b\,c\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$
$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+2\,i\,3^{3/4}\,a^{2/3}\,\left(-5\,b\,c+2\,a^{2/3}\,(-b)^{1/3}\,e\right)$$
$$\left.\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 433: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 509 leaves, 5 steps) :

$$\frac{2\,e\,\sqrt{a+b\,x^3}}{3\,b}+\frac{2\,d\,\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$
$$\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$
$$\left(b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$
$$\left(2\,\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,c-\left(1-\sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$
$$\left(3^{1/4}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 305 leaves) :

$$-\frac{1}{3\,(-b)^{5/3}\sqrt{a+b\,x^3}}\Bigg(2\,(-b)^{2/3}\,e\,\left(a+b\,x^3\right)-$$

$$6\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,b\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+2\,i\,3^{3/4}$$

$$a^{1/3}\,b\,\left((-b)^{1/3}\,c+a^{1/3}\,d\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\Bigg)$$

Problem 434: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2}{x\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 518 leaves, 7 steps) :

$$\frac{2\,e\,\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,d-\left(1-\sqrt{3}\right)a^{1/3}\,e\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 493 leaves) :

$$-\frac{2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-$$

$$\left(2\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(b^{1/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-\left(2\,\sqrt{2}\,a^{1/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/$$

$$\left(b^{2/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\right)$$

- **Problem 435: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2}{x^2\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 547 leaves, 8 steps) :

$$-\frac{c\sqrt{a+b x^3}}{a x}+\frac{b^{1/3} c\sqrt{a+b x^3}}{a\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{2 d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\; b^{1/3} c\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(2 a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)-$$

$$\left(\sqrt{2+\sqrt{3}}\left(\left(1-\sqrt{3}\right) b^{2/3} c-2 a^{2/3} e\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4} a^{2/3} b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 513 leaves) :

$$-\frac{c\sqrt{a+b\,x^3}}{a\,x}-\frac{2\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-$$

$$\left(2\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\right)-\left(\sqrt{2}\,b^{1/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/$$

$$\left(a^{2/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\right)$$

- **Problem 436: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2}{x^3\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 569 leaves, 9 steps) :

$$-\frac{c\sqrt{a+b x^3}}{2 a x^2}-\frac{d\sqrt{a+b x^3}}{a x}+\frac{b^{1/3} d\sqrt{a+b x^3}}{a\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{2 e \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\; b^{1/3} d\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(2 a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)-$$

$$\left(\sqrt{2+\sqrt{3}}\; b^{1/3}\left(b^{1/3} c+2\left(1-\sqrt{3}\right) a^{1/3} d\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(2\times 3^{1/4} a\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 525 leaves) :

$$-\frac{(c+2\,d\,x)\,\sqrt{a+b\,x^3}}{2\,a\,x^2}-\frac{2\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}+$$

$$\left(b^{2/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(2\,a\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-\left(\sqrt{2}\,b^{1/3}\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/$$

$$\left(a^{2/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)$$

- **Problem 437: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^5\,\left(c+d\,x+e\,x^2\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 594 leaves, 8 steps):

$$\frac{2\,x\,\left(a\,d+a\,e\,x-b\,c\,x^{2}\right)}{3\,b^{2}\,\sqrt{a+b\,x^{3}}}+\frac{4\,c\,\sqrt{a+b\,x^{3}}}{3\,b^{2}}+\frac{2\,d\,x\,\sqrt{a+b\,x^{3}}}{5\,b^{2}}+\frac{2\,e\,x^{2}\,\sqrt{a+b\,x^{3}}}{7\,b^{2}}-\frac{80\,a\,e\,\sqrt{a+b\,x^{3}}}{21\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(40\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^{2}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^{2}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{3/4}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^{2}}}\,\sqrt{a+b\,x^{3}}\right)-\left(16\,\sqrt{2+\sqrt{3}}\,a\,\left(14\,b^{1/3}\,d-25\,\left(1-\sqrt{3}\right)\,a^{1/3}\,e\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^{2}}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^{2}}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(105\times 3^{1/4}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^{2}}}\,\sqrt{a+b\,x^{3}}\right)$$

Result (type 4, 334 leaves) :

$$\frac{1}{315\,(-b)^{8/3}\,\sqrt{a+b\,x^{3}}}\left(6\,(-b)^{2/3}\,\left(a\,\left(70\,c+56\,d\,x+50\,e\,x^{2}\right)+b\,x^{3}\,(35\,c+3\,x\,(7\,d+5\,e\,x))\right)+\right.$$

$$1200\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,e\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^{2}}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$16\,i\,3^{3/4}\,a^{4/3}\,\left(14\,(-b)^{1/3}\,d+25\,a^{1/3}\,e\right)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^{2}}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 438: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^{4}\,\left(c+d\,x+e\,x^{2}\right)}{\left(a+b\,x^{3}\right)^{3/2}}\,dx$$

Optimal (type 4, 574 leaves, 7 steps) :

$$\frac{2x\left(a e-b c x-b d x^2\right)}{3 b^2\sqrt{a+b x^3}}+\frac{4 d\sqrt{a+b x^3}}{3 b^2}+\frac{2 e x\sqrt{a+b x^3}}{5 b^2}+\frac{8 c\sqrt{a+b x^3}}{3 b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\left(4\sqrt{2-\sqrt{3}}\ a^{1/3} c\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{3/4} b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \sqrt{a+b x^3}\right)-\left(8\sqrt{2+\sqrt{3}}\ a^{1/3}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+4a^{2/3}e\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(15\times 3^{1/4} b^{7/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \sqrt{a+b x^3}\right)$$

Result (type 4, 330 leaves) :

$$\frac{1}{45(-b)^{8/3}\sqrt{a+b x^3}}$$

$$\left(6(-b)^{2/3}\left(2a(5d+4ex)+b x^2\left(-5c+5dx+3ex^2\right)\right)-120(-1)^{2/3}3^{1/4}a^{2/3}b c\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]-8\,i\,3^{3/4}a^{2/3}\left(-5bc+4a^{2/3}(-b)^{1/3}e\right)$$

$$\left.\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 439: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^3\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 542 leaves, 6 steps) :

$$-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}+\frac{4e\sqrt{a+bx^3}}{3b^2}+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\left(4\sqrt{2-\sqrt{3}}\,a^{1/3}d\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{3/4}b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)+$$

$$\left(4\sqrt{2+\sqrt{3}}\left(b^{1/3}c-2\left(1-\sqrt{3}\right)a^{1/3}d\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3\times 3^{1/4}b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 319 leaves):

$$\frac{1}{9(-b)^{8/3}\sqrt{a+bx^3}}\left(6(-b)^{2/3}\left(2ae+bx\left(-c-dx+ex^2\right)\right)-\right.$$

$$24(-1)^{2/3}3^{1/4}a^{2/3}bd\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+4i\,3^{3/4}$$

$$\left.a^{1/3}b\left((-b)^{1/3}c+2a^{1/3}d\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 440: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 522 leaves, 4 steps):

$$-\frac{2\left(c+d x+e x^2\right)}{3 b \sqrt{a+b x^3}}+\frac{8 e \sqrt{a+b x^3}}{3 b^{5/3}\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(4 \sqrt{2-\sqrt{3}}\, a^{1/3} e\left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(3^{3/4} b^{5/3} \sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(4 \sqrt{2+\sqrt{3}}\left(b^{1/3} d-2\left(1-\sqrt{3}\right) a^{1/3} e\right)\left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(3 \times 3^{1/4} b^{5/3} \sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 305 leaves):

$$\frac{1}{9 (-b)^{5/3} \sqrt{a+b x^3}}\left(6 (-b)^{2/3} (c+x (d+e x))+\right.$$

$$24 (-1)^{2/3} 3^{1/4} a^{2/3} e \sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]-4\, i\, 3^{3/4}$$

$$\left.a^{1/3}\left((-b)^{1/3} d+2 a^{1/3} e\right) \sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 441: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,\left(c+d\,x+e\,x^2\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 561 leaves, 6 steps):

$$-\frac{2\,x\,\left(a\,e-b\,c\,x-b\,d\,x^2\right)}{3\,a\,b\,\sqrt{a+b\,x^3}}-\frac{2\,d\,\sqrt{a+b\,x^3}}{3\,a\,b}-\frac{2\,c\,\sqrt{a+b\,x^3}}{3\,a\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\frac{\sqrt{2-\sqrt{3}}\;c\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{3^{3/4}\,a^{2/3}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}}+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\;\left(b^{2/3}\,\left(c-\sqrt{3}\;c\right)+2\,a^{2/3}\,e\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(3\times 3^{1/4}\,a^{2/3}\,b^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 317 leaves):

$$-\frac{1}{9\,a\,(-b)^{5/3}\,\sqrt{a+b\,x^3}}\left(6\,(-b)^{2/3}\,\left(b\,c\,x^2-a\,(d+e\,x)\right)+6\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,b\,c\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+2\,i\,3^{3/4}\,a^{2/3}\,\left(-b\,c+2\,a^{2/3}\,(-b)^{1/3}\,e\right)$$

$$\left.\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 442: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x+e\,x^2}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 532 leaves, 4 steps):

$$-\frac{2\,d\,\sqrt{a+b\,x^3}}{3\,a\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,\left(a\,e-b\,x\,\left(c+d\,x\right)\right)}{3\,a\,b\,\sqrt{a+b\,x^3}}+$$

$$\frac{\sqrt{2-\sqrt{3}}\;d\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]}{3^{3/4}\,a^{2/3}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}}+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,c+\left(1-\sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(3\times 3^{1/4}\,a\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 314 leaves):

$$-\frac{1}{9\,a\,\left(-b\right)^{5/3}\,\sqrt{a+b\,x^3}}\left(6\,\left(-b\right)^{2/3}\,\left(-a\,e+b\,x\,\left(c+d\,x\right)\right)+\right.$$

$$6\,\left(-1\right)^{2/3}\,3^{1/4}\,a^{2/3}\,b\,d\,\sqrt{\left(-1\right)^{5/6}\,\left(-1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]+2\,i\,3^{3/4}$$

$$\left.a^{1/3}\,b\,\left(\left(-b\right)^{1/3}\,c-a^{1/3}\,d\right)\,\sqrt{\frac{\left(-1\right)^{5/6}\,\left(-a^{1/3}+\left(-b\right)^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]\right)$$

Problem 443: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c + d x + e x^2}{x \left(a + b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 579 leaves, 10 steps):

$$\frac{2 x \left(a d + a e x - b c x^2\right)}{3 a^2 \sqrt{a + b x^3}} + \frac{2 c \sqrt{a + b x^3}}{3 a^2} - \frac{2 e \sqrt{a + b x^3}}{3 a b^{2/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} - \frac{2 c \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 a^{3/2}} +$$

$$\frac{\sqrt{2 - \sqrt{3}}\, e \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3^{3/4} a^{2/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}} +$$

$$\left(2 \sqrt{2 + \sqrt{3}} \left(b^{1/3} d + \left(1 - \sqrt{3}\right) a^{1/3} e\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(3 \times 3^{1/4} a b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 518 leaves):

$$\frac{1}{3a}2\left(\frac{c+x\,(d+e\,x)}{\sqrt{a+b\,x^3}}-\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{\sqrt{a}}-\right.$$

$$\left(d\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{(-1)^{1/3}a^{1/3}-(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\right)+\left(\sqrt{2}\,a^{1/3}e\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}a^{1/3}-(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/$$

$$\left.\left(b^{2/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\right)\right)$$

- **Problem 444: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2}{x^2\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 607 leaves, 11 steps):

$$\frac{2x\left(a e-b c x-b d x^2\right)}{3a^2\sqrt{a+b x^3}}+\frac{2d\sqrt{a+b x^3}}{3a^2}-\frac{c\sqrt{a+b x^3}}{a^2 x}+\frac{5b^{1/3}c\sqrt{a+b x^3}}{3a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{2d\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3a^{3/2}}-$$

$$\left(5\sqrt{2-\sqrt{3}}\;b^{1/3}c\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(2\times 3^{3/4}a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+b x^3}\right)-$$

$$\left(\sqrt{2+\sqrt{3}}\;\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3\times 3^{1/4}a^{5/3}b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+b x^3}\right)$$

Result (type 4, 542 leaves) :

$$\frac{-3\,a\,c-5\,b\,c\,x^3+2\,a\,x\,(d+e\,x)}{3\,a^2\,x\,\sqrt{a+b\,x^3}}-\frac{1}{6\,a^2}\left(4\,\sqrt{a}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+\right.$$

$$\left(4\,a\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(b^{1/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)+\left(10\,\sqrt{2}\,a^{1/3}\,b^{1/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right.$$

$$\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)\right)$$

- **Problem 445: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3\,\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)\,dx$$

Optimal (type 4, 733 leaves, 13 steps):

$$-\frac{4 a^2 e \sqrt{a+b x^3}}{45 b^2}+\frac{6 a (17 b c-8 a f) x \sqrt{a+b x^3}}{935 b^2}+\frac{6 a (19 b d-10 a g) x^2 \sqrt{a+b x^3}}{1729 b^2}+\frac{2 a e x^3 \sqrt{a+b x^3}}{45 b}+\frac{6 a f x^4 \sqrt{a+b x^3}}{187 b}+$$

$$\frac{6 a g x^5 \sqrt{a+b x^3}}{247 b}-\frac{24 a^2 (19 b d-10 a g) \sqrt{a+b x^3}}{1729 b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 x^3 \sqrt{a+b x^3} \left(62985 c x+53295 d x^2+46189 e x^3+40755 f x^4+36465 g x^5\right)}{692835}+$$

$$\left(12\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{7/3} (19 b d-10 a g) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(4\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^2 \left(1729 b^{1/3} (17 b c-8 a f)-1870 \left(1-\sqrt{3}\right) a^{1/3} (19 b d-10 a g)\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(1616615 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 433 leaves) :

$$\frac{1}{14549535\,(-b)^{8/3}\sqrt{a+b x^3}}\Bigg(2\,(-b)^{2/3}\left(a+b x^3\right)\left(-2 a^2\,(323323 e+27 x\,(6916 f+4675 g x))+21 b^2 x^4\left(62985 c+11 x\left(4845 d+13 x\left(323 e+285 f x+255 g x^2\right)\right)\right)+\right.$$

$$\left. a b x\,(793611 c+x\,(479655 d+7 x\,(46189 e+135 x\,(247 f+187 g x))))\right)+201960\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,(19 b d-10 a g)$$

$$\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]-$$

$$36\,i\,3^{3/4}\,a^{7/3}\left(323 b\left(91\,(-b)^{1/3} c+110 a^{1/3} d\right)-4\left(3458 a\,(-b)^{1/3} f+4675 a^{4/3} g\right)\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\Bigg)$$

- **Problem 446: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2\sqrt{a+b x^3}\,\left(c+d x+e x^2+f x^3+g x^4\right)\,dx$$

Optimal (type 4, 681 leaves, 11 steps):

$$\frac{2\,a\,(5\,b\,c-2\,a\,f)\,\sqrt{a+b\,x^3}}{45\,b^2}+\frac{6\,a\,(17\,b\,d-8\,a\,g)\,x\,\sqrt{a+b\,x^3}}{935\,b^2}+\frac{6\,a\,e\,x^2\,\sqrt{a+b\,x^3}}{91\,b}+\frac{2\,a\,f\,x^3\,\sqrt{a+b\,x^3}}{45\,b}+$$

$$\frac{6\,a\,g\,x^4\,\sqrt{a+b\,x^3}}{187\,b}-\frac{24\,a^2\,e\,\sqrt{a+b\,x^3}}{91\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+\frac{2\,x^2\,\sqrt{a+b\,x^3}\,\left(12\,155\,c\,x+9945\,d\,x^2+8415\,e\,x^3+7293\,f\,x^4+6435\,g\,x^5\right)}{109\,395}+$$

$$\left(12\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{7/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(91\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-\left(4\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a^2\,\left(1547\,b\,d-1870\,\left(1-\sqrt{3}\right)\,a^{1/3}\,b^{2/3}\,e-728\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(85\,085\,b^{7/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 399 leaves):

$$\frac{1}{765\,765\,(-b)^{8/3}\,\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,\left(a+b\,x^3\right)\,\left(-182\,a^2\,(187\,f+108\,g\,x)+\right.\right.$$

$$\left.7\,b^2\,x^3\,\left(12\,155\,c+9945\,d\,x+33\,x^2\,(255\,e+13\,x\,(17\,f+15\,g\,x))\right)+a\,b\,\left(85\,085\,c+x\,\left(41\,769\,d+x\,\left(25\,245\,e+17\,017\,f\,x+12\,285\,g\,x^2\right)\right)\right)\right)+$$

$$201\,960\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,b\,e\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$36\,i\,3^{3/4}\,a^{7/3}\,\left(17\,b\,\left(91\,(-b)^{1/3}\,d+110\,a^{1/3}\,e\right)-728\,a\,(-b)^{1/3}\,g\right)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 447: Result unnecessarily involves imaginary or complex numbers.

$$\int x \sqrt{a+b x^3} \left(c+d x+e x^2+f x^3+g x^4\right) dx$$

Optimal (type 4, 667 leaves, 9 steps):

$$\frac{2 a (5 b d-2 a g) \sqrt{a+b x^3}}{45 b^2}+\frac{6 a e x \sqrt{a+b x^3}}{55 b}+\frac{6 a f x^2 \sqrt{a+b x^3}}{91 b}+\frac{2 a g x^3 \sqrt{a+b x^3}}{45 b}+$$
$$\frac{6 a (13 b c-4 a f) \sqrt{a+b x^3}}{91 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 x \sqrt{a+b x^3} \left(6435 c x+5005 d x^2+4095 e x^3+3465 f x^4+3003 g x^5\right)}{45045}-$$
$$\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{4/3} (13 b c-4 a f) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(91 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\left(2\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^{4/3} \left(182 a^{2/3} b^{1/3} e+55 \left(1-\sqrt{3}\right) (13 b c-4 a f)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(5005 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 390 leaves):

$$\frac{1}{45045\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\left(a+b\,x^3\right)\left(-2002\,a^2\,g+b^2\,x^2\left(6435\,c+7\,x\left(715\,d+585\,e\,x+495\,f\,x^2+429\,g\,x^3\right)\right)+a\,b\,(5005\,d+x\,(2457\,e+11\,x\,(135\,f+91\,g\,x)))\right)-\right.$$

$$2970\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,b\,(13\,b\,c-4\,a\,f)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]-18\,i\,3^{3/4}\,a^{5/3}\,b\left(-715\,b\,c+182\,a^{2/3}\,(-b)^{1/3}\,e+220\,a\,f\right)$$

$$\left.\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 448: Result unnecessarily involves imaginary or complex numbers.**

$$\int\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)\,dx$$

Optimal (type 4, 639 leaves, 8 steps) :

$$\frac{2 a e \sqrt{a+b x^3}}{9 b}+\frac{6 a f x \sqrt{a+b x^3}}{55 b}+\frac{6 a g x^2 \sqrt{a+b x^3}}{91 b}+$$

$$\frac{6 a (13 b d-4 a g) \sqrt{a+b x^3}}{91 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 \sqrt{a+b x^3} \left(9009 c x+6435 d x^2+5005 e x^3+4095 f x^4+3465 g x^5\right)}{45045}-$$

$$\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\; a^{4/3} (13 b d-4 a g) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)+$$

$$\left(2\times 3^{3/4} \sqrt{2+\sqrt{3}}\; a \left(91 b^{1/3} (11 b c-2 a f)-55 \left(1-\sqrt{3}\right) a^{1/3} (13 b d-4 a g)\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(5005 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)$$

Result (type 4, 393 leaves) :

$$-\frac{1}{45045\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\left(a+b\,x^3\right)\left(a\,(5005\,e+27\,x\,(91\,f+55\,g\,x))+b\,x\left(9009\,c+5\,x\left(1287\,d+7\,x\left(143\,e+117\,f\,x+99\,g\,x^2\right)\right)\right)\right)-2970\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\right.$$

$$(13\,b\,d-4\,a\,g)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$18\,i\,3^{3/4}\,a^{4/3}\left(143\,b\left(7\,(-b)^{1/3}\,c+5\,a^{1/3}\,d\right)-2\,a\left(91\,(-b)^{1/3}\,f+110\,a^{1/3}\,g\right)\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 449: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x}\,dx$$

Optimal (type 4, 620 leaves, 11 steps) :

$$\frac{2\,a\,f\,\sqrt{a+b\,x^3}}{9\,b}+\frac{6\,a\,g\,x\,\sqrt{a+b\,x^3}}{55\,b}+\frac{6\,a\,e\,\sqrt{a+b\,x^3}}{7\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\frac{2\,\sqrt{a+b\,x^3}\,\left(1155\,c\,x+693\,d\,x^2+495\,e\,x^3+385\,f\,x^4+315\,g\,x^5\right)}{3465\,x}-\frac{2}{3}\,\sqrt{a}\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(7\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(2\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a\,\left(77\,b\,d-55\,\left(1-\sqrt{3}\right)\,a^{1/3}\,b^{2/3}\,e-14\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(385\,b^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 714 leaves) :

$$\frac{2\sqrt{a+b x^3}\ \left(1155 b c+7 a\ (55 f+27 g x)+b x\ (693 d+5 x\ (99 e+7 x\ (11 f+9 g x)))\right)}{3465 b}-$$

$$\frac{1}{1155 b^{4/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}}\ 2\sqrt{a}\ \left(385 b^{4/3} c\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\ \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+693\sqrt{a}\ b d\ \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\right.$$

$$\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],\ (-1)^{1/3}\right]-126 a^{3/2} g$$

$$\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],\ (-1)^{1/3}\right]-$$

$$495\sqrt{2}\ a^{5/6} b^{2/3} e\ \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 450: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b x^3}\ \left(c+d x+e x^2+f x^3+g x^4\right)}{x^2}\,dx$$

Optimal (type 4, 638 leaves, 11 steps) :

$$\frac{2\,a\,g\,\sqrt{a+b\,x^3}}{9\,b}-\frac{3\,c\,\sqrt{a+b\,x^3}}{x}+\frac{3\,(7\,b\,c+2\,a\,f)\,\sqrt{a+b\,x^3}}{7\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\frac{2\,\sqrt{a+b\,x^3}\,\left(315\,c\,x+105\,d\,x^2+63\,e\,x^3+45\,f\,x^4+35\,g\,x^5\right)}{315\,x^2}-\frac{2}{3}\,\sqrt{a}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,(7\,b\,c+2\,a\,f)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(14\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,a^{1/3}\,\left(14\,a^{2/3}\,b^{1/3}\,e-5\,\left(1-\sqrt{3}\right)\,(7\,b\,c+2\,a\,f)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(35\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 810 leaves) :

$$\frac{1}{315\,b\,x\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}}\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\left(a+b\,x^3\right)\,(-315\,b\,c+70\,a\,g\,x+2\,b\,x\,(105\,d+x\,(63\,e+5\,x\,(9\,f+7\,g\,x))))-\right.$$

$$210\,\sqrt{a}\,b\,d\,x\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-378\,a\,b^{2/3}\,e\,x\,\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)$$

$$\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]+$$

$$945\,\sqrt{2}\,a^{1/3}\,b^{4/3}\,c\,x\,\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$270\,\sqrt{2}\,a^{4/3}\,b^{1/3}\,f\,x\,\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 451: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^3}\,dx$$

Optimal (type 4, 640 leaves, 10 steps) :

$$\frac{3\,c\,\sqrt{a+b\,x^3}}{2\,x^2}-\frac{3\,d\,\sqrt{a+b\,x^3}}{x}+\frac{3\,(7\,b\,d+2\,a\,g)\,\sqrt{a+b\,x^3}}{7\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-$$

$$\frac{2\,\sqrt{a+b\,x^3}\,\left(105\,c\,x-105\,d\,x^2-35\,e\,x^3-21\,f\,x^4-15\,g\,x^5\right)}{105\,x^3}-\frac{2}{3}\,\sqrt{a}\;e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(3\times3^{1/4}\,\sqrt{2-\sqrt{3}}\;a^{1/3}\,(7\,b\,d+2\,a\,g)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(14\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\;\left(7\,b^{1/3}\,(5\,b\,c+4\,a\,f)-10\,\left(1-\sqrt{3}\right)a^{1/3}\,(7\,b\,d+2\,a\,g)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(70\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 962 leaves) :

$$\frac{1}{210\, b^{2/3}\, x^2 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{a+b x^3}}$$

$$\left(b^{2/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \left(a+b x^3\right) \left(-105\, c+2\, x \left(-105\, d+70\, e\, x+42\, f\, x^2+30\, g\, x^3\right)\right)-140 \sqrt{a}\, b^{2/3}\, e\, x^2 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right.$$

$$\sqrt{a+b x^3}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-315\, b^{4/3}\, c\, x^2 \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right]-252\, a\, b^{1/3}\, f\, x^2 \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}$$

$$\sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right]+$$

$$630 \sqrt{2}\, a^{1/3}\, b\, d\, x^2 \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3\, i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$180 \sqrt{2}\, a^{4/3}\, g\, x^2 \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3\, i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 452: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+b x^3}\,\left(c+d x+e x^2+f x^3+g x^4\right)}{x^4}\,dx$$

Optimal (type 4, 637 leaves, 11 steps):

$$\frac{c\sqrt{a+b x^3}}{3 x^3}+\frac{3 d\sqrt{a+b x^3}}{2 x^2}-\frac{3 e\sqrt{a+b x^3}}{x}+\frac{3 b^{1/3} e\sqrt{a+b x^3}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}-$$

$$\frac{2\sqrt{a+b x^3}\,\left(5 c x+15 d x^2-15 e x^3-5 f x^4-3 g x^5\right)}{15 x^4}-\frac{(b c+2 a f)\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3} b^{1/3} e\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\sqrt{a+b x^3}\right)+\left(3^{3/4}\sqrt{2+\sqrt{3}}\,\left(5 b d-10\left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e+4 a g\right)\left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(10\, b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\sqrt{a+b x^3}\right)$$

Result (type 4, 769 leaves):

$$\sqrt{a+b x^3}\left(\frac{2 f}{3}-\frac{10 c+3 x\left(5 d+10 e x-4 g x^3\right)}{30 x^3}\right)-\frac{b\, c\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 \sqrt{a}}-\frac{2}{3} \sqrt{a}\, f\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$\left(3 b^{2/3} d\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right) \Big/$$

$$\left(2 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)-$$

$$\left(6\, a\, g\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right) \Big/$$

$$\left(5 b^{1/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)-\left(3 \sqrt{2}\, a^{1/3} b^{1/3} e\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right) \Big/$$

$$\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)$$

- **Problem 453: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+b x^3}\left(c+d x+e x^2+f x^3+g x^4\right)}{x^5}\, dx$$

Optimal (type 4, 694 leaves, 12 steps) :

$$\frac{3\,c\,\sqrt{a+b\,x^3}}{20\,x^4}+\frac{d\,\sqrt{a+b\,x^3}}{3\,x^3}+\frac{3\,e\,\sqrt{a+b\,x^3}}{2\,x^2}-\frac{3\,(b\,c+8\,a\,f)\,\sqrt{a+b\,x^3}}{8\,a\,x}+$$

$$\frac{3\,b^{1/3}\,(b\,c+8\,a\,f)\,\sqrt{a+b\,x^3}}{8\,a\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,\sqrt{a+b\,x^3}\,\left(3\,c\,x+5\,d\,x^2+15\,e\,x^3-15\,f\,x^4-5\,g\,x^5\right)}{15\,x^5}-\frac{(b\,d+2\,a\,g)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,b^{1/3}\,(b\,c+8\,a\,f)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(16\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{1/3}\,\left(4\,a^{2/3}\,b^{1/3}\,e-\left(1-\sqrt{3}\right)\,(b\,c+8\,a\,f)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(8\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 855 leaves) :

$$\frac{\sqrt{a+b x^3}\left(-6 a c-9 b c x^3-4 a x\left(2 d+x\left(3 e+6 f x-4 g x^2\right)\right)\right)}{24 a x^4}-\frac{1}{24 a \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}}$$

$$\left(8 \sqrt{a}\, b\, d \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+16 a^{3/2} g \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+\right.$$

$$36 a b^{2/3} e\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]-$$

$$9 \sqrt{2}\, a^{1/3} b^{4/3} c\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$72 \sqrt{2}\, a^{4/3} b^{1/3} f\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 454: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+b x^3}\left(c+d x+e x^2+f x^3+g x^4\right)}{x^6}\, dx$$

Optimal (type 4, 652 leaves, 10 steps) :

$$-\frac{1}{60}\left(\frac{12\,c}{x^5}+\frac{15\,d}{x^4}+\frac{20\,e}{x^3}+\frac{30\,f}{x^2}+\frac{60\,g}{x}\right)\sqrt{a+b\,x^3}-\frac{3\,b\,c\,\sqrt{a+b\,x^3}}{20\,a\,x^2}-\frac{3\,b\,d\,\sqrt{a+b\,x^3}}{8\,a\,x}+\frac{3\,b^{1/3}\,(b\,d+8\,a\,g)\,\sqrt{a+b\,x^3}}{8\,a\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{b\,e\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,b^{1/3}\,(b\,d+8\,a\,g)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(16\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{1/3}\,\left(2\,b^{1/3}\,(b\,c-10\,a\,f)+5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,(b\,d+8\,a\,g)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(40\,a\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 934 leaves) :

$$-\frac{\sqrt{a+b x^3}\,\left(24 a c+9 b x^3 (2 c+5 d x)+10 a x \left(3 d+4 e x+6 x^2 (f+2 g x)\right)\right)}{120 a x^5}-$$

$$\frac{1}{120 a \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\sqrt{a+b x^3}}\,b^{1/3}\left(40 \sqrt{a}\, b^{2/3} e \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\sqrt{a+b x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\right.$$

$$18 b^{4/3} c \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]+$$

$$180 a b^{1/3} f \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\text{EllipticF}\Big[$$

$$\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\Big]-45 \sqrt{2}\, a^{1/3} b d \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$360 \sqrt{2}\, a^{4/3} g \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 455: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+b x^3}\,\left(c+d x+e x^2+f x^3+g x^4\right)}{x^7}\,dx$$

Optimal (type 4, 659 leaves, 11 steps) :

$$-\frac{1}{60}\left(\frac{10\,c}{x^6}+\frac{12\,d}{x^5}+\frac{15\,e}{x^4}+\frac{20\,f}{x^3}+\frac{30\,g}{x^2}\right)\sqrt{a+b\,x^3}-\frac{b\,c\,\sqrt{a+b\,x^3}}{12\,a\,x^3}-$$

$$\frac{3\,b\,d\,\sqrt{a+b\,x^3}}{20\,a\,x^2}-\frac{3\,b\,e\,\sqrt{a+b\,x^3}}{8\,a\,x}+\frac{3\,b^{4/3}\,e\,\sqrt{a+b\,x^3}}{8\,a\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+\frac{b\,(b\,c-4\,a\,f)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{12\,a^{3/2}}-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,b^{4/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(16\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)-\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,\,b^{2/3}\,\left(2\,b\,d+5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,b^{2/3}\,e-20\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(40\,a\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 800 leaves) :

$$-\frac{\sqrt{a+b x^3}\ \left(b x^3\ (10 c+9 x\ (2 d+5 e x))+a\left(20 c+2 x\left(12 d+5 x\left(3 e+4 f x+6 g x^2\right)\right)\right)\right)}{120\ a\ x^6}+$$

$$\frac{1}{80\ a} b\left(\frac{20\ b\ c\ \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-\frac{80}{3}\sqrt{a}\ f\ \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+\left(12\ b^{2/3}\ d\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right.\right.$$

$$\left.\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\bigg/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\right)-$$

$$\left(120\ a\ g\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\bigg/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\right)-\left(30\sqrt{2}\ a^{1/3} b^{1/3} e\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right.$$

$$\sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\bigg/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\right)\right)$$

- **Problem 456: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\sqrt{a+b x^3}\ \left(c+d x+e x^2+f x^3+g x^4\right)}{x^8}\,dx$$

Optimal (type 4, 711 leaves, 12 steps):

$$-\frac{1}{420}\left(\frac{60\,c}{x^7}+\frac{70\,d}{x^6}+\frac{84\,e}{x^5}+\frac{105\,f}{x^4}+\frac{140\,g}{x^3}\right)\sqrt{a+b\,x^3}-\frac{3\,b\,c\,\sqrt{a+b\,x^3}}{56\,a\,x^4}-\frac{b\,d\,\sqrt{a+b\,x^3}}{12\,a\,x^3}-$$

$$\frac{3\,b\,e\,\sqrt{a+b\,x^3}}{20\,a\,x^2}+\frac{3\,b\,(5\,b\,c-14\,a\,f)\,\sqrt{a+b\,x^3}}{112\,a^2\,x}-\frac{3\,b^{4/3}\,(5\,b\,c-14\,a\,f)\,\sqrt{a+b\,x^3}}{112\,a^2\,\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{b\,(b\,d-4\,a\,g)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{12\,a^{3/2}}+$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,b^{4/3}\,(5\,b\,c-14\,a\,f)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(224\,a^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{4/3}\,\left(28\,a^{2/3}\,b^{1/3}\,e-5\,\left(1-\sqrt{3}\right)(5\,b\,c-14\,a\,f)\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(560\,a^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 892 leaves) :

$$\frac{1}{1680\, a^2 x^7}\sqrt{a+b x^3}\, \left(225 b^2 c x^6-2 a b x^3 (45 c+7 x (10 d+9 x (2 e+5 f x)))-4 a^2 (60 c+7 x (10 d+x (12 e+5 x (3 f+4 g x))))\right)+$$

$$\frac{1}{1680\, a^2 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{a+b x^3}}$$

$$b \left(140 \sqrt{a}\, b\, d \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{a+b x^3}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-560\, a^{3/2} g \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{a+b x^3}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+252\, a\right.$$

$$b^{2/3} e \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],\, (-1)^{1/3}\right]-$$

$$225 \sqrt{2}\, a^{1/3} b^{4/3} c \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$630 \sqrt{2}\, a^{4/3} b^{1/3} f \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 457: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a+b x^3}\, \left(c+d x+e x^2+f x^3+g x^4\right)}{x^9}\, dx$$

Optimal (type 4, 743 leaves, 13 steps) :

$$-\frac{1}{840}\left(\frac{105\,c}{x^8}+\frac{120\,d}{x^7}+\frac{140\,e}{x^6}+\frac{168\,f}{x^5}+\frac{210\,g}{x^4}\right)\sqrt{a+b\,x^3}-\frac{3\,b\,c\,\sqrt{a+b\,x^3}}{80\,a\,x^5}-\frac{3\,b\,d\,\sqrt{a+b\,x^3}}{56\,a\,x^4}-\frac{b\,e\,\sqrt{a+b\,x^3}}{12\,a\,x^3}+$$

$$\frac{3\,b\,(7\,b\,c-16\,a\,f)\,\sqrt{a+b\,x^3}}{320\,a^2\,x^2}+\frac{3\,b\,(5\,b\,d-14\,a\,g)\,\sqrt{a+b\,x^3}}{112\,a^2\,x}-\frac{3\,b^{4/3}\,(5\,b\,d-14\,a\,g)\,\sqrt{a+b\,x^3}}{112\,a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{b^2\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{12\,a^{3/2}}+$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,b^{4/3}\,(5\,b\,d-14\,a\,g)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(224\,a^{5/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{4/3}\left(7\,b^{1/3}\,(7\,b\,c-16\,a\,f)+20\left(1-\sqrt{3}\right)a^{1/3}\,(5\,b\,d-14\,a\,g)\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2240\,a^2\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 979 leaves) :

$$\frac{1}{6720\,a^2\,x^8}\sqrt{a+b\,x^3}\,\left(9\,b^2\,x^6\,(49\,c+100\,d\,x)-4\,a\,b\,x^3\,(63\,c+2\,x\,(45\,d+7\,x\,(10\,e+9\,x\,(2\,f+5\,g\,x))))-8\,a^2\,(105\,c+2\,x\,(60\,d+7\,x\,(10\,e+3\,x\,(4\,f+5\,g\,x))))\right)+$$

$$\frac{1}{6720\,a^2\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$b^{4/3}\left(560\,\sqrt{a}\,b^{2/3}\,e\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-441\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right.$$

$$\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]+1008\,a\,b^{1/3}\,f\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)$$

$$\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$900\,\sqrt{2}\,a^{1/3}\,b\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$2520\,\sqrt{2}\,a^{4/3}\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 458: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right) \, dx$$

Optimal (type 4, 791 leaves, 14 steps):

$$-\frac{4 a^3 e \sqrt{a+b x^3}}{105 b^2}+\frac{54 a^2 (23 b c-8 a f) x \sqrt{a+b x^3}}{21505 b^2}+\frac{54 a^2 (5 b d-2 a g) x^2 \sqrt{a+b x^3}}{8645 b^2}+\frac{2 a^2 e x^3 \sqrt{a+b x^3}}{105 b}+\frac{54 a^2 f x^4 \sqrt{a+b x^3}}{4301 b}+$$

$$\frac{54 a^2 g x^5 \sqrt{a+b x^3}}{6175 b}-\frac{216 a^3 (5 b d-2 a g) \sqrt{a+b x^3}}{8645 b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 x^3 \left(a+b x^3\right)^{3/2} \left(229425 c x+205275 d x^2+185725 e x^3+169575 f x^4+156009 g x^5\right)}{3900225}+$$

$$\frac{2 a x^3 \sqrt{a+b x^3} \left(8947575 c x+6774075 d x^2+5311735 e x^3+4279275 f x^4+3522519 g x^5\right)}{185910725}+$$

$$\left(108\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{10/3} (5 b d-2 a g) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(8645 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(36\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^3 \left(1729 b^{1/3} (23 b c-8 a f)-8602 \left(1-\sqrt{3}\right) a^{1/3} (5 b d-2 a g)\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(37182145 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 466 leaves):

$$\frac{1}{557\,732\,175\,(-b)^{8/3}\sqrt{a+b x^3}}\Bigg(2\,(-b)^{2/3}\left(a+b x^3\right)$$

$$\Big(-10 a^3\,(1\,062\,347\,e+81\,x\,(6916\,f+4301\,g\,x))+a^2 b\,x\left(16\,105\,635\,c+x\left(8\,709\,525\,d+5\,311\,735\,e\,x+3\,501\,225\,f\,x^2+2\,438\,667\,g\,x^3\right)\right)+$$
$$143\,b^3 x^7\left(229\,425\,c+17\,x\left(12\,075\,d+19\,x\left(575\,e+525\,f\,x+483\,g\,x^2\right)\right)\right)+$$
$$2\,a\,b^2 x^4\,(29\,825\,250\,c+11\,x\,(2\,258\,025\,d+13\,x\,(148\,580\,e+21\,x\,(6175\,f+5474\,g\,x))))\Big)+$$

$$13\,935\,240\,(-1)^{2/3}\,3^{1/4}\,a^{11/3}\,(5\,b\,d-2\,a\,g)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$540\,i\,3^{3/4}\,a^{10/3}\left(39\,767\,(-b)^{1/3}\,b\,c+43\,010\,a^{1/3}\,b\,d-13\,832\,a\,(-b)^{1/3}\,f-17\,204\,a^{4/3}\,g\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\Bigg)$$

- **Problem 459: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2\left(a+b x^3\right)^{3/2}\left(c+d x+e x^2+f x^3+g x^4\right)dx$$

Optimal (type 4, 742 leaves, 12 steps) :

$$\frac{2\,a^2\,(7\,b\,c-2\,a\,f)\,\sqrt{a+b\,x^3}}{105\,b^2}+\frac{54\,a^2\,(23\,b\,d-8\,a\,g)\,x\,\sqrt{a+b\,x^3}}{21\,505\,b^2}+\frac{54\,a^2\,e\,x^2\,\sqrt{a+b\,x^3}}{1729\,b}+\frac{2\,a^2\,f\,x^3\,\sqrt{a+b\,x^3}}{105\,b}+\frac{54\,a^2\,g\,x^4\,\sqrt{a+b\,x^3}}{4301\,b}-$$

$$\frac{216\,a^3\,e\,\sqrt{a+b\,x^3}}{1729\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+\frac{2\,x^2\,\left(a+b\,x^3\right)^{3/2}\,\left(52\,003\,c\,x+45\,885\,d\,x^2+41\,055\,e\,x^3+37\,145\,f\,x^4+33\,915\,g\,x^5\right)}{780\,045}+$$

$$\frac{2\,a\,x^2\,\sqrt{a+b\,x^3}\,\left(7\,436\,429\,c\,x+5\,368\,545\,d\,x^2+4\,064\,445\,e\,x^3+3\,187\,041\,f\,x^4+2\,567\,565\,g\,x^5\right)}{111\,546\,435}+$$

$$\left(108\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{10/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(1729\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(36\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a^3\,\left(43\,010\,\left(1-\sqrt{3}\right)\,a^{1/3}\,b^{2/3}\,e-1729\,(23\,b\,d-8\,a\,g)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(37\,182\,145\,b^{7/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 436 leaves) :

$$\frac{1}{111\,546\,435\,(-b)^{8/3}\sqrt{a+b x^3}}\Bigg(2\,(-b)^{2/3}\left(a+b x^3\right)\left(-494\,a^3\,(4301\,f+2268\,g\,x)+143\,b^3\,x^6\left(52\,003\,c+5\,x\left(9177\,d+17\,x\left(483\,e+437\,f\,x+399\,g\,x^2\right)\right)\right)+a^2\right.$$

$$b\left(7\,436\,429\,c+x\left(3\,221\,127\,d+x\left(1\,741\,905\,e+1\,062\,347\,f\,x+700\,245\,g\,x^2\right)\right)\right)+$$

$$\left.2\,a\,b^2\,x^3\,(7\,436\,429\,c+x\,(5\,965\,050\,d+11\,x\,(451\,605\,e+247\,x\,(1564\,f+1365\,g\,x))))\right)+$$

$$13\,935\,240\,(-1)^{2/3}\,3^{1/4}\,a^{11/3}\,b\,e\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$108\,i\,3^{3/4}\,a^{10/3}\left(39\,767\,(-b)^{1/3}\,b\,d+43\,010\,a^{1/3}\,b\,e-13\,832\,a\,(-b)^{1/3}\,g\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\Bigg)$$

- **Problem 460: Result unnecessarily involves imaginary or complex numbers.**

$$\int x\left(a+b x^3\right)^{3/2}\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)dx$$

Optimal (type 4, 723 leaves, 10 steps) :

$$\frac{2 a^2 (7 b d - 2 a g) \sqrt{a + b x^3}}{105 b^2} + \frac{54 a^2 e x \sqrt{a + b x^3}}{935 b} + \frac{54 a^2 f x^2 \sqrt{a + b x^3}}{1729 b} + \frac{2 a^2 g x^3 \sqrt{a + b x^3}}{105 b} +$$

$$\frac{54 a^2 (19 b c - 4 a f) \sqrt{a + b x^3}}{1729 b^{5/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} + \frac{2 x \left(a + b x^3\right)^{3/2} \left(33915 c x + 29393 d x^2 + 25935 e x^3 + 23205 f x^4 + 20995 g x^5\right)}{440895} +$$

$$\frac{2 a x \sqrt{a + b x^3} \left(479655 c x + 323323 d x^2 + 233415 e x^3 + 176715 f x^4 + 138567 g x^5\right)}{4849845} -$$

$$\left(27 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^{7/3} (19 b c - 4 a f) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) - \left(18 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a^{7/3} \left(3458 a^{2/3} b^{1/3} e + 935 \left(1 - \sqrt{3}\right) (19 b c - 4 a f)\right) \left(a^{1/3} + b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(1616615 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 429 leaves) :

$$\frac{1}{4849845\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\left(a+b\,x^3\right)\right.$$

$$\left(-92378\,a^3\,g+a^2\,b\,(323323\,d+x\,(140049\,e+187\,x\,(405\,f+247\,g\,x)))+11\,b^3\,x^5\left(33915\,c+13\,x\left(2261\,d+5\,x\left(399\,e+357\,f\,x+323\,g\,x^2\right)\right)\right)+\right.$$
$$\left.2\,a\,b^2\,x^2\left(426360\,c+x\left(323323\,d+x\left(259350\,e+215985\,f\,x+184756\,g\,x^2\right)\right)\right)\right)-151470\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,b\,(19\,b\,c-4\,a\,f)$$

$$\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]-$$

$$54\,i\,3^{3/4}\,a^{8/3}\,b\left(-17765\,b\,c+3458\,a^{2/3}\,(-b)^{1/3}\,e+3740\,a\,f\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

- **Problem 461: Result unnecessarily involves imaginary or complex numbers.**

$$\int\left(a+b\,x^3\right)^{3/2}\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)\,dx$$

Optimal (type 4, 694 leaves, 9 steps) :

$$\frac{2\,a^2\,e\,\sqrt{a+b\,x^3}}{15\,b}+\frac{54\,a^2\,f\,x\,\sqrt{a+b\,x^3}}{935\,b}+\frac{54\,a^2\,g\,x^2\,\sqrt{a+b\,x^3}}{1729\,b}+$$

$$\frac{54\,a^2\,(19\,b\,d-4\,a\,g)\,\sqrt{a+b\,x^3}}{1729\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{2\,\left(a+b\,x^3\right)^{3/2}\left(62\,985\,c\,x+53\,295\,d\,x^2+46\,189\,e\,x^3+40\,755\,f\,x^4+36\,465\,g\,x^5\right)}{692\,835}+$$

$$\frac{2\,a\,\sqrt{a+b\,x^3}\,\left(793\,611\,c\,x+479\,655\,d\,x^2+323\,323\,e\,x^3+233\,415\,f\,x^4+176\,715\,g\,x^5\right)}{4\,849\,845}-$$

$$\left(27\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{7/3}\,(19\,b\,d-4\,a\,g)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(1729\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(18\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a^2\,\left(1729\,b^{1/3}\,(17\,b\,c-2\,a\,f)-935\,\left(1-\sqrt{3}\right)a^{1/3}\,(19\,b\,d-4\,a\,g)\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(1\,616\,615\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 429 leaves) :

$$-\frac{1}{4849845\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\left(a+b\,x^3\right)\left(a^2\,(323323\,e+81\,x\,(1729\,f+935\,g\,x))+7\,b^2\,x^4\left(62985\,c+11\,x\left(4845\,d+13\,x\left(323\,e+285\,f\,x+255\,g\,x^2\right)\right)\right)+\right.\right.$$

$$\left.2\,a\,b\,x\left(617253\,c+x\left(426360\,d+7\,x\left(46189\,e+37050\,f\,x+30855\,g\,x^2\right)\right)\right)\right)-151470\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,(19\,b\,d-4\,a\,g)$$

$$\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$54\,i\,3^{3/4}\,a^{7/3}\left(323\,b\left(91\,(-b)^{1/3}\,c+55\,a^{1/3}\,d\right)-3458\,a\,(-b)^{1/3}\,f-3740\,a^{4/3}\,g\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

- **Problem 462: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x}\,dx$$

Optimal (type 4, 676 leaves, 12 steps) :

$$\frac{2\,a^2 f\sqrt{a+b\,x^3}}{15\,b}+\frac{54\,a^2 g\,x\sqrt{a+b\,x^3}}{935\,b}+\frac{54\,a^2 e\sqrt{a+b\,x^3}}{91\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{2\left(a+b\,x^3\right)^{3/2}\left(12\,155\,c\,x+9945\,d\,x^2+8415\,e\,x^3+7293\,f\,x^4+6435\,g\,x^5\right)}{109\,395\,x}+$$

$$\frac{2\,a\sqrt{a+b\,x^3}\left(85\,085\,c\,x+41\,769\,d\,x^2+25\,245\,e\,x^3+17\,017\,f\,x^4+12\,285\,g\,x^5\right)}{255\,255\,x}-\frac{2}{3}a^{3/2}\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(27\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{7/3}\,e\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)+\left(18\times 3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\left(1547\,b\,d-935\left(1-\sqrt{3}\right)a^{1/3}\,b^{2/3}\,e-182\,a\,g\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(85\,085\,b^{4/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 753 leaves) :

$$\frac{1}{765765\,b}2\sqrt{a+b\,x^3}\,\left(273\,a^2\,(187\,f+81\,g\,x)+2\,a\,b\,\left(170170\,c+97461\,d\,x+67320\,e\,x^2+51051\,f\,x^3+40950\,g\,x^4\right)+7\,b^2\,x^3\,\left(12155\,c+9945\,d\,x+33\,x^2\,(255\,e+13\,x\,(17\,f+15\,g\,x))\right)\right)-$$

$$\frac{1}{255255\,b^{4/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}2\,a^{3/2}\left(85085\,b^{4/3}\,c\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+125307\,\sqrt{a}\,b\,d\right.$$

$$\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-14742$$

$$a^{3/2}\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$75735\,\sqrt{2}\,a^{5/6}\,b^{2/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 463: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^2}\,dx$$

Optimal (type 4, 692 leaves, 12 steps) :

$$\frac{2 a^2 g \sqrt{a+b x^3}}{15 b}-\frac{27 a c \sqrt{a+b x^3}}{7 x}+\frac{27 a (13 b c+2 a f) \sqrt{a+b x^3}}{91 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 a \sqrt{a+b x^3} \left(19305 c x+5005 d x^2+2457 e x^3+1485 f x^4+1001 g x^5\right)}{15015 x^2}+$$

$$\frac{2 \left(a+b x^3\right)^{3/2} \left(6435 c x+5005 d x^2+4095 e x^3+3465 f x^4+3003 g x^5\right)}{45045 x^2}-\frac{2}{3} a^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{4/3} (13 b c+2 a f) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(182 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^{4/3} \left(182 a^{2/3} b^{1/3} e-55 \left(1-\sqrt{3}\right) (13 b c+2 a f)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(5005 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 817 leaves) :

$$\frac{1}{45045\, b\, x}\sqrt{a+b x^3}\left(6006 a^2 g x+2 b^2 x^3 \left(6435 c+7 x \left(715 d+585 e x+495 f x^2+429 g x^3\right)\right)+a b \left(-45045 c+4 x \left(10010 d+5733 e x+33 x^2 (120 f+91 g x)\right)\right)\right)-$$

$$\frac{1}{15015\, b^{2/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}}\, a \left(10010 \sqrt{a}\, b^{2/3} d \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\, \mathrm{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+14742\, a\, b^{1/3} e\right.$$

$$\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right]-$$

$$57915 \sqrt{2}\, a^{1/3} b\, c \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right) \mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$8910 \sqrt{2}\, a^{4/3} f \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right) \mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 464: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^3}\, dx$$

Optimal (type 4, 694 leaves, 11 steps) :

$$\frac{27\,a\,c\,\sqrt{a+b\,x^3}}{10\,x^2}-\frac{27\,a\,d\,\sqrt{a+b\,x^3}}{7\,x}+\frac{27\,a\,(13\,b\,d+2\,a\,g)\,\sqrt{a+b\,x^3}}{91\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,a\,\sqrt{a+b\,x^3}\,\left(27027\,c\,x-19305\,d\,x^2-5005\,e\,x^3-2457\,f\,x^4-1485\,g\,x^5\right)}{15015\,x^3}+$$

$$\frac{2\,\left(a+b\,x^3\right)^{3/2}\,\left(9009\,c\,x+6435\,d\,x^2+5005\,e\,x^3+4095\,f\,x^4+3465\,g\,x^5\right)}{45045\,x^3}-\frac{2}{3}\,a^{3/2}\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(27\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,(13\,b\,d+2\,a\,g)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(182\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a\,\left(91\,b^{1/3}\,(11\,b\,c+4\,a\,f)-110\,\left(1-\sqrt{3}\right)\,a^{1/3}\,(13\,b\,d+2\,a\,g)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(10010\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 952 leaves) :

$$\frac{1}{90090\,x^2}\sqrt{a+b\,x^3}\,\left(a\left(-45045\,c-90090\,d\,x+8\,x^2\,(10010\,e+9\,x\,(637\,f+440\,g\,x))\right)+4\,b\,x^3\left(9009\,c+5\,x\left(1287\,d+7\,x\left(143\,e+117\,f\,x+99\,g\,x^2\right)\right)\right)\right)-$$

$$\frac{1}{30030\,b^{2/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$a\left(20020\sqrt{a}\,b^{2/3}\,e\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+81081\,b^{4/3}\,c\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right.$$

$$\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],(-1)^{1/3}\right]+29484\,a\,b^{1/3}\,f\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)$$

$$\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],(-1)^{1/3}\right]-$$

$$115830\sqrt{2}\,a^{1/3}\,b\,d\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$17820\sqrt{2}\,a^{4/3}\,g\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 465: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^4} \, dx$$

Optimal (type 4, 692 leaves, 12 steps):

$$\frac{a c \sqrt{a+b x^3}}{x^3}+\frac{27 a d \sqrt{a+b x^3}}{10 x^2}-\frac{27 a e \sqrt{a+b x^3}}{7 x}+\frac{27 a b^{1/3} e \sqrt{a+b x^3}}{7 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{2 a \sqrt{a+b x^3} \left(1155 c x+2079 d x^2-1485 e x^3-385 f x^4-189 g x^5\right)}{1155 x^4}+\frac{2 \left(a+b x^3\right)^{3/2} \left(1155 c x+693 d x^2+495 e x^3+385 f x^4+315 g x^5\right)}{3465 x^4}-\frac{1}{3} \sqrt{a} \left(3 b c+2 a f\right) \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{4/3} b^{1/3} e \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(14 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a \left(77 b d-110 \left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e+28 a g\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(770 b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 813 leaves):

$$\sqrt{a+b x^3}\left(a\left(\frac{8 f}{9}-\frac{c}{3 x^3}-\frac{d}{2 x^2}-\frac{e}{x}+\frac{28 g x}{55}\right)+b\left(\frac{2 c}{3}+\frac{2 d x}{5}+\frac{2 e x^2}{7}+\frac{2 f x^3}{9}+\frac{2 g x^4}{11}\right)\right)-$$

$$\sqrt{a}\, b\, c\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\frac{2}{3} a^{3/2} f\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$\left(27 a b^{2/3} d\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/$$

$$\left(10\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\sqrt{a+b x^3}\right)-$$

$$\left(54 a^2 g\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/$$

$$\left(55 b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\sqrt{a+b x^3}\right)-\left(27\sqrt{2}\, a^{4/3} b^{1/3} e\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/\left(7\right.$$

$$\left.\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\sqrt{a+b x^3}\right)$$

▪ Problem 466: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^3\right)^{3/2}\left(c+d x+e x^2+f x^3+g x^4\right)}{x^5}\,dx$$

Optimal (type 4, 741 leaves, 13 steps):

$$\frac{27\,a\,c\,\sqrt{a+b\,x^3}}{20\,x^4}+\frac{a\,d\,\sqrt{a+b\,x^3}}{x^3}+\frac{27\,a\,e\,\sqrt{a+b\,x^3}}{10\,x^2}-\frac{27\,(7\,b\,c+8\,a\,f)\,\sqrt{a+b\,x^3}}{56\,x}+$$

$$\frac{27\,b^{1/3}\,(7\,b\,c+8\,a\,f)\,\sqrt{a+b\,x^3}}{56\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,a\,\sqrt{a+b\,x^3}\,\left(189\,c\,x+105\,d\,x^2+189\,e\,x^3-135\,f\,x^4-35\,g\,x^5\right)}{105\,x^5}+$$

$$\frac{2\,\left(a+b\,x^3\right)^{3/2}\,\left(315\,c\,x+105\,d\,x^2+63\,e\,x^3+45\,f\,x^4+35\,g\,x^5\right)}{315\,x^5}-\frac{1}{3}\,\sqrt{a}\,(3\,b\,d+2\,a\,g)\,\mathrm{ArcTanh}\!\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(27\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,b^{1/3}\,(7\,b\,c+8\,a\,f)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\mathrm{EllipticE}\!\left[\mathrm{ArcSin}\!\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(112\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a^{1/3}\,b^{1/3}\,\left(28\,a^{2/3}\,b^{1/3}\,e-5\,\left(1-\sqrt{3}\right)\,(7\,b\,c+8\,a\,f)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\mathrm{EllipticF}\!\left[\mathrm{ArcSin}\!\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(280\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 878 leaves) :

$$\frac{1}{2520\,x^4}\sqrt{a+b\,x^3}\,\left(-70\,a\,(9\,c+2\,x\,(6\,d+x\,(9\,e+2\,x\,(9\,f-8\,g\,x))))+b\,x^3\,(-3465\,c+16\,x\,(105\,d+x\,(63\,e+5\,x\,(9\,f+7\,g\,x))))\right)-$$

$$\sqrt{a}\,b\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-\frac{2}{3}\,a^{3/2}\,g\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(27\,a\,b^{2/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(10\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-\left(27\,a^{1/3}\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/$$

$$\left(4\,\sqrt{2}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-\left(27\,\sqrt{2}\,a^{4/3}\,b^{1/3}\,f\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/\left(7\right.$$

$$\left.\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\right)$$

- **Problem 467: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^6}\,dx$$

Optimal (type 4, 689 leaves, 11 steps):

$$\frac{27\,b\,c\,\sqrt{a+b\,x^3}}{20\,x^2}-\frac{27\,b\,d\,\sqrt{a+b\,x^3}}{8\,x}+\frac{27\,b^{1/3}\,(7\,b\,d+8\,a\,g)\,\sqrt{a+b\,x^3}}{56\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{1}{60}\left(\frac{12\,c}{x^5}+\frac{15\,d}{x^4}+\frac{20\,e}{x^3}+\frac{30\,f}{x^2}+\frac{60\,g}{x}\right)\left(a+b\,x^3\right)^{3/2}-$$

$$\frac{b\,\sqrt{a+b\,x^3}\,\left(252\,c\,x-315\,d\,x^2-140\,e\,x^3-126\,f\,x^4-180\,g\,x^5\right)}{140\,x^3}-\sqrt{a}\,b\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(27\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,b^{1/3}\,(7\,b\,d+8\,a\,g)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(112\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{1/3}\,\left(14\,b^{1/3}\,(b\,c+2\,a\,f)-5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,(7\,b\,d+8\,a\,g)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(280\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 949 leaves) :

$$-\frac{1}{840\,x^5}\sqrt{a+b\,x^3}\,\left(14\,a\left(12\,c+5\,x\left(3\,d+4\,e\,x+6\,x^2\,(f+2\,g\,x)\right)\right)+b\,x^3\,(546\,c+x\,(1155\,d-16\,x\,(35\,e+3\,x\,(7\,f+5\,g\,x))))\right)-$$

$$\frac{1}{280\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$b^{1/3}\left(280\,\sqrt{a}\,b^{2/3}\,e\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+378\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right.$$

$$\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]+756\,a\,b^{1/3}\,f\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)$$

$$\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$945\,\sqrt{2}\,a^{1/3}\,b\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\mathrm{i}\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,\mathrm{i}+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$1080\,\sqrt{2}\,a^{4/3}\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\mathrm{i}\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,\mathrm{i}+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 468: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^7} \, dx$$

Optimal (type 4, 692 leaves, 12 steps):

$$\frac{b c \sqrt{a+b x^3}}{4 x^3}+\frac{27 b d \sqrt{a+b x^3}}{20 x^2}-\frac{27 b e \sqrt{a+b x^3}}{8 x}+\frac{27 b^{4/3} e \sqrt{a+b x^3}}{8 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{1}{60} \left(\frac{10 c}{x^6}+\frac{12 d}{x^5}+\frac{15 e}{x^4}+\frac{20 f}{x^3}+\frac{30 g}{x^2}\right) \left(a+b x^3\right)^{3/2}-\frac{b \sqrt{a+b x^3} \left(10 c x+36 d x^2-45 e x^3-20 f x^4-18 g x^5\right)}{20 x^4}-\frac{b (b c+4 a f) \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{4 \sqrt{a}}-\left(27 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^{1/3} b^{4/3} e \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(16 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(9 \times 3^{3/4} \sqrt{2+\sqrt{3}} b^{2/3} \left(2 b d-5 \left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e+4 a g\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(40 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 805 leaves):

$$-\frac{1}{120\,x^6}\sqrt{a+b\,x^3}\,\left(b\,x^3\,\left(50\,c+x\,\left(78\,d+x\,\left(165\,e-80\,f\,x-48\,g\,x^2\right)\right)\right)+a\,\left(20\,c+2\,x\,\left(12\,d+5\,x\,\left(3\,e+4\,f\,x+6\,g\,x^2\right)\right)\right)\right)+$$

$$\frac{3}{80}\,b\,\left(-\frac{20\,b\,c\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-\frac{80}{3}\,\sqrt{a}\,f\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-\left(36\,b^{2/3}\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right.\right.$$

$$\left.\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]\right)\Big/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(72\,a\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]\right)\Big/$$

$$\left(b^{1/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-\left(90\,\sqrt{2}\,a^{1/3}\,b^{1/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right.$$

$$\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\,\left(\left(-1+(-1)^{2/3}\right)\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\right.$$

$$\left.\left.\left.\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)\right)$$

- **Problem 469: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^8}\,dx$$

Optimal (type 4, 746 leaves, 13 steps) :

$$\frac{27\,b\,c\,\sqrt{a+b\,x^3}}{280\,x^4}+\frac{b\,d\,\sqrt{a+b\,x^3}}{4\,x^3}+\frac{27\,b\,e\,\sqrt{a+b\,x^3}}{20\,x^2}-\frac{27\,b\,(b\,c+14\,a\,f)\,\sqrt{a+b\,x^3}}{112\,a\,x}+$$

$$\frac{27\,b^{4/3}\,(b\,c+14\,a\,f)\,\sqrt{a+b\,x^3}}{112\,a\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{1}{420}\left(\frac{60\,c}{x^7}+\frac{70\,d}{x^6}+\frac{84\,e}{x^5}+\frac{105\,f}{x^4}+\frac{140\,g}{x^3}\right)\left(a+b\,x^3\right)^{3/2}-$$

$$\frac{b\,\sqrt{a+b\,x^3}\,\left(36\,c\,x+70\,d\,x^2+252\,e\,x^3-315\,f\,x^4-140\,g\,x^5\right)}{140\,x^5}-\frac{b\,(b\,d+4\,a\,g)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{4\,\sqrt{a}}-$$

$$\left(27\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,b^{4/3}\,(b\,c+14\,a\,f)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(224\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{4/3}\,\left(28\,a^{2/3}\,b^{1/3}\,e-5\,\left(1-\sqrt{3}\right)\,(b\,c+14\,a\,f)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(560\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 897 leaves) :

$$-\frac{1}{1680\,a\,x^7}\sqrt{a+b\,x^3}\,\left(405\,b^2\,c\,x^6+2\,a\,b\,x^3\,\left(255\,c+7\,x\,\left(50\,d+x\,\left(78\,e+165\,f\,x-80\,g\,x^2\right)\right)\right)+4\,a^2\,\left(60\,c+7\,x\,\left(10\,d+x\,\left(12\,e+5\,x\,\left(3\,f+4\,g\,x\right)\right)\right)\right)\right)-$$

$$\frac{1}{560\,a\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$b\left(140\,\sqrt{a}\,b\,d\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+560\,a^{3/2}\,g\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+756\,a\right.$$

$$b^{2/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$135\,\sqrt{2}\,a^{1/3}\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$1890\,\sqrt{2}\,a^{4/3}\,b^{1/3}\,f\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 470: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^9}\,dx$$

Optimal (type 4, 705 leaves, 11 steps) :

$$-\frac{1}{560} b \left(\frac{63 c}{x^5}+\frac{90 d}{x^4}+\frac{140 e}{x^3}+\frac{252 f}{x^2}+\frac{630 g}{x}\right) \sqrt{a+b x^3}-\frac{27 b^2 c \sqrt{a+b x^3}}{320 a x^2}-\frac{27 b^2 d \sqrt{a+b x^3}}{112 a x}+$$

$$\frac{27 b^{4/3} (b d+14 a g) \sqrt{a+b x^3}}{112 a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{1}{840} \left(\frac{105 c}{x^8}+\frac{120 d}{x^7}+\frac{140 e}{x^6}+\frac{168 f}{x^5}+\frac{210 g}{x^4}\right) \left(a+b x^3\right)^{3/2}-\frac{b^2 e \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{4 \sqrt{a}}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\; b^{4/3} (b d+14 a g) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(224 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)-\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\; b^{4/3} \left(7 b^{1/3} (b c-16 a f)+20 \left(1-\sqrt{3}\right) a^{1/3} (b d+14 a g)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(2240 a \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)$$

Result (type 4, 978 leaves) :

$$-\frac{1}{6720\,a\,x^8}\sqrt{a+b\,x^3}\,\left(81\,b^2\,x^6\,(7\,c+20\,d\,x)+4\,a\,b\,x^3\,\left(399\,c+2\,x\,\left(255\,d+7\,x\,\left(50\,e+78\,f\,x+165\,g\,x^2\right)\right)\right)+8\,a^2\,(105\,c+2\,x\,(60\,d+7\,x\,(10\,e+3\,x\,(4\,f+5\,g\,x))))\right)-$$

$$\frac{1}{2240\,a\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}\,b^{4/3}\left(560\,\sqrt{a}\,b^{2/3}\,e\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-189\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\right.$$

$$\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]+3024\,a\,b^{1/3}\,f$$

$$\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$540\,\sqrt{2}\,a^{1/3}\,b\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$7560\,\sqrt{2}\,a^{4/3}\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 471: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^{10}}\,dx$$

Optimal (type 4, 714 leaves, 12 steps) :

$$-\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680}-\frac{b^2c\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2d\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2e\sqrt{a+bx^3}}{112ax}+$$

$$\frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)\left(a+bx^3\right)^{3/2}}{2520}+\frac{b^2(bc-6af)\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]}{24a^{3/2}}-$$

$$\left(27\times3^{1/4}\sqrt{2-\sqrt{3}}\,b^{7/3}e\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(224a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)-\left(9\times3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(7bd+20\left(1-\sqrt{3}\right)a^{1/3}b^{2/3}e-112ag\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2240a\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 844 leaves) :

$$-\frac{1}{20160\,a\,x^9}\sqrt{a+b\,x^3}\,\left(3\,b^2\,x^6\,(280\,c+81\,x\,(7\,d+20\,e\,x))+4\,a\,b\,x^3\left(980\,c+3\,x\left(399\,d+510\,e\,x+28\,x^2\,(25\,f+39\,g\,x)\right)\right)+8\,a^2\,(280\,c+3\,x\,(105\,d+4\,x\,(30\,e+7\,x\,(5\,f+6\,g\,x))))\right)+$$

$$\frac{1}{6720\,a^{3/2}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}\left(280\,b^3\,c\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-\right.$$

$$1680\,a\,b^2\,f\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+567\,\sqrt{a}\,b^{8/3}\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}$$

$$\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-9072\,a^{3/2}\,b^{5/3}\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)$$

$$\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$1620\,\sqrt{2}\,a^{5/6}\,b^{7/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 472: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^{11}}\,dx$$

Optimal (type 4, 764 leaves, 13 steps):

$$-\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680}-\frac{27b^2c\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2d\sqrt{a+bx^3}}{24ax^3}-\frac{27b^2e\sqrt{a+bx^3}}{320ax^2}+\frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x}-$$

$$\frac{27b^{7/3}(bc-4af)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)\left(a+bx^3\right)^{3/2}}{2520}+\frac{b^2(bd-6ag)\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]}{24a^{3/2}}+$$

$$\left(27\times3^{1/4}\sqrt{2-\sqrt{3}}\;b^{7/3}(bc-4af)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(896a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+bx^3}\right)-\left(9\times3^{3/4}\sqrt{2+\sqrt{3}}\;b^{7/3}\left(7a^{2/3}b^{1/3}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2240a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+bx^3}\right)$$

Result (type 4, 930 leaves) :

$$-\frac{1}{20160\,a^2\,x^{10}}\sqrt{a+b\,x^3}\,\left(-1215\,b^3\,c\,x^9+8\,a^3\,\left(252\,c+5\,x\,\left(56\,d+63\,e\,x+72\,f\,x^2+84\,g\,x^3\right)\right)+3\,a\,b^2\,x^6\,\left(162\,c+x\,\left(280\,d+81\,x\,\left(7\,e+20\,f\,x\right)\right)\right)+4\,a^2\,b\,x^3\,\left(828\,c+x\,\left(980\,d+3\,x\,\left(399\,e+510\,f\,x+700\,g\,x^2\right)\right)\right)\right)+\frac{1}{6720\,a^2\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$b^2\left(280\,\sqrt{a}\,b\,d\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-1680\,a^{3/2}\,g\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+567\right.$$

$$a\,b^{2/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$405\,\sqrt{2}\,a^{1/3}\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$1620\,\sqrt{2}\,a^{4/3}\,b^{1/3}\,f\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

- **Problem 473: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^{12}}\,dx$$

Optimal (type 4, 796 leaves, 14 steps) :

$$-\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480}-\frac{27b^2c\sqrt{a+bx^3}}{1760ax^5}-\frac{27b^2d\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2e\sqrt{a+bx^3}}{24ax^3}+\frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2}+$$

$$\frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(bd-4ag)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720}+\frac{b^3e\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]}{24a^{3/2}}+$$

$$\left(27\times 3^{1/4}\sqrt{2-\sqrt{3}}\,b^{7/3}(bd-4ag)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(896a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)+\left(9\times 3^{3/4}\sqrt{2+\sqrt{3}}\,b^{7/3}\left(7b^{1/3}(7bc-22af)+110\left(1-\sqrt{3}\right)a^{1/3}(bd-4ag)\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(49280a^2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 1017 leaves) :

$$-\frac{1}{443520\,a^2\,x^{11}}\sqrt{a+b\,x^3}\;\Big(-243\,b^3\,x^9\,(49\,c+110\,d\,x)+16\,a^3\,(2520\,c+11\,x\,(252\,d+5\,x\,(56\,e+9\,x\,(7\,f+8\,g\,x))))+$$
$$6\,a\,b^2\,x^6\,(1134\,c+11\,x\,(162\,d+x\,(280\,e+81\,x\,(7\,f+20\,g\,x))))+8\,a^2\,b\,x^3\,(7875\,c+11\,x\,(828\,d+x\,(980\,e+9\,x\,(133\,f+170\,g\,x))))\Big)+$$

$$\frac{1}{147840\,a^2\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}}\,b^{7/3}\left(6160\sqrt{a}\,b^{2/3}\,e\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\Big]-3969\,b^{4/3}\,c\right.$$

$$\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\Big[\text{ArcSin}\Big[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\Big],(-1)^{1/3}\Big]+12474$$

$$a\,b^{1/3}\,f\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\Big[\text{ArcSin}\Big[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\Big],(-1)^{1/3}\Big]-$$

$$8910\sqrt{2}\,a^{1/3}\,b\,d\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]-\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]\right)+$$

$$35640\sqrt{2}\,a^{4/3}\,g\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left.\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]-\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]\right)\right)$$

- **Problem 495: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}\,dx$$

Optimal (type 4, 418 leaves, 14 steps):

$$\frac{2 a c x \sqrt{a+b x^4}}{21 b}-\frac{a d x^2 \sqrt{a+b x^4}}{16 b}+\frac{2 a e x^3 \sqrt{a+b x^4}}{45 b}-\frac{2 a^2 e x \sqrt{a+b x^4}}{15 b^{3/2} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{1}{63} x^5 \left(9 c+7 e x^2\right) \sqrt{a+b x^4}+\frac{f x^4 \left(a+b x^4\right)^{3/2}}{10 b}-$$

$$\frac{\left(8 a f-15 b d x^2\right) \left(a+b x^4\right)^{3/2}}{120 b^2}-\frac{a^2 d \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 b^{3/2}}+\frac{2 a^{9/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 b^{7/4} \sqrt{a+b x^4}}-$$

$$\frac{a^{7/4} \left(5 \sqrt{b} c+7 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{105 b^{7/4} \sqrt{a+b x^4}}$$

Result (type 4, 296 leaves) :

$$\frac{1}{5040 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^2 \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(-\left(a+b x^4\right) \left(336 a^2 f-2 b^2 x^5 \left(360 c+7 x \left(45 d+40 e x+36 f x^2\right)\right)-a b x (480 c+7 x (45 d+8 x (4 e+3 f x)))\right)-\right.\right.$$

$$\left.315 a^2 \sqrt{b} d \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)-672 a^{5/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]+$$

$$\left.96 a^2 \sqrt{b} \left(5 i \sqrt{b} c+7 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)$$

- **Problem 496: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3 \left(c+d x+e x^2+f x^3\right) \sqrt{a+b x^4} \, dx$$

Optimal (type 4, 394 leaves, 13 steps) :

$$\frac{2 a d x \sqrt{a+b x^4}}{21 b}-\frac{a e x^2 \sqrt{a+b x^4}}{16 b}+\frac{2 a f x^3 \sqrt{a+b x^4}}{45 b}-\frac{2 a^2 f x \sqrt{a+b x^4}}{15 b^{3/2} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{1}{63} x^5 \left(9 d+7 f x^2\right) \sqrt{a+b x^4}+$$

$$\frac{\left(4 c+3 e x^2\right) \left(a+b x^4\right)^{3/2}}{24 b}-\frac{a^2 e \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 b^{3/2}}+\frac{2 a^{9/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 b^{7/4} \sqrt{a+b x^4}}-$$

$$\frac{a^{7/4} \left(5 \sqrt{b} d+7 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{105 b^{7/4} \sqrt{a+b x^4}}$$

Result (type 4, 275 leaves) :

$$\frac{1}{5040 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^{3/2} \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\sqrt{b} \left(a+b x^4\right) \left(10 b x^4 (84 c+x (72 d+7 x (9 e+8 f x)))+a (840 c+x (480 d+7 x (45 e+32 f x)))\right)-315 a^2 e \right.\right.$$

$$\left.\sqrt{a+b x^4} \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)-672 a^{5/2} f \sqrt{1+\frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]+$$

$$\left.96 a^2 \left(5 i \sqrt{b} d+7 \sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)$$

- **Problem 497: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 \left(c+d x+e x^2+f x^3\right) \sqrt{a+b x^4} \, dx$$

Optimal (type 4, 369 leaves, 12 steps) :

$$\frac{2\,a\,e\,x\,\sqrt{a+b\,x^4}}{21\,b}-\frac{a\,f\,x^2\,\sqrt{a+b\,x^4}}{16\,b}+\frac{2\,a\,c\,x\,\sqrt{a+b\,x^4}}{5\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{35}\,x^3\,\left(7\,c+5\,e\,x^2\right)\,\sqrt{a+b\,x^4}+$$

$$\frac{\left(4\,d+3\,f\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}}{24\,b}-\frac{a^2\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{16\,b^{3/2}}-\frac{2\,a^{5/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,b^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{a^{5/4}\,\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{105\,b^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 280 leaves) :

$$\frac{1}{1680\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\sqrt{b}\,\left(a+b\,x^4\right)\,\left(5\,a\,\left(56\,d+x\,\left(32\,e+21\,f\,x\right)\right)+2\,b\,x^3\,\left(168\,c+5\,x\,\left(28\,d+3\,x\,\left(8\,e+7\,f\,x\right)\right)\right)\right)-105\,a^2\,f\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+\right.$$

$$672\,a^{3/2}\,b\,c\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+$$

$$\left.32\,i\,a^{3/2}\,\sqrt{b}\,\left(21\,i\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 498: Result unnecessarily involves imaginary or complex numbers.**

$$\int x\,\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}\,dx$$

Optimal (type 4, 354 leaves, 12 steps) :

$$\frac{2\,a\,f\,x\,\sqrt{a+b\,x^4}}{21\,b}+\frac{1}{4}\,c\,x^2\,\sqrt{a+b\,x^4}+\frac{2\,a\,d\,x\,\sqrt{a+b\,x^4}}{5\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{35}\,x^3\,\left(7\,d+5\,f\,x^2\right)\,\sqrt{a+b\,x^4}+$$

$$\frac{e\,\left(a+b\,x^4\right)^{3/2}}{6\,b}+\frac{a\,c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,\sqrt{b}}-\frac{2\,a^{5/4}\,d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,b^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{a^{5/4}\,\left(21\,\sqrt{b}\,d-5\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{105\,b^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 266 leaves) :

$$\frac{1}{420\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\left(a+b\,x^4\right)\,\left(10\,a\,(7\,e+4\,f\,x)+b\,x^2\,\left(105\,c+84\,d\,x+70\,e\,x^2+60\,f\,x^3\right)\right)+105\,a\,\sqrt{b}\,c\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+\right.$$

$$168\,a^{3/2}\,\sqrt{b}\,d\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+$$

$$\left.8\,i\,a^{3/2}\,\left(21\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 499: Result unnecessarily involves imaginary or complex numbers.**

$$\int\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}\,dx$$

Optimal (type 4, 331 leaves, 11 steps) :

$$\frac{1}{4} d x^2 \sqrt{a+b x^4}+\frac{2 a e x \sqrt{a+b x^4}}{5 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{1}{15} x \left(5 c+3 e x^2\right) \sqrt{a+b x^4}+\frac{f \left(a+b x^4\right)^{3/2}}{6 b}+$$

$$\frac{a d \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{4 \sqrt{b}}-\frac{2 a^{5/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{5 b^{3/4} \sqrt{a+b x^4}}+$$

$$\frac{a^{3/4} \left(5 \sqrt{b} c+3 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 b^{3/4} \sqrt{a+b x^4}}$$

Result (type 4, 257 leaves) :

$$\frac{1}{60 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\left(a+b x^4\right) \left(10 a f+b x \left(20 c+x \left(15 d+2 x \left(6 e+5 f x\right)\right)\right)\right)+15 a \sqrt{b} d \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+24 a^{3/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}}\right.$$

$$\left.\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-8 a \sqrt{b} \left(5 i \sqrt{b} c+3 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

- **Problem 500: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \sqrt{a+b x^4}}{x} \, dx$$

Optimal (type 4, 345 leaves, 14 steps) :

$$\frac{2\,a\,f\,x\,\sqrt{a+b\,x^4}}{5\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}\,\left(2\,c+e\,x^2\right)\,\sqrt{a+b\,x^4}+\frac{1}{15}\,x\,\left(5\,d+3\,f\,x^2\right)\,\sqrt{a+b\,x^4}+\frac{a\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,\sqrt{b}}-$$

$$\frac{1}{2}\,\sqrt{a}\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{2\,a^{5/4}\,f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,b^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{a^{3/4}\,\left(5\,\sqrt{b}\,d+3\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{15\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 280 leaves) :

$$\frac{1}{60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(15\,a\,e\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+\sqrt{b}\,\left(\left(a+b\,x^4\right)\,\left(30\,c+x\,\left(20\,d+3\,x\,\left(5\,e+4\,f\,x\right)\right)\right)-30\,\sqrt{a}\,c\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)\right)+\right.$$

$$\left.24\,a^{3/2}\,f\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]-8\,a\,\left(5\,i\,\sqrt{b}\,d+3\,\sqrt{a}\,f\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 501: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}}{x^2}\,dx$$

Optimal (type 4, 341 leaves, 14 steps) :

$$\frac{2\sqrt{b}\,c\,x\sqrt{a+b\,x^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{\left(3\,c-e\,x^2\right)\sqrt{a+b\,x^4}}{3\,x}+\frac{1}{4}\left(2\,d+f\,x^2\right)\sqrt{a+b\,x^4}+\frac{a\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\sqrt{b}}-$$

$$\frac{1}{2}\sqrt{a}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{2\,a^{1/4}\,b^{1/4}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{a+b\,x^4}}+$$

$$\frac{a^{1/4}\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{3\,b^{1/4}\sqrt{a+b\,x^4}}$$

Result (type 4, 355 leaves) :

$$\left(\frac{d}{2}-\frac{c}{x}+\frac{e\,x}{3}+\frac{f\,x^2}{4}\right)\sqrt{a+b\,x^4}+\frac{1}{6}\left(\frac{3\,a\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\sqrt{b}}-3\sqrt{a}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]+\frac{1}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\,x^4}}\right.$$

$$12\sqrt{a}\,\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)-$$

$$\left.\frac{4\,i\,a\,e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\,x^4}}\right)$$

- **Problem 502: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^3}\,dx$$

Optimal (type 4, 342 leaves, 14 steps) :

$$\frac{2\sqrt{b}\,d\,x\sqrt{a+b\,x^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{\left(c-e\,x^2\right)\sqrt{a+b\,x^4}}{2\,x^2}-\frac{\left(3\,d-f\,x^2\right)\sqrt{a+b\,x^4}}{3\,x}+\frac{1}{2}\sqrt{b}\,c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-$$

$$\frac{1}{2}\sqrt{a}\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{2\,a^{1/4}\,b^{1/4}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{a+b\,x^4}}+$$

$$\frac{a^{1/4}\left(3\sqrt{b}\,d+\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{3\,b^{1/4}\sqrt{a+b\,x^4}}$$

Result (type 4, 296 leaves) :

$$\frac{1}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^2\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(-3\,c+x\left(-6\,d+3\,e\,x+2\,f\,x^2\right)\right)+3\sqrt{b}\,c\,x^2\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-3\sqrt{a}\,e\,x^2\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$12\sqrt{a}\,\sqrt{b}\,d\,x^2\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.4\,i\sqrt{a}\left(-3\,i\sqrt{b}\,d+\sqrt{a}\,f\right)x^2\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 503: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^4}\,dx$$

Optimal (type 4, 357 leaves, 15 steps) :

$$-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}\,e\,x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3}-\frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2}+\frac{1}{2}\sqrt{b}\,d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]-$$

$$\frac{1}{2}\sqrt{a}\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right]-\frac{2a^{1/4}b^{1/4}e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{a+bx^4}}+$$

$$\frac{b^{1/4}\left(\sqrt{b}\,c+3\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{3a^{1/4}\sqrt{a+bx^4}}$$

Result (type 4, 295 leaves) :

$$\frac{1}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^3\sqrt{a+bx^4}}$$

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(2c+3x\left(d+2ex-fx^2\right)\right)-3\sqrt{b}\,d\,x^3\sqrt{a+bx^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]+3\sqrt{a}\,f\,x^3\sqrt{a+bx^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right]\right)+\right.$$

$$12\sqrt{a}\,\sqrt{b}\,e\,x^3\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.4\sqrt{b}\left(i\sqrt{b}\,c+3\sqrt{a}\,e\right)x^3\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 504: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^5}\,dx$$

Optimal (type 4, 329 leaves, 13 steps) :

$$-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}\,f\,x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}\,x^2}+\frac{1}{2}\sqrt{b}\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]-$$

$$\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right]}{4\sqrt{a}}-\frac{2\,a^{1/4}\,b^{1/4}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{\sqrt{a+bx^4}}+$$

$$\frac{b^{1/4}\left(\sqrt{b}\,d+3\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{3\,a^{1/4}\sqrt{a+bx^4}}$$

Result (type 4, 267 leaves) :

$$\frac{1}{12}\left(-\frac{\sqrt{a+bx^4}\left(3c+4dx+6x^2(e+2fx)\right)}{x^4}+6\sqrt{b}\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]-\right.$$

$$\frac{3\,b\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right]}{\sqrt{a}}-\frac{24\,i\,a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,f\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{a+bx^4}}-$$

$$\left.\frac{8\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\,d-3\,i\sqrt{a}\,f\right)\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{a+bx^4}}\right)$$

- **Problem 505: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^6}\,dx$$

Optimal (type 4, 360 leaves, 14 steps) :

$$-\frac{1}{60}\left(\frac{12\,c}{x^5}+\frac{15\,d}{x^4}+\frac{20\,e}{x^3}+\frac{30\,f}{x^2}\right)\sqrt{a+b\,x^4}-\frac{2\,b\,c\,\sqrt{a+b\,x^4}}{5\,a\,x}+\frac{2\,b^{3/2}\,c\,x\,\sqrt{a+b\,x^4}}{5\,a\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{2}\sqrt{b}\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-$$

$$\frac{b\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{4\,\sqrt{a}}-\frac{2\,b^{5/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{b^{3/4}\,\left(3\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{15\,a^{3/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 314 leaves) :

$$\frac{1}{60\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^5\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(12\,a\,c+24\,b\,c\,x^4+5\,a\,x\,\left(3\,d+4\,e\,x+6\,f\,x^2\right)\right)-30\,a\,\sqrt{b}\,f\,x^5\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+15\,\sqrt{a}\right.\right.$$

$$\left.b\,d\,x^5\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+24\,\sqrt{a}\,b^{3/2}\,c\,x^5\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.8\,i\,\sqrt{a}\,b\,\left(-3\,i\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\,x^5\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 506: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}}{x^7}\,dx$$

Optimal (type 4, 352 leaves, 12 steps) :

$$-\frac{1}{60}\left(\frac{10\,c}{x^6}+\frac{12\,d}{x^5}+\frac{15\,e}{x^4}+\frac{20\,f}{x^3}\right)\sqrt{a+b\,x^4}-\frac{b\,c\,\sqrt{a+b\,x^4}}{6\,a\,x^2}-\frac{2\,b\,d\,\sqrt{a+b\,x^4}}{5\,a\,x}+\frac{2\,b^{3/2}\,d\,x\,\sqrt{a+b\,x^4}}{5\,a\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-$$

$$\frac{b\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{4\,\sqrt{a}}-\frac{2\,b^{5/4}\,d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{b^{3/4}\,\left(3\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{15\,a^{3/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 277 leaves):

$$\frac{1}{60\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^6\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\left(a+b\,x^4\right)\,\left(10\,a\,c+2\,b\,x^4\,(5\,c+12\,d\,x)+a\,x\,(12\,d+5\,x\,(3\,e+4\,f\,x))\right)+15\,\sqrt{a}\,b\,e\,x^6\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$24\,\sqrt{a}\,b^{3/2}\,d\,x^6\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]-$$

$$\left.8\,i\,\sqrt{a}\,b\,\left(-3\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,x^6\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 507: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}}{x^8}\,dx$$

Optimal (type 4, 375 leaves, 13 steps):

$$-\frac{1}{420}\left(\frac{60\,c}{x^7}+\frac{70\,d}{x^6}+\frac{84\,e}{x^5}+\frac{105\,f}{x^4}\right)\sqrt{a+b\,x^4}-\frac{2\,b\,c\,\sqrt{a+b\,x^4}}{21\,a\,x^3}-\frac{b\,d\,\sqrt{a+b\,x^4}}{6\,a\,x^2}-\frac{2\,b\,e\,\sqrt{a+b\,x^4}}{5\,a\,x}+$$

$$\frac{2\,b^{3/2}\,e\,x\,\sqrt{a+b\,x^4}}{5\,a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{4\,\sqrt{a}}-\frac{2\,b^{5/4}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{b^{5/4}\left(5\,\sqrt{b}\,c-21\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{105\,a^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 283 leaves) :

$$\frac{1}{420\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^7\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(2\,b\,x^4\,(20\,c+7\,x\,(5\,d+12\,e\,x))+a\,(60\,c+7\,x\,(10\,d+3\,x\,(4\,e+5\,f\,x)))\right)+105\,\sqrt{a}\,b\,f\,x^7\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$168\,\sqrt{a}\,b^{3/2}\,e\,x^7\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.8\,b^{3/2}\left(-5\,i\,\sqrt{b}\,c+21\,\sqrt{a}\,e\right)x^7\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 508: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^9}\,dx$$

Optimal (type 4, 400 leaves, 14 steps) :

$$-\frac{1}{840}\left(\frac{105\,c}{x^8}+\frac{120\,d}{x^7}+\frac{140\,e}{x^6}+\frac{168\,f}{x^5}\right)\sqrt{a+b\,x^4}-\frac{b\,c\,\sqrt{a+b\,x^4}}{16\,a\,x^4}-\frac{2\,b\,d\,\sqrt{a+b\,x^4}}{21\,a\,x^3}-\frac{b\,e\,\sqrt{a+b\,x^4}}{6\,a\,x^2}-\frac{2\,b\,f\,\sqrt{a+b\,x^4}}{5\,a\,x}+$$

$$\frac{2\,b^{3/2}\,f\,x\,\sqrt{a+b\,x^4}}{5\,a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{b^2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{16\,a^{3/2}}-\frac{2\,b^{5/4}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{b^{5/4}\left(5\,\sqrt{b}\,d-21\,\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{105\,a^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 293 leaves) :

$$\frac{1}{1680\,a^{3/2}\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x^8\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\,\left(a+b\,x^4\right)\left(b\,x^4\left(105\,c+8\,x\left(20\,d+35\,e\,x+84\,f\,x^2\right)\right)+a\,\left(210\,c+8\,x\,\left(30\,d+7\,x\,\left(5\,e+6\,f\,x\right)\right)\right)\right)+105\,b^2\right.\right.$$

$$\left.c\,x^8\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+672\,a\,b^{3/2}\,f\,x^8\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]-$$

$$\left.32\,\sqrt{a}\;b^{3/2}\left(-5\,i\,\sqrt{b}\;d+21\,\sqrt{a}\;f\right)x^8\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

- **Problem 509: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^{10}}\,dx$$

Optimal (type 4, 425 leaves, 15 steps) :

$$-\frac{1}{504}\left(\frac{56\,c}{x^9}+\frac{63\,d}{x^8}+\frac{72\,e}{x^7}+\frac{84\,f}{x^6}\right)\sqrt{a+b\,x^4}-\frac{2\,b\,c\,\sqrt{a+b\,x^4}}{45\,a\,x^5}-\frac{b\,d\,\sqrt{a+b\,x^4}}{16\,a\,x^4}-\frac{2\,b\,e\,\sqrt{a+b\,x^4}}{21\,a\,x^3}-\frac{b\,f\,\sqrt{a+b\,x^4}}{6\,a\,x^2}+\frac{2\,b^2\,c\,\sqrt{a+b\,x^4}}{15\,a^2\,x}-$$

$$\frac{2\,b^{5/2}\,c\,x\,\sqrt{a+b\,x^4}}{15\,a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{b^2\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{16\,a^{3/2}}+\frac{2\,b^{9/4}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{15\,a^{7/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{b^{7/4}\left(7\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{105\,a^{7/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 305 leaves) :

$$\frac{1}{5040\,a^2\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^9\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(-\left(a+b\,x^4\right)\left(-672\,b^2\,c\,x^8+10\,a^2\left(56\,c+63\,d\,x+72\,e\,x^2+84\,f\,x^3\right)+a\,b\,x^4\,\left(224\,c+15\,x\,\left(21\,d+8\,x\,\left(4\,e+7\,f\,x\right)\right)\right)\right)+\right.\right.$$

$$\left.315\,\sqrt{a}\,b^2\,d\,x^9\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)-672\,\sqrt{a}\,b^{5/2}\,c\,x^9\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$\left.96\,\sqrt{a}\,b^2\left(7\,\sqrt{b}\,c+5\,i\,\sqrt{a}\,e\right)x^9\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 510: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}\,dx$$

Optimal (type 4, 476 leaves, 16 steps) :

$$\frac{4\,a^2\,c\,x\,\sqrt{a+b\,x^4}}{77\,b}-\frac{a^2\,d\,x^2\,\sqrt{a+b\,x^4}}{32\,b}+\frac{4\,a^2\,e\,x^3\,\sqrt{a+b\,x^4}}{195\,b}-\frac{4\,a^3\,e\,x\,\sqrt{a+b\,x^4}}{65\,b^{3/2}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2\,a\,x^5\,\left(117\,c+77\,e\,x^2\right)\,\sqrt{a+b\,x^4}}{3003}-$$

$$\frac{a\,d\,x^2\,\left(a+b\,x^4\right)^{3/2}}{48\,b}+\frac{1}{143}\,x^5\,\left(13\,c+11\,e\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}+\frac{f\,x^4\,\left(a+b\,x^4\right)^{5/2}}{14\,b}-\frac{\left(12\,a\,f-35\,b\,d\,x^2\right)\,\left(a+b\,x^4\right)^{5/2}}{420\,b^2}-$$

$$\frac{a^3\,d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{32\,b^{3/2}}+\frac{4\,a^{13/4}\,e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{65\,b^{7/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{2\,a^{11/4}\,\left(65\,\sqrt{b}\,c+77\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5005\,b^{7/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 327 leaves) :

$$\frac{1}{480480\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^2\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(-\left(a+b\,x^4\right)\,\left(13728\,a^3\,f-40\,b^3\,x^9\,\left(1092\,c+11\,x\,\left(91\,d+84\,e\,x+78\,f\,x^2\right)\right)-a^2\,b\,x\,\left(24960\,c+11\,x\,\left(1365\,d+896\,e\,x+624\,f\,x^2\right)\right)-\right.\right.\right.$$

$$\left.\left.2\,a\,b^2\,x^5\,\left(40560\,c+11\,x\,\left(3185\,d+2800\,e\,x+2496\,f\,x^2\right)\right)\right)-15015\,a^3\,\sqrt{b}\,d\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)-$$

$$29568\,a^{7/2}\,\sqrt{b}\,e\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+384\,a^3\,\sqrt{b}\,\left(65\,i\,\sqrt{b}\,c+77\,\sqrt{a}\,e\right)$$

$$\left.\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 511: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3\,\left(c+d\,x+e\,x^2+f\,x^3\right)\,\left(a+b\,x^4\right)^{3/2}\,dx$$

Optimal (type 4, 452 leaves, 15 steps):

$$\frac{4 a^2 d x \sqrt{a+b x^4}}{77 b}-\frac{a^2 e x^2 \sqrt{a+b x^4}}{32 b}+\frac{4 a^2 f x^3 \sqrt{a+b x^4}}{195 b}-\frac{4 a^3 f x \sqrt{a+b x^4}}{65 b^{3/2} \left(\sqrt{a}+\sqrt{b} x^2\right)}+$$
$$\frac{2 a x^5 \left(117 d+77 f x^2\right) \sqrt{a+b x^4}}{3003}-\frac{a e x^2 \left(a+b x^4\right)^{3/2}}{48 b}+\frac{1}{143} x^5 \left(13 d+11 f x^2\right) \left(a+b x^4\right)^{3/2}+\frac{\left(6 c+5 e x^2\right) \left(a+b x^4\right)^{5/2}}{60 b}-$$
$$\frac{a^3 e \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{32 b^{3/2}}+\frac{4 a^{13/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{65 b^{7/4} \sqrt{a+b x^4}}-$$
$$\frac{2 a^{11/4} \left(65 \sqrt{b} d+77 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5005 b^{7/4} \sqrt{a+b x^4}}$$

Result (type 4, 306 leaves):

$$\frac{1}{480480 b^2 \sqrt{a+b x^4}} \Bigg(b \left(a+b x^4\right) \left(56 b^2 x^8 \left(858 c+780 d x+715 e x^2+660 f x^3\right)+\right.$$
$$\left.2 a b x^4 (48048 c+5 x (8112 d+77 x (91 e+80 f x)))+a^2 (48048 c+x (24960 d+77 x (195 e+128 f x)))\right)-$$
$$15015 a^3 \sqrt{b} e \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]+29568 i a^4 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} f \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]+$$
$$384 a^{7/2} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(65 \sqrt{b} d-77 i \sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\Bigg)$$

- **Problem 512: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 \left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2} \, dx$$

Optimal (type 4, 427 leaves, 14 steps):

$$\frac{4 a^2 e x \sqrt{a+b x^4}}{77 b}-\frac{a^2 f x^2 \sqrt{a+b x^4}}{32 b}+\frac{4 a^2 c x \sqrt{a+b x^4}}{15 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{2 a x^3 \left(77 c+45 e x^2\right) \sqrt{a+b x^4}}{1155}-$$

$$\frac{a f x^2 \left(a+b x^4\right)^{3/2}}{48 b}+\frac{1}{99} x^3 \left(11 c+9 e x^2\right) \left(a+b x^4\right)^{3/2}+\frac{\left(6 d+5 f x^2\right) \left(a+b x^4\right)^{5/2}}{60 b}-$$

$$\frac{a^3 f \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{32 b^{3/2}}-\frac{4 a^{9/4} c \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 b^{3/4} \sqrt{a+b x^4}}+$$

$$\frac{2 a^{9/4} \left(77 \sqrt{b} c-15 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{1155 b^{5/4} \sqrt{a+b x^4}}$$

Result (type 4, 325 leaves):

$$\frac{1}{110880 b}\sqrt{a+b x^4}$$
$$\left(9 a^2 (1232 d+5 x (128 e+77 f x))+56 b^2 x^7 \left(220 c+3 x \left(66 d+60 e x+55 f x^2\right)\right)+2 a b x^3 (13552 c+3 x (3696 d+5 x (624 e+539 f x)))\right)-$$

$$\frac{a^3 f \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{32 b^{3/2}}+\frac{4 i a^2 c \sqrt{1+\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)}{15 \left(\frac{i \sqrt{b}}{\sqrt{a}}\right)^{3/2} \sqrt{a+b x^4}}+$$

$$\frac{4 i a^3 e \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{77 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}}$$

- **Problem 513: Result unnecessarily involves imaginary or complex numbers.**

$$\int x \left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2} dx$$

Optimal (type 4, 409 leaves, 14 steps):

$$\frac{4 a^2 f x \sqrt{a+b x^4}}{77 b}+\frac{3}{16} a c x^2 \sqrt{a+b x^4}+\frac{4 a^2 d x \sqrt{a+b x^4}}{15 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+$$

$$\frac{2 a x^3 \left(77 d+45 f x^2\right) \sqrt{a+b x^4}}{1155}+\frac{1}{8} c x^2 \left(a+b x^4\right)^{3/2}+\frac{1}{99} x^3 \left(11 d+9 f x^2\right) \left(a+b x^4\right)^{3/2}+\frac{e \left(a+b x^4\right)^{5/2}}{10 b}+$$

$$\frac{3 a^2 c \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-\frac{4 a^{9/4} d \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 b^{3/4} \sqrt{a+b x^4}}+$$

$$\frac{2 a^{9/4} \left(77 \sqrt{b} d-15 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{1155 b^{5/4} \sqrt{a+b x^4}}$$

Result (type 4, 302 leaves) :

$$\frac{1}{55440 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\left(a+b x^4\right) \left(72 a^2 (77 e+40 f x)+14 b^2 x^6 \left(495 c+4 x \left(110 d+99 e x+90 f x^2\right)\right)+a b x^2 (17325 c+16 x (847 d+9 x (77 e+65 f x)))\right)+\right.\right.$$

$$\left.10395 a^2 \sqrt{b} c \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+14784 a^{5/2} \sqrt{b} d \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]+$$

$$\left.192 i a^{5/2} \left(77 i \sqrt{b} d+15 \sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

- **Problem 514: Result unnecessarily involves imaginary or complex numbers.**

$$\int \left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2} dx$$

Optimal (type 4, 382 leaves, 13 steps) :

$$\frac{3}{16} a d x^2 \sqrt{a+b x^4}+\frac{4 a^2 e x \sqrt{a+b x^4}}{15 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{2}{105} a x \left(15 c+7 e x^2\right) \sqrt{a+b x^4}+\frac{1}{8} d x^2 \left(a+b x^4\right)^{3/2}+\frac{1}{63} x \left(9 c+7 e x^2\right) \left(a+b x^4\right)^{3/2}+$$

$$\frac{f \left(a+b x^4\right)^{5/2}}{10 b}+\frac{3 a^2 d \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-\frac{4 a^{9/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 b^{3/4} \sqrt{a+b x^4}}+$$

$$\frac{2 a^{7/4} \left(15 \sqrt{b} c+7 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{105 b^{3/4} \sqrt{a+b x^4}}$$

Result (type 4, 294 leaves) :

$$\frac{1}{5040 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\left(a+b x^4\right) \left(504 a^2 f+2 b^2 x^5 \left(360 c+7 x \left(45 d+40 e x+36 f x^2\right)\right)+a b x \left(2160 c+7 x \left(225 d+16 x \left(11 e+9 f x\right)\right)\right)\right)+\right.\right.$$

$$\left.945 a^2 \sqrt{b} d \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+1344 a^{5/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-$$

$$\left.192 a^2 \sqrt{b} \left(15 i \sqrt{b} c+7 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

- **Problem 515: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2}}{x} \, dx$$

Optimal (type 4, 403 leaves, 16 steps) :

$$\frac{4 a^2 f x \sqrt{a+b x^4}}{15 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{1}{16} a \left(8 c+3 e x^2\right) \sqrt{a+b x^4}+\frac{2}{105} a x \left(15 d+7 f x^2\right) \sqrt{a+b x^4}+$$

$$\frac{1}{24} \left(4 c+3 e x^2\right) \left(a+b x^4\right)^{3/2}+\frac{1}{63} x \left(9 d+7 f x^2\right) \left(a+b x^4\right)^{3/2}+\frac{3 a^2 e \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-$$

$$\frac{1}{2} a^{3/2} c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]-\frac{4 a^{9/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 b^{3/4} \sqrt{a+b x^4}}+$$

$$\frac{2 a^{7/4} \left(15 \sqrt{b} d+7 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{105 b^{3/4} \sqrt{a+b x^4}}$$

Result (type 4, 319 leaves):

$$\frac{\sqrt{a+b x^4} \left(10 b x^4 (84 c+x (72 d+7 x (9 e+8 f x)))+a (3360 c+x (2160 d+7 x (225 e+176 f x)))\right)}{5040}+\frac{3 a^2 e \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-$$

$$\frac{1}{2} a^{3/2} c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]+\frac{4 i a^2 f \sqrt{1+\frac{b x^4}{a}} \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)}{15 \left(\frac{i \sqrt{b}}{\sqrt{a}}\right)^{3/2} \sqrt{a+b x^4}}-$$

$$\frac{4 i a^2 d \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]}{7 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{a+b x^4}}$$

- **Problem 516: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2}}{x^2} \, dx$$

Optimal (type 4, 404 leaves, 16 steps):

$$\frac{12\,a\sqrt{b}\,c\,x\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2}{35}\,x\left(5\,a\,e+21\,b\,c\,x^2\right)\sqrt{a+b\,x^4}+\frac{1}{16}\,a\left(8\,d+3\,f\,x^2\right)\sqrt{a+b\,x^4}-\frac{\left(7\,c-e\,x^2\right)\left(a+b\,x^4\right)^{3/2}}{7\,x}+\frac{1}{24}\left(4\,d+3\,f\,x^2\right)\left(a+b\,x^4\right)^{3/2}+$$

$$\frac{3\,a^2\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{16\,\sqrt{b}}-\frac{1}{2}\,a^{3/2}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{12\,a^{5/4}\,b^{1/4}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,\sqrt{a+b\,x^4}}+$$

$$\frac{2\,a^{5/4}\left(21\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{35\,b^{1/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 328 leaves):

$$\sqrt{a+b\,x^4}\left(a\left(\frac{2\,d}{3}-\frac{c}{x}+\frac{3\,e\,x}{7}+\frac{5\,f\,x^2}{16}\right)+b\left(\frac{c\,x^3}{5}+\frac{d\,x^4}{6}+\frac{e\,x^5}{7}+\frac{f\,x^6}{8}\right)\right)+\frac{3\,a^2\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{16\,\sqrt{b}}-\frac{1}{2}\,a^{3/2}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]+$$

$$\frac{12\,i\,a\,b\,c\,\sqrt{1+\frac{b\,x^4}{a}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)}{5\left(\frac{i\,\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+b\,x^4}}-$$

$$\frac{4\,i\,a^2\,e\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{7\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\,x^4}}$$

- **Problem 517: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^3}\,dx$$

Optimal (type 4, 406 leaves, 16 steps):

$$\frac{12\,a\,\sqrt{b}\,d\,x\,\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}\left(2\,a\,e+3\,b\,c\,x^2\right)\sqrt{a+b\,x^4}+\frac{2}{35}\,x\left(5\,a\,f+21\,b\,d\,x^2\right)\sqrt{a+b\,x^4}-\frac{\left(3\,c-e\,x^2\right)\left(a+b\,x^4\right)^{3/2}}{6\,x^2}-\frac{\left(7\,d-f\,x^2\right)\left(a+b\,x^4\right)^{3/2}}{7\,x}+$$

$$\frac{3}{4}\,a\,\sqrt{b}\,c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{1}{2}\,a^{3/2}\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{12\,a^{5/4}\,b^{1/4}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,\sqrt{a+b\,x^4}}+$$

$$\frac{2\,a^{5/4}\left(21\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{35\,b^{1/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 326 leaves) :

$$\frac{1}{420\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^2\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(-210\,a\,c+b\,x^4\left(105\,c+84\,d\,x+70\,e\,x^2+60\,f\,x^3\right)+20\,a\,x\,\left(-21\,d+x\,\left(14\,e+9\,f\,x\right)\right)\right)+315\,a\,\sqrt{b}\,c\,x^2\,\sqrt{a+b\,x^4}\right.\right.$$

$$\left.\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-210\,a^{3/2}\,e\,x^2\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+1008\,a^{3/2}\,\sqrt{b}\,d\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}$$

$$\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-48\,i\,a^{3/2}\left(-21\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 518: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 408 leaves, 16 steps) :

$$\frac{12\,a\,\sqrt{b}\,e\,x\,\sqrt{a+b\,x^4}}{5\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2\,\left(9\,a\,e-5\,b\,c\,x^2\right)\,\sqrt{a+b\,x^4}}{15\,x}+\frac{1}{4}\,\left(2\,a\,f+3\,b\,d\,x^2\right)\,\sqrt{a+b\,x^4}-\frac{\left(5\,c-3\,e\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}}{15\,x^3}-\frac{\left(3\,d-f\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}}{6\,x^2}+$$

$$\frac{3}{4}\,a\,\sqrt{b}\,d\,\text{ArcTanh}\Big[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\Big]-\frac{1}{2}\,a^{3/2}\,f\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\Big]-\frac{12\,a^{5/4}\,b^{1/4}\,e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\Big[2\,\text{ArcTan}\Big[\frac{b^{1/4}\,x}{a^{1/4}}\Big],\,\frac{1}{2}\Big]}{5\,\sqrt{a+b\,x^4}}+$$

$$\frac{2\,a^{3/4}\,b^{1/4}\,\left(5\,\sqrt{b}\,c+9\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{b^{1/4}\,x}{a^{1/4}}\Big],\,\frac{1}{2}\Big]}{15\,\sqrt{a+b\,x^4}}$$

Result (type 4, 327 leaves) :

$$\frac{1}{60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^3\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\left(a+b\,x^4\right)\,\left(-10\,a\,\left(2\,c+x\,\left(3\,d+6\,e\,x-4\,f\,x^2\right)\right)+b\,x^4\,\left(20\,c+x\,\left(15\,d+2\,x\,\left(6\,e+5\,f\,x\right)\right)\right)\right)+45\,a\,\sqrt{b}\,d\,x^3\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\Big[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\Big]-\right.\right.$$

$$\left.30\,a^{3/2}\,f\,x^3\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\Big]\right)+144\,a^{3/2}\,\sqrt{b}\,e\,x^3\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\Big],\,-1\Big]-$$

$$\left.16\,a\,\sqrt{b}\,\left(5\,i\,\sqrt{b}\,c+9\,\sqrt{a}\,e\right)\,x^3\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\Big],\,-1\Big]\right)$$

- **Problem 519: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\left(a+b\,x^4\right)^{3/2}}{x^5}\,dx$$

Optimal (type 4, 386 leaves, 15 steps) :

$$\frac{12\,a\sqrt{b}\,f\,x\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{3}{4}\,b\left(c+e\,x^2\right)\sqrt{a+b\,x^4}+\frac{2}{15}\,b\,x\left(5\,d+9\,f\,x^2\right)\sqrt{a+b\,x^4}-\frac{1}{12}\left(\frac{3\,c}{x^4}+\frac{4\,d}{x^3}+\frac{6\,e}{x^2}+\frac{12\,f}{x}\right)\left(a+b\,x^4\right)^{3/2}+$$

$$\frac{3}{4}\,a\sqrt{b}\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{3}{4}\sqrt{a}\,b\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{12\,a^{5/4}\,b^{1/4}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\sqrt{a+b\,x^4}}+$$

$$\frac{2\,a^{3/4}\,b^{1/4}\left(5\sqrt{b}\,d+9\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{15\sqrt{a+b\,x^4}}$$

Result (type 4, 329 leaves) :

$$\frac{1}{60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^4\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(a+b\,x^4\right)\left(5\,a\left(3\,c+4\,d\,x+6\,x^2\left(e+2\,f\,x\right)\right)-b\,x^4\left(30\,c+x\left(20\,d+3\,x\left(5\,e+4\,f\,x\right)\right)\right)\right)+45\,a\sqrt{b}\,e\,x^4\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\right.\right.$$

$$\left.45\sqrt{a}\,b\,c\,x^4\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+144\,a^{3/2}\sqrt{b}\,f\,x^4\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.16\,a\sqrt{b}\left(5\,i\sqrt{b}\,d+9\sqrt{a}\,f\right)x^4\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 520: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^6}\,dx$$

Optimal (type 4, 387 leaves, 15 steps) :

$$\frac{12 b^{3/2} c x \sqrt{a+b x^4}}{5 \left(\sqrt{a}+\sqrt{b} x^2\right)}-\frac{2 b \left(9 c-5 e x^2\right) \sqrt{a+b x^4}}{15 x}+\frac{3}{4} b \left(d+f x^2\right) \sqrt{a+b x^4}-\frac{1}{60} \left(\frac{12 c}{x^5}+\frac{15 d}{x^4}+\frac{20 e}{x^3}+\frac{30 f}{x^2}\right) \left(a+b x^4\right)^{3/2}+$$

$$\frac{3}{4} a \sqrt{b} f \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]-\frac{3}{4} \sqrt{a} b d \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]-\frac{12 a^{1/4} b^{5/4} c \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{a+b x^4}}+$$

$$\frac{2 a^{1/4} b^{3/4} \left(9 \sqrt{b} c+5 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 \sqrt{a+b x^4}}$$

Result (type 4, 331 leaves) :

$$\frac{1}{60 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x^5 \sqrt{a+b x^4}}$$

$$\left(-\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\left(a+b x^4\right) \left(12 a c+84 b c x^4+5 a x \left(3 d+4 e x+6 f x^2\right)-5 b x^5 (6 d+x (4 e+3 f x))\right)-45 a \sqrt{b} f x^5 \sqrt{a+b x^4} \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]+\right.\right.$$

$$\left.45 \sqrt{a} b d x^5 \sqrt{a+b x^4} \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+144 \sqrt{a} b^{3/2} c x^5 \sqrt{1+\frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]-$$

$$\left.16 i \sqrt{a} b \left(-9 i \sqrt{b} c+5 \sqrt{a} e\right) x^5 \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)$$

- **Problem 521: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2}}{x^7} \, dx$$

Optimal (type 4, 392 leaves, 15 steps) :

$$\frac{12\,b^{3/2}\,d\,x\,\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b\left(2\,c-3\,e\,x^2\right)\sqrt{a+b\,x^4}}{4\,x^2}-\frac{2\,b\left(9\,d-5\,f\,x^2\right)\sqrt{a+b\,x^4}}{15\,x}-\frac{1}{60}\left(\frac{10\,c}{x^6}+\frac{12\,d}{x^5}+\frac{15\,e}{x^4}+\frac{20\,f}{x^3}\right)\left(a+b\,x^4\right)^{3/2}+$$

$$\frac{1}{2}\,b^{3/2}\,c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{3}{4}\,\sqrt{a}\,b\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{12\,a^{1/4}\,b^{5/4}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,\sqrt{a+b\,x^4}}+$$

$$\frac{2\,a^{1/4}\,b^{3/4}\left(9\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{15\,\sqrt{a+b\,x^4}}$$

Result (type 4, 331 leaves) :

$$\frac{1}{60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^6\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(2\,b\,x^4\,(20\,c+x\,(42\,d-5\,x\,(3\,e+2\,f\,x)))+a\,(10\,c+x\,(12\,d+5\,x\,(3\,e+4\,f\,x)))\right)-30\,b^{3/2}\,c\,x^6\,\sqrt{a+b\,x^4}\right.\right.$$

$$\left.\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+45\,\sqrt{a}\,b\,e\,x^6\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+144\,\sqrt{a}\,b^{3/2}\,d\,x^6\,\sqrt{1+\frac{b\,x^4}{a}}$$

$$\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-16\,i\,\sqrt{a}\,b\left(-9\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)x^6\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 522: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^8}\,dx$$

Optimal (type 4, 412 leaves, 16 steps) :

$$-\frac{12\,b\,e\,\sqrt{a+b\,x^4}}{5\,x}+\frac{12\,b^{3/2}\,e\,x\,\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2\,b\left(5\,c-21\,e\,x^2\right)\sqrt{a+b\,x^4}}{35\,x^3}-\frac{b\left(2\,d-3\,f\,x^2\right)\sqrt{a+b\,x^4}}{4\,x^2}-\frac{1}{420}\left(\frac{60\,c}{x^7}+\frac{70\,d}{x^6}+\frac{84\,e}{x^5}+\frac{105\,f}{x^4}\right)\left(a+b\,x^4\right)^{3/2}+$$
$$\frac{1}{2}\,b^{3/2}\,d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{3}{4}\,\sqrt{a}\,b\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{12\,a^{1/4}\,b^{5/4}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5\,\sqrt{a+b\,x^4}}+$$
$$\frac{2\,b^{5/4}\left(5\,\sqrt{b}\,c+21\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{35\,a^{1/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 330 leaves) :

$$\frac{1}{420\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^7\,\sqrt{a+b\,x^4}}$$
$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(2\,b\,x^4\,\left(90\,c+7\,x\,\left(20\,d+3\,x\,\left(14\,e-5\,f\,x\right)\right)\right)+a\,\left(60\,c+7\,x\,\left(10\,d+3\,x\,\left(4\,e+5\,f\,x\right)\right)\right)\right)-210\,b^{3/2}\,d\,x^7\,\sqrt{a+b\,x^4}\right.\right.$$
$$\left.\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+315\,\sqrt{a}\,b\,f\,x^7\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+1008\,\sqrt{a}\,b^{3/2}\,e\,x^7\,\sqrt{1+\frac{b\,x^4}{a}}$$
$$\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-48\,b^{3/2}\left(5\,i\,\sqrt{b}\,c+21\,\sqrt{a}\,e\right)x^7\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 523: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^9}\,dx$$

Optimal (type 4, 377 leaves, 14 steps) :

$$-\frac{1}{560} b \left(\frac{105 c}{x^4}+\frac{160 d}{x^3}+\frac{280 e}{x^2}+\frac{672 f}{x}\right) \sqrt{a+b x^4}+\frac{12 b^{3/2} f x \sqrt{a+b x^4}}{5 \left(\sqrt{a}+\sqrt{b} x^2\right)}-\frac{1}{840} \left(\frac{105 c}{x^8}+\frac{120 d}{x^7}+\frac{140 e}{x^6}+\frac{168 f}{x^5}\right) \left(a+b x^4\right)^{3/2}+$$

$$\frac{1}{2} b^{3/2} e \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]-\frac{3 b^2 c \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16 \sqrt{a}}-\frac{12 a^{1/4} b^{5/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 \sqrt{a+b x^4}}+$$

$$\frac{2 b^{5/4} \left(5 \sqrt{b} d+21 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{35 a^{1/4} \sqrt{a+b x^4}}$$

Result (type 4, 309 leaves) :

$$-\frac{\sqrt{a+b x^4} \left(b x^4 \left(525 c+16 x \left(45 d+70 e x+147 f x^2\right)\right)+a (210 c+8 x (30 d+7 x (5 e+6 f x)))\right)}{1680 x^8}+$$

$$\frac{1}{2} b^{3/2} e \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]-\frac{3 b^2 c \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16 \sqrt{a}}-\frac{12 i a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b f \sqrt{1+\frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{5 \sqrt{a+b x^4}}-$$

$$\frac{4 b^{3/2} \left(5 i \sqrt{b} d+21 \sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]}{35 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{a+b x^4}}$$

- **Problem 524: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2}}{x^{10}} \, dx$$

Optimal (type 4, 405 leaves, 15 steps) :

$$-\frac{b\left(\frac{224 c}{x^5}+\frac{315 d}{x^4}+\frac{480 e}{x^3}+\frac{840 f}{x^2}\right)\sqrt{a+b x^4}}{1680}-\frac{4 b^2 c\sqrt{a+b x^4}}{15 a x}+\frac{4 b^{5/2} c x\sqrt{a+b x^4}}{15 a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}-\frac{1}{504}\left(\frac{56 c}{x^9}+\frac{63 d}{x^8}+\frac{72 e}{x^7}+\frac{84 f}{x^6}\right)\left(a+b x^4\right)^{3/2}+$$

$$\frac{1}{2} b^{3/2} f \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]-\frac{3 b^2 d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16\sqrt{a}}-\frac{4 b^{9/4} c\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 a^{3/4}\sqrt{a+b x^4}}+$$

$$\frac{2 b^{7/4}\left(7\sqrt{b}\, c+15\sqrt{a}\, e\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{105 a^{3/4}\sqrt{a+b x^4}}$$

Result (type 4, 351 leaves) :

$$\frac{1}{5040 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x^9\sqrt{a+b x^4}}$$

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+b x^4\right)\left(1344 b^2 c x^8+10 a^2\left(56 c+63 d x+72 e x^2+84 f x^3\right)+a b x^4\left(1232 c+15 x\left(105 d+16 x\left(9 e+14 f x\right)\right)\right)\right)-\right.\right.$$

$$\left.2520 a b^{3/2} f x^9\sqrt{a+b x^4}\, \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]+945\sqrt{a}\, b^2 d x^9\sqrt{a+b x^4}\, \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+1344\sqrt{a}\, b^{5/2} c x^9\sqrt{1+\frac{b x^4}{a}}$$

$$\left.\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-192 i\sqrt{a}\, b^2\left(-7 i\sqrt{b}\, c+15\sqrt{a}\, e\right) x^9\sqrt{1+\frac{b x^4}{a}}\, \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

- **Problem 525: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}}{x^{11}}\, dx$$

Optimal (type 4, 399 leaves, 13 steps) :

$$-\frac{b\left(\frac{168 c}{x^6}+\frac{224 d}{x^5}+\frac{315 e}{x^4}+\frac{480 f}{x^3}\right)\sqrt{a+b x^4}}{1680}-\frac{b^2 c\sqrt{a+b x^4}}{10 a x^2}-\frac{4 b^2 d\sqrt{a+b x^4}}{15 a x}+\frac{4 b^{5/2} d x\sqrt{a+b x^4}}{15 a\left(\sqrt{a}+\sqrt{b} x^2\right)}-$$

$$\frac{\left(\frac{252 c}{x^{10}}+\frac{280 d}{x^9}+\frac{315 e}{x^8}+\frac{360 f}{x^7}\right)\left(a+b x^4\right)^{3/2}}{2520}-\frac{3 b^2 e \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16\sqrt{a}}-\frac{4 b^{9/4} d\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 a^{3/4}\sqrt{a+b x^4}}+$$

$$\frac{2 b^{7/4}\left(7\sqrt{b} d+15\sqrt{a} f\right)\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{105 a^{3/4}\sqrt{a+b x^4}}$$

Result (type 4, 314 leaves) :

$$\frac{1}{5040 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x^{10}\sqrt{a+b x^4}}$$

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+b x^4\right)\left(168 b^2 x^8(3 c+8 d x)+a^2(504 c+10 x(56 d+9 x(7 e+8 f x)))+a b x^4(1008 c+x(1232 d+45 x(35 e+48 f x)))\right)+\right.\right.$$

$$\left.945\sqrt{a} b^2 e x^{10}\sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+1344\sqrt{a} b^{5/2} d x^{10}\sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right],-1\right]-$$

$$\left.192 i\sqrt{a} b^2\left(-7 i\sqrt{b} d+15\sqrt{a} f\right) x^{10}\sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

- **Problem 526: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}}{x^{12}}\,dx$$

Optimal (type 4, 424 leaves, 14 steps) :

$$-\frac{b\left(\frac{1440 c}{x^7}+\frac{1848 d}{x^6}+\frac{2464 e}{x^5}+\frac{3465 f}{x^4}\right)\sqrt{a+b x^4}}{18480}-\frac{4 b^2 c\sqrt{a+b x^4}}{77 a x^3}-\frac{b^2 d\sqrt{a+b x^4}}{10 a x^2}-\frac{4 b^2 e\sqrt{a+b x^4}}{15 a x}+\frac{4 b^{5/2} e x\sqrt{a+b x^4}}{15 a\left(\sqrt{a}+\sqrt{b} x^2\right)}-$$

$$\frac{\left(\frac{360 c}{x^{11}}+\frac{396 d}{x^{10}}+\frac{440 e}{x^9}+\frac{495 f}{x^8}\right)\left(a+b x^4\right)^{3/2}}{3960}-\frac{3 b^2 f \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16\sqrt{a}}-\frac{4 b^{9/4} e\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 a^{3/4}\sqrt{a+b x^4}}-$$

$$\frac{2 b^{9/4}\left(15\sqrt{b} c-77\sqrt{a} e\right)\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{1155 a^{5/4}\sqrt{a+b x^4}}$$

Result (type 4, 317 leaves) :

$$\frac{1}{55440 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x^{11}\sqrt{a+b x^4}}\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.$$

$$\left(\left(a+b x^4\right)\left(24 b^2 x^8 (120 c+77 x (3 d+8 e x))+a b x^4\left(9360 c+77 x\left(144 d+176 e x+225 f x^2\right)\right)+14 a^2 (360 c+11 x (36 d+5 x (8 e+9 f x)))\right)+\right.$$

$$\left.10395\sqrt{a} b^2 f x^{11}\sqrt{a+b x^4}\operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+14784\sqrt{a} b^{5/2} e x^{11}\sqrt{1+\frac{b x^4}{a}}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right],-1\right]-$$

$$\left.192 b^{5/2}\left(-15 i\sqrt{b} c+77\sqrt{a} e\right) x^{11}\sqrt{1+\frac{b x^4}{a}}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 527: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}}{x^{13}}\,dx$$

Optimal (type 4, 449 leaves, 15 steps) :

$$-\frac{b\left(\frac{1155 c}{x^8}+\frac{1440 d}{x^7}+\frac{1848 e}{x^6}+\frac{2464 f}{x^5}\right)\sqrt{a+b x^4}}{18480}-\frac{b^2 c\sqrt{a+b x^4}}{32 a x^4}-\frac{4 b^2 d\sqrt{a+b x^4}}{77 a x^3}-\frac{b^2 e\sqrt{a+b x^4}}{10 a x^2}-\frac{4 b^2 f\sqrt{a+b x^4}}{15 a x}+\frac{4 b^{5/2} f x\sqrt{a+b x^4}}{15 a\left(\sqrt{a}+\sqrt{b} x^2\right)}-$$

$$\frac{\left(\frac{165 c}{x^{12}}+\frac{180 d}{x^{11}}+\frac{198 e}{x^{10}}+\frac{220 f}{x^9}\right)\left(a+b x^4\right)^{3/2}}{1980}+\frac{b^3 c\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{32 a^{3/2}}-\frac{4 b^{9/4} f\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{15 a^{3/4}\sqrt{a+b x^4}}-$$

$$\frac{2 b^{9/4}\left(15\sqrt{b} d-77\sqrt{a} f\right)\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{1155 a^{5/4}\sqrt{a+b x^4}}$$

Result (type 4, 328 leaves):

$$\frac{1}{110880 a^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^{12}\sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\left(a+b x^4\right)\left(56 a^2\left(165 c+2 x\left(90 d+99 e x+110 f x^2\right)\right)+3 b^2 x^8\left(1155 c+16 x\left(120 d+77 x\left(3 e+8 f x\right)\right)\right)+\right.\right.\right.$$

$$\left.\left.2 a b x^4\left(8085 c+16 x\left(585 d+77 x\left(9 e+11 f x\right)\right)\right)\right)+3465 b^3 c x^{12}\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+$$

$$29568 a b^{5/2} f x^{12}\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-384\sqrt{a}\,b^{5/2}\left(-15 i\sqrt{b} d+77\sqrt{a} f\right)x^{12}$$

$$\left.\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 528: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}}{x^{14}}\,dx$$

Optimal (type 4, 474 leaves, 16 steps):

$$-\frac{b\left(\frac{12320\,c}{x^9}+\frac{15015\,d}{x^8}+\frac{18720\,e}{x^7}+\frac{24024\,f}{x^6}\right)\sqrt{a+b\,x^4}}{240240}-\frac{4\,b^2\,c\,\sqrt{a+b\,x^4}}{195\,a\,x^5}-\frac{b^2\,d\,\sqrt{a+b\,x^4}}{32\,a\,x^4}-$$

$$\frac{4\,b^2\,e\,\sqrt{a+b\,x^4}}{77\,a\,x^3}-\frac{b^2\,f\,\sqrt{a+b\,x^4}}{10\,a\,x^2}+\frac{4\,b^3\,c\,\sqrt{a+b\,x^4}}{65\,a^2\,x}-\frac{4\,b^{7/2}\,c\,x\,\sqrt{a+b\,x^4}}{65\,a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\left(\frac{660\,c}{x^{13}}+\frac{715\,d}{x^{12}}+\frac{780\,e}{x^{11}}+\frac{858\,f}{x^{10}}\right)\left(a+b\,x^4\right)^{3/2}}{8580}+$$

$$\frac{b^3\,d\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{32\,a^{3/2}}+\frac{4\,b^{13/4}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{65\,a^{7/4}\sqrt{a+b\,x^4}}-$$

$$\frac{2\,b^{11/4}\left(77\sqrt{b}\,c+65\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{5005\,a^{7/4}\sqrt{a+b\,x^4}}$$

Result (type 4, 339 leaves) :

$$\frac{1}{480480\,a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x^{13}\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(a+b\,x^4\right)\left(-29568\,b^3\,c\,x^{12}+56\,a^3\left(660\,c+13\,x\left(55\,d+60\,e\,x+66\,f\,x^2\right)\right)+a\,b^2\,x^8\,(9856\,c+39\,x\,(385\,d+16\,x\,(40\,e+77\,f\,x)))+\right.\right.\right.$$

$$\left.\left.2\,a^2\,b\,x^4\,(30800\,c+13\,x\,(2695\,d+48\,x\,(65\,e+77\,f\,x)))\right)+15015\sqrt{a}\,b^3\,d\,x^{13}\sqrt{a+b\,x^4}\;\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)-$$

$$29568\sqrt{a}\,b^{7/2}\,c\,x^{13}\sqrt{1+\frac{b\,x^4}{a}}\;\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]+384\sqrt{a}\,b^3\left(77\sqrt{b}\,c+65\,i\sqrt{a}\,e\right)x^{13}$$

$$\left.\sqrt{1+\frac{b\,x^4}{a}}\;\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

Problem 529: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \left(c+d x+e x^2+f x^3\right)}{\sqrt{a+b x^4}} \, dx$$

Optimal (type 4, 361 leaves, 12 steps):

$$\frac{c x \sqrt{a+b x^4}}{3 b}+\frac{e x^3 \sqrt{a+b x^4}}{5 b}+\frac{f x^4 \sqrt{a+b x^4}}{6 b}-\frac{3 a e x \sqrt{a+b x^4}}{5 b^{3/2} \left(\sqrt{a}+\sqrt{b} x^2\right)}-\frac{\left(4 a f-3 b d x^2\right) \sqrt{a+b x^4}}{12 b^2}-$$
$$\frac{a d \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{4 b^{3/2}}+\frac{3 a^{5/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{5 b^{7/4} \sqrt{a+b x^4}}-$$
$$\frac{a^{3/4} \left(5 \sqrt{b} c+9 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{30 b^{7/4} \sqrt{a+b x^4}}$$

Result (type 4, 259 leaves):

$$\frac{1}{60 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^2 \sqrt{a+b x^4}}$$
$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(-\left(a+b x^4\right) (20 a f-b x (20 c+x (15 d+2 x (6 e+5 f x))))-15 a \sqrt{b} d \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)-36 a^{3/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}}\right.$$
$$\left.\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]+4 a \sqrt{b} \left(5 i \sqrt{b} c+9 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)$$

Problem 530: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(c+d x+e x^2+f x^3\right)}{\sqrt{a+b x^4}} \, dx$$

Optimal (type 4, 336 leaves, 11 steps):

$$\frac{d\,x\,\sqrt{a+b\,x^4}}{3\,b}+\frac{f\,x^3\,\sqrt{a+b\,x^4}}{5\,b}-\frac{3\,a\,f\,x\,\sqrt{a+b\,x^4}}{5\,b^{3/2}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\left(2\,c+e\,x^2\right)\,\sqrt{a+b\,x^4}}{4\,b}-$$

$$\frac{a\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,b^{3/2}}+\frac{3\,a^{5/4}\,f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{5\,b^{7/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{a^{3/4}\,\left(5\,\sqrt{b}\,d+9\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{30\,b^{7/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 241 leaves) :

$$\frac{1}{60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\sqrt{b}\,\left(a+b\,x^4\right)\,\left(30\,c+x\,\left(20\,d+3\,x\,\left(5\,e+4\,f\,x\right)\right)\right)-15\,a\,e\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)-\right.$$

$$\left.36\,a^{3/2}\,f\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+4\,a\,\left(5\,i\,\sqrt{b}\,d+9\,\sqrt{a}\,f\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 531: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^2\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 308 leaves, 10 steps) :

$$\frac{e\,x\,\sqrt{a+b\,x^4}}{3\,b}+\frac{c\,x\,\sqrt{a+b\,x^4}}{\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\left(2\,d+f\,x^2\right)\,\sqrt{a+b\,x^4}}{4\,b}-$$

$$\frac{a\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,b^{3/2}}-\frac{a^{1/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{b^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{a^{1/4}\,\left(3\,\sqrt{b}\,c-\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{6\,b^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 245 leaves) :

$$\frac{1}{12\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\sqrt{b}\,\left(6\,d+4\,e\,x+3\,f\,x^2\right)\,\left(a+b\,x^4\right)-3\,a\,f\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+\right.$$

$$12\,\sqrt{a}\,b\,c\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+$$

$$\left.4\,i\,\sqrt{a}\,\sqrt{b}\,\left(3\,i\,\sqrt{b}\,c+\sqrt{a}\,e\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

- **Problem 532: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 299 leaves, 10 steps) :

$$\frac{e\sqrt{a+b x^4}}{2 b}+\frac{f x\sqrt{a+b x^4}}{3 b}+\frac{d x\sqrt{a+b x^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b x^4}}\right]}{2\sqrt{b}}-\frac{a^{1/4} d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{a+b x^4}}+$$

$$\frac{a^{1/4}\left(3\sqrt{b}\,d-\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{6\,b^{5/4}\sqrt{a+b x^4}}$$

Result (type 4, 235 leaves):

$$\frac{1}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b\sqrt{a+b x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((3 e+2 f x)\left(a+b x^4\right)+3\sqrt{b}\,c\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b x^4}}\right]\right)+\right.$$

$$\left.6\sqrt{a}\sqrt{b}\,d\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+2\,i\sqrt{a}\left(3\,i\sqrt{b}\,d+\sqrt{a}\,f\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 533: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d x+e x^2+f x^3}{\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 276 leaves, 9 steps):

$$\frac{f\sqrt{a+b x^4}}{2 b}+\frac{e x\sqrt{a+b x^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b x^4}}\right]}{2\sqrt{b}}-\frac{a^{1/4} e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{a+b x^4}}+$$

$$\frac{a^{1/4}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,b^{3/4}\sqrt{a+b x^4}}$$

Result (type 4, 225 leaves):

$$\frac{1}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, b\sqrt{a+b x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(a f+b f x^4+\sqrt{b}\, d\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]\right)+\right.$$

$$\left.2\sqrt{a}\sqrt{b}\, e\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-2\sqrt{b}\left(i\sqrt{b}\, c+\sqrt{a}\, e\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

- **Problem 534: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d x+e x^2+f x^3}{x\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 285 leaves, 12 steps):

$$\frac{f x\sqrt{a+b x^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\, x^2\right)}+\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{2\sqrt{b}}-\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2\sqrt{a}}-\frac{a^{1/4} f\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{a+b x^4}}+$$

$$\frac{a^{1/4}\left(\frac{\sqrt{b}\, d}{\sqrt{a}}+f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 b^{3/4}\sqrt{a+b x^4}}$$

Result (type 4, 235 leaves):

$$-\frac{1}{2 b\sqrt{a+b x^4}}\, i\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b x^4}\left(\sqrt{a}\, e\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]-\sqrt{b}\, c\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$\left.2 a f\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-2\sqrt{a}\left(i\sqrt{b}\, d+\sqrt{a}\, f\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

- **Problem 535: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d x+e x^2+f x^3}{x^2\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 309 leaves, 13 steps):

$$-\frac{c\sqrt{a+b x^4}}{a x}+\frac{\sqrt{b}\, c\, x\sqrt{a+b x^4}}{a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}+\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{2\sqrt{b}}-$$

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2\sqrt{a}}-\frac{b^{1/4} c\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\sqrt{a+b x^4}}+$$

$$\frac{\left(\sqrt{b}\, c+\sqrt{a}\, e\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{3/4} b^{1/4}\sqrt{a+b x^4}}$$

Result (type 4, 250 leaves) :

$$\frac{1}{2}\left(-\frac{2 c\sqrt{a+b x^4}}{a x}+\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{\sqrt{b}}-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{\sqrt{a}}\right)-$$

$$\frac{i\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, c\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]}{\sqrt{a+b x^4}}-\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-i\sqrt{b}\, c+\sqrt{a}\, e\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]}{\sqrt{b}\sqrt{a+b x^4}}$$

- **Problem 536: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d x+e x^2+f x^3}{x^3\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 300 leaves, 11 steps) :

$$-\frac{c\sqrt{a+b x^4}}{2 a x^2}-\frac{d\sqrt{a+b x^4}}{a x}+\frac{\sqrt{b}\, d\, x\sqrt{a+b x^4}}{a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}-\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2\sqrt{a}}-\frac{b^{1/4} d\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\sqrt{a+b x^4}}+$$

$$\frac{\left(\sqrt{b}\, d+\sqrt{a}\, f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{3/4} b^{1/4}\sqrt{a+b x^4}}$$

Result (type 4, 242 leaves) :

$$\frac{1}{2\,a\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^2\,\sqrt{a+b\,x^4}}\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\left((c+2\,d\,x)\,\left(a+b\,x^4\right)+\sqrt{a}\,e\,x^2\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+2\,\sqrt{a}\,\sqrt{b}\,d\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-2\,i\,\sqrt{a}\,\left(-i\,\sqrt{b}\,d+\sqrt{a}\,f\right)\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

■ Problem 537: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x^4\,\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 323 leaves, 12 steps) :

$$-\frac{c\,\sqrt{a+b\,x^4}}{3\,a\,x^3}-\frac{d\,\sqrt{a+b\,x^4}}{2\,a\,x^2}-\frac{e\,\sqrt{a+b\,x^4}}{a\,x}+\frac{\sqrt{b}\,e\,x\,\sqrt{a+b\,x^4}}{a\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,\sqrt{a}}-\frac{b^{1/4}\,e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\,\sqrt{a+b\,x^4}}-\frac{b^{1/4}\,\left(\sqrt{b}\,c-3\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{6\,a^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 249 leaves) :

$$\frac{1}{6\,a\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^3\,\sqrt{a+b\,x^4}}\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\,(2\,c+3\,x\,(d+2\,e\,x))+3\,\sqrt{a}\,f\,x^3\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$6\,\sqrt{a}\,\sqrt{b}\,e\,x^3\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.2\,\sqrt{b}\,\left(-i\,\sqrt{b}\,c+3\,\sqrt{a}\,e\right)\,x^3\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 538: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x^5\,\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 346 leaves, 13 steps):

$$-\frac{c\,\sqrt{a+b\,x^4}}{4\,a\,x^4}-\frac{d\,\sqrt{a+b\,x^4}}{3\,a\,x^3}-\frac{e\,\sqrt{a+b\,x^4}}{2\,a\,x^2}-\frac{f\,\sqrt{a+b\,x^4}}{a\,x}+\frac{\sqrt{b}\,f\,x\,\sqrt{a+b\,x^4}}{a\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{b\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{4\,a^{3/2}}-\frac{b^{1/4}\,f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{b^{1/4}\,\left(\sqrt{b}\,d-3\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{6\,a^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 259 leaves):

$$\left(\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\,(a+b x^4)\,(3 c+4 d x+6 x^2 (e+2 f x))+3 b c x^4\sqrt{a+b x^4}\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$12 a\sqrt{b}\, f x^4\sqrt{1+\frac{b x^4}{a}}\,\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.4\sqrt{a}\,\sqrt{b}\,\left(-\mathrm{i}\sqrt{b}\,d+3\sqrt{a}\,f\right) x^4\sqrt{1+\frac{b x^4}{a}}\,\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Bigg/\left(12 a^{3/2}\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x^4\sqrt{a+b x^4}\right)$$

Problem 539: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d x+e x^2+f x^3}{x^6\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 377 leaves, 14 steps):

$$-\frac{c\sqrt{a+b x^4}}{5 a x^5}-\frac{d\sqrt{a+b x^4}}{4 a x^4}-\frac{e\sqrt{a+b x^4}}{3 a x^3}-\frac{f\sqrt{a+b x^4}}{2 a x^2}+\frac{3 b c\sqrt{a+b x^4}}{5 a^2 x}-\frac{3 b^{3/2} c x\sqrt{a+b x^4}}{5 a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{b d\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{4 a^{3/2}}+\frac{3 b^{5/4} c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{5 a^{7/4}\sqrt{a+b x^4}}-$$

$$\frac{b^{3/4}\left(9\sqrt{b}\,c+5\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{30 a^{7/4}\sqrt{a+b x^4}}$$

Result (type 4, 268 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(a+b x^4\right)\left(12 a c-36 b c x^4+5 a x\left(3 d+4 e x+6 f x^2\right)\right)+15 \sqrt{a}\, b\, d\, x^5 \sqrt{a+b x^4}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)-\right.$$

$$36 \sqrt{a}\, b^{3/2} c\, x^5 \sqrt{1+\frac{b x^4}{a}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$\left.4 \sqrt{a}\, b\left(9 \sqrt{b}\, c+5 i \sqrt{a}\, e\right) x^5 \sqrt{1+\frac{b x^4}{a}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)\Big/\left(60 a^2 \sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x^5 \sqrt{a+b x^4}\right)$$

- **Problem 540: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^6\left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 365 leaves, 12 steps):

$$\frac{x\left(a e+a f x-b c x^2-b d x^3\right)}{2 b^2 \sqrt{a+b x^4}}+\frac{d \sqrt{a+b x^4}}{b^2}+\frac{e x \sqrt{a+b x^4}}{3 b^2}+\frac{f x^2 \sqrt{a+b x^4}}{4 b^2}+\frac{3 c x \sqrt{a+b x^4}}{2 b^{3/2}\left(\sqrt{a}+\sqrt{b}\, x^2\right)}-$$

$$\frac{3 a f\, \text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{4 b^{5/2}}-\frac{3 a^{1/4} c\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 b^{7/4} \sqrt{a+b x^4}}+$$

$$\frac{a^{1/4}\left(9 \sqrt{b}\, c-5 \sqrt{a}\, e\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{12 b^{9/4} \sqrt{a+b x^4}}$$

Result (type 4, 267 leaves):

$$\frac{1}{12\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, b^{5/2}\sqrt{a+b x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(a\left(12 d+10 e x+9 f x^2\right)+b x^3\left(-6 c+6 d x+4 e x^2+3 f x^3\right)\right)-9 a f\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]\right)+18\sqrt{a}\, b\, c\sqrt{1+\frac{b x^4}{a}}\right.$$
$$\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]+2 i\sqrt{a}\sqrt{b}\left(9 i\sqrt{b}\, c+5\sqrt{a}\, e\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

Problem 541: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^5\left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\frac{x\left(a f-b c x-b d x^2-b e x^3\right)}{2 b^2\sqrt{a+b x^4}}+\frac{e\sqrt{a+b x^4}}{b^2}+\frac{f x\sqrt{a+b x^4}}{3 b^2}+\frac{3 d x\sqrt{a+b x^4}}{2 b^{3/2}\left(\sqrt{a}+\sqrt{b}\, x^2\right)}+$$
$$\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{2 b^{3/2}}-\frac{3 a^{1/4} d\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 b^{7/4}\sqrt{a+b x^4}}+$$
$$\frac{a^{1/4}\left(9\sqrt{b}\, d-5\sqrt{a}\, f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{12 b^{9/4}\sqrt{a+b x^4}}$$

Result (type 4, 255 leaves):

$$\frac{1}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^2\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(a\,(6\,e+5\,f\,x)+b\,x^2\,\left(-3\,c-3\,d\,x+3\,e\,x^2+2\,f\,x^3\right)+3\,\sqrt{b}\,c\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+\right.$$

$$\left.9\,\sqrt{a}\,\sqrt{b}\,d\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+i\,\sqrt{a}\,\left(9\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 542: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{x^4\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 314 leaves, 10 steps):

$$-\frac{x\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{2\,b\,\sqrt{a+b\,x^4}}+\frac{f\,\sqrt{a+b\,x^4}}{b^2}+\frac{3\,e\,x\,\sqrt{a+b\,x^4}}{2\,b^{3/2}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\,b^{3/2}}-\frac{3\,a^{1/4}\,e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,b^{7/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{\left(\sqrt{b}\,c+3\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{1/4}\,b^{7/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 243 leaves):

$$\frac{1}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^2\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(2\,a\,f+b\,x\,\left(-c-d\,x-e\,x^2+f\,x^3\right)+\sqrt{b}\,d\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+\right.$$

$$\left.3\,\sqrt{a}\,\sqrt{b}\,e\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\sqrt{b}\,\left(i\,\sqrt{b}\,c+3\,\sqrt{a}\,e\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 543: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 297 leaves, 9 steps):

$$-\frac{c+d x+e x^2+f x^3}{2 b \sqrt{a+b x^4}}+\frac{3 f x \sqrt{a+b x^4}}{2 b^{3/2} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{e \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{2 b^{3/2}}-\frac{3 a^{1/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 b^{7/4} \sqrt{a+b x^4}}+$$
$$\frac{\left(\sqrt{b} d+3 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{4 a^{1/4} b^{7/4} \sqrt{a+b x^4}}$$

Result (type 4, 224 leaves):

$$\frac{1}{2 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^{3/2} \sqrt{a+b x^4}}\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(-\sqrt{b} \left(c+x \left(d+x \left(e+f x\right)\right)\right)+e \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+\right.$$
$$\left.3 \sqrt{a} f \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\left(i \sqrt{b} d+3 \sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 544: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 333 leaves, 10 steps):

$$-\frac{x \left(a e+a f x-b c x^2-b d x^3\right)}{2 a b \sqrt{a+b x^4}}-\frac{d \sqrt{a+b x^4}}{2 a b}-\frac{c x \sqrt{a+b x^4}}{2 a \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{f \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{2 b^{3/2}}+$$
$$\frac{c \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{3/4} b^{3/4} \sqrt{a+b x^4}}-\frac{\left(\sqrt{b} c-\sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{4 a^{3/4} b^{5/4} \sqrt{a+b x^4}}$$

Result (type 4, 242 leaves) :

$$\frac{1}{2\,a\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\,\left(b\,c\,x^3-a\,(d+x\,(e+f\,x))\right)+a\,f\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)-\sqrt{a}\,b\,c\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\sqrt{a}\,\sqrt{b}\,\left(\sqrt{b}\,c-i\,\sqrt{a}\,e\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 545: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 303 leaves, 7 steps) :

$$-\frac{x\,\left(a\,f-b\,c\,x-b\,d\,x^2-b\,e\,x^3\right)}{2\,a\,b\,\sqrt{a+b\,x^4}}-\frac{e\,\sqrt{a+b\,x^4}}{2\,a\,b}-\frac{d\,x\,\sqrt{a+b\,x^4}}{2\,a\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}-\frac{\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{3/4}\,b^{5/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 197 leaves) :

$$\frac{1}{2\,a\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(b\,x^2\,(c+d\,x)-a\,(e+f\,x)\right)-\sqrt{a}\,\sqrt{b}\,d\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\sqrt{a}\,\left(\sqrt{b}\,d-i\,\sqrt{a}\,f\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 546: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 275 leaves, 4 steps) :

$$-\frac{e\,x\,\sqrt{a+b\,x^4}}{2\,a\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{a\,f-b\,x\,\left(c+d\,x+e\,x^2\right)}{2\,a\,b\,\sqrt{a+b\,x^4}}+\frac{e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{5/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 195 leaves) :

$$\frac{1}{2\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(-a\,f+b\,x\,\left(c+x\,\left(d+e\,x\right)\right)\right)-\sqrt{a}\,\sqrt{b}\,e\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\sqrt{b}\,\left(-i\,\sqrt{b}\,c+\sqrt{a}\,e\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 547: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x\,\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 323 leaves, 11 steps) :

$$\frac{x\,\left(a\,d+a\,e\,x+a\,f\,x^2-b\,c\,x^3\right)}{2\,a^2\,\sqrt{a+b\,x^4}}+\frac{c\,\sqrt{a+b\,x^4}}{2\,a^2}-\frac{f\,x\,\sqrt{a+b\,x^4}}{2\,a\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,a^{3/2}}+\frac{f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}+\frac{\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{5/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 225 leaves) :

$$\frac{1}{2\,a^{3/2}\,b\,\sqrt{a+b\,x^4}}\left(\sqrt{a}\,b\,(c+x\,(d+x\,(e+f\,x)))-b\,c\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]+\right.$$

$$\left.\mathrm{i}\,a^{3/2}\,\sqrt{\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{a}}}\,f\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\frac{b\left(\sqrt{b}\,d+\mathrm{i}\,\sqrt{a}\,f\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\left(\frac{\mathrm{i}\,\sqrt{b}}{\sqrt{a}}\right)^{3/2}}\right)$$

- **Problem 548: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x^2\,\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 344 leaves, 13 steps):

$$\frac{x\,\left(a\,e+a\,f\,x-b\,c\,x^2-b\,d\,x^3\right)}{2\,a^2\,\sqrt{a+b\,x^4}}+\frac{d\,\sqrt{a+b\,x^4}}{2\,a^2}-\frac{c\,\sqrt{a+b\,x^4}}{a^2\,x}+\frac{3\,\sqrt{b}\,c\,x\,\sqrt{a+b\,x^4}}{2\,a^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-$$

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,a^{3/2}}-\frac{3\,b^{1/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{7/4}\,\sqrt{a+b\,x^4}}+$$

$$\frac{\left(3\,\sqrt{b}\,c+\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{7/4}\,b^{1/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 245 leaves):

$$\frac{1}{2\,a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-2\,a\,c-3\,b\,c\,x^4+a\,x\,(d+x\,(e+f\,x))-\sqrt{a}\;d\,x\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$3\sqrt{a}\;\sqrt{b}\;c\,x\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]-$$

$$\left.i\sqrt{a}\;\left(-3\,i\sqrt{b}\;c+\sqrt{a}\;e\right)x\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

- **Problem 549: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x^3\,\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 367 leaves, 15 steps):

$$\frac{x\,\left(a\,f-b\,c\,x-b\,d\,x^2-b\,e\,x^3\right)}{2\,a^2\sqrt{a+b\,x^4}}+\frac{e\sqrt{a+b\,x^4}}{2\,a^2}-\frac{c\sqrt{a+b\,x^4}}{2\,a^2\,x^2}-\frac{d\sqrt{a+b\,x^4}}{a^2\,x}+\frac{3\sqrt{b}\;d\,x\,\sqrt{a+b\,x^4}}{2\,a^2\left(\sqrt{a}+\sqrt{b}\;x^2\right)}-$$

$$\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,a^{3/2}}-\frac{3\,b^{1/4}\,d\,\left(\sqrt{a}+\sqrt{b}\;x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{7/4}\sqrt{a+b\,x^4}}+$$

$$\frac{\left(3\sqrt{b}\;d+\sqrt{a}\;f\right)\left(\sqrt{a}+\sqrt{b}\;x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{4\,a^{7/4}\,b^{1/4}\sqrt{a+b\,x^4}}$$

Result (type 4, 259 leaves):

$$\frac{1}{2\,a^2\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^2\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(b\,x^4\,(2\,c+3\,d\,x)+a\,\left(c+2\,d\,x-x^2\,(e+f\,x)\right)+\sqrt{a}\,e\,x^2\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+3\,\sqrt{a}\,\sqrt{b}\,d\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}\right.$$

$$\left.\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-i\,\sqrt{a}\,\left(-3\,i\,\sqrt{b}\,d+\sqrt{a}\,f\right)\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

- **Problem 550: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x^4\,\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 387 leaves, 17 steps):

$$-\frac{x\,\left(b\,c+b\,d\,x+b\,e\,x^2+b\,f\,x^3\right)}{2\,a^2\,\sqrt{a+b\,x^4}}+\frac{f\,\sqrt{a+b\,x^4}}{2\,a^2}-\frac{c\,\sqrt{a+b\,x^4}}{3\,a^2\,x^3}-\frac{d\,\sqrt{a+b\,x^4}}{2\,a^2\,x^2}-\frac{e\,\sqrt{a+b\,x^4}}{a^2\,x}+$$

$$\frac{3\,\sqrt{b}\,e\,x\,\sqrt{a+b\,x^4}}{2\,a^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,a^{3/2}}-\frac{3\,b^{1/4}\,e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{7/4}\,\sqrt{a+b\,x^4}}-$$

$$\frac{b^{1/4}\,\left(5\,\sqrt{b}\,c-9\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{12\,a^{9/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 267 leaves):

$$\frac{1}{6\,a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x^3\sqrt{a+b\,x^4}}\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(2\,a\,c+b\,x^4\left(5\,c+6\,d\,x+9\,e\,x^2\right)+3\,a\,x\,\left(d+x\,\left(2\,e-f\,x\right)\right)+3\,\sqrt{a}\;f\,x^3\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+9\,\sqrt{a}\;\sqrt{b}\;e\,x^3\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]-\sqrt{b}\;\left(-5\,i\,\sqrt{b}\;c+9\,\sqrt{a}\;e\right)\,x^3\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x\right],-1\right]\right)$$

- **Problem 557: Result more than twice size of optimal antiderivative.**

$$\int\frac{81+36\,x^2+16\,x^4}{729-64\,x^6}\,dx$$

Optimal (type 3, 10 leaves, 2 steps) :

$$\frac{1}{6}\,\text{ArcTanh}\left[\frac{2\,x}{3}\right]$$

Result (type 3, 21 leaves) :

$$-\frac{1}{12}\,\text{Log}\,[3-2\,x]+\frac{1}{12}\,\text{Log}\,[3+2\,x]$$

- **Problem 567: Result unnecessarily involves imaginary or complex numbers.**

$$\int\frac{81+36\,x^2+16\,x^4}{\left(729-64\,x^6\right)^2}\,dx$$

Optimal (type 3, 81 leaves, 8 steps) :

$$\frac{1}{17\,496\,\left(3-2\,x\right)}-\frac{1}{17\,496\,\left(3+2\,x\right)}-\frac{\text{ArcTan}\left[\frac{3-4\,x}{3\,\sqrt{3}}\right]}{13\,122\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{3+4\,x}{3\,\sqrt{3}}\right]}{13\,122\,\sqrt{3}}+\frac{\text{ArcTanh}\left[\frac{2\,x}{3}\right]}{8748}$$

Result (type 3, 122 leaves) :

$$\frac{1}{157464}\left(\frac{36\,x}{9-4\,x^2}+3\,\sqrt{3}\,\text{ArcTan}\left[\frac{1}{3}\left(-i+\sqrt{3}\right)x\right]+4\,i\,\sqrt{3}\,\text{ArcTanh}\left[\frac{1}{3}\left(1-i\,\sqrt{3}\right)x\right]+\left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\,\sqrt{3}\right)}}\right)\text{ArcTanh}\left[\frac{1}{3}\left(x+i\,\sqrt{3}\,x\right)\right]-9\,\text{Log}[3-2\,x]+9\,\text{Log}[3+2\,x]\right)$$

- **Problem 583: Result more than twice size of optimal antiderivative.**

$$\int\frac{c+d\,x^{-1+n}}{\left(a+b\,x^n\right)^3}\,dx$$

Optimal (type 5, 46 leaves, 3 steps):

$$-\frac{d}{2\,b\,n\,\left(a+b\,x^n\right)^2}+\frac{c\,x\,\text{Hypergeometric2F1}\left[3,\frac{1}{n},1+\frac{1}{n},-\frac{b\,x^n}{a}\right]}{a^3}$$

Result (type 5, 108 leaves):

$$\frac{x\,\left(c+d\,x^{-1+n}\right)\left(\frac{a^2\,n\,(-a\,d+b\,c\,x)}{b\,(a+b\,x^n)^2}+\frac{a\,c\,(-1+2\,n)\,x}{a+b\,x^n}+c\,\left(1-3\,n+2\,n^2\right)x\,\text{Hypergeometric2F1}\left[1,\frac{1}{n},1+\frac{1}{n},-\frac{b\,x^n}{a}\right]\right)}{2\,a^3\,n^2\,(c\,x+d\,x^n)}$$

- **Problem 590: Result unnecessarily involves higher level functions.**

$$\int\frac{1+x^3}{\left(1-x^4\right)\left(1+x^4\right)^{1/4}}\,dx$$

Optimal (type 3, 103 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[\frac{2^{1/4}\,x}{\left(1+x^4\right)^{1/4}}\right]}{2\times2^{1/4}}-\frac{\text{ArcTan}\left[\frac{\left(1+x^4\right)^{1/4}}{2^{1/4}}\right]}{2\times2^{1/4}}+\frac{\text{ArcTanh}\left[\frac{2^{1/4}\,x}{\left(1+x^4\right)^{1/4}}\right]}{2\times2^{1/4}}+\frac{\text{ArcTanh}\left[\frac{\left(1+x^4\right)^{1/4}}{2^{1/4}}\right]}{2\times2^{1/4}}$$

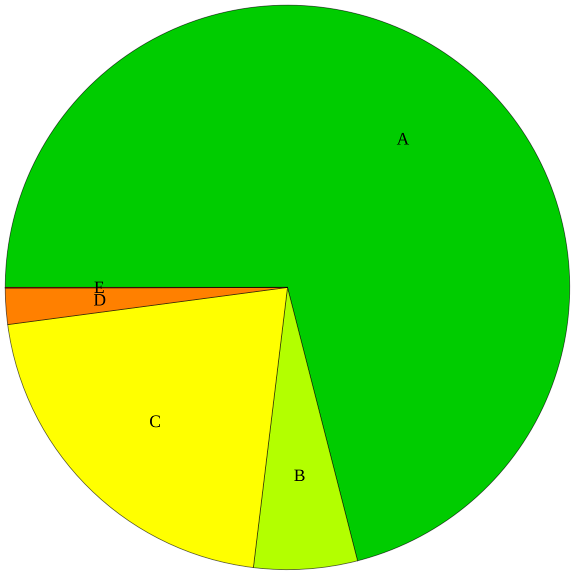
Result (type 6, 166 leaves):

$$-\frac{2\,x^4\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^4,x^4\right]}{\left(-1+x^4\right)\left(1+x^4\right)^{1/4}\left(8\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^4,x^4\right]+x^4\left(4\,\text{AppellF1}\left[2,\frac{1}{4},2,3,-x^4,x^4\right]-\text{AppellF1}\left[2,\frac{5}{4},1,3,-x^4,x^4\right]\right)\right)}+$$

$$\frac{2\,\text{ArcTan}\left[\frac{2^{1/4}\,x}{\left(1+x^4\right)^{1/4}}\right]-\text{Log}\left[1-\frac{2^{1/4}\,x}{\left(1+x^4\right)^{1/4}}\right]+\text{Log}\left[1+\frac{2^{1/4}\,x}{\left(1+x^4\right)^{1/4}}\right]}{4\times2^{1/4}}$$

Summary of Integration Test Results

5184 integration problems



A - 3681 optimal antiderivatives

B - 307 more than twice size of optimal antiderivatives

C - 1087 unnecessarily complex antiderivatives

D - 107 unable to integrate problems

E - 2 integration timeouts